# Preliminary Design for Pilot: Bridging Conceptual Gaps in Orthogonal Projection in Applied Linear Algebra Through Principal Component Analysis (PCA) Visualization

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Students often encounter orthogonal projection as a formula-driven procedure, leading to disconnects between geometric intuition, vector decomposition, and symbolic manipulation. (Córdova-Esparza & Piñon, 2025). Leveraging PCA as a visual means, this study aims to bridge students coordinate image, vector, and symbol for deeper conceptual understanding. Research Questions: RQ1. How do PCA-based visuals support undergraduates in developing a robust understanding of orthogonal projection? RQ2. Which instructional features foster stronger conceptual explanations and reduce misconception?

PCA technique offers a visual framework links geometric intuition with algebraic operations, making orthogonal projection more accessible (Konyalioglu et al., 2011; Pandit et al., 2020). In undergraduate linear algebra, PCA connects dimensionality reduction, visualization, and real-world applications (Gewers et al., 2018; Greenacre et al., 2022), Its foundation in orthogonal projection enables students to visualize high-dimensional data on lower-dimensional subspaces (Espadoto et al., 2021; Görtler et al., 2019). PCA visualization and interactive activities connect theory and practice, supporting conceptual reasoning (Glen & Zazkis, 2020; Trigueros, 2018). Conventional learning creates gaps between symbolic manipulation and conceptual understanding (Kontorovich, 2020; Zhang, 2024). Few studies address how PCA-based activities coordinate geometric, vector, symbolic reasoning, an explicit gap this project addresses.

This study draws on Multiple External Representations (MERs) theory, which strengthens conceptual understanding by coordinating visual, geometric, and symbolic forms (Hahn, L., & Klein, P.2023). PCA-based visualization, vector decomposition, and symbolic projection activities foster representational fluency (Fonger et al., 2018). Guided by constructivism theory (Greenfield & Bruner, 1966), the design uses hands-on tasks to bridge image ↔vector ↔symbol. A conjecture map as a design-based research approach (Oktaç, 2019), systematically links instructional features to student reasoning and evidence of conceptual change. These frameworks guide activities to bridge conceptual gaps in orthogonal projection.

This activity applies PCA as a data-driven context to make orthogonal projection visible and discussable. Students work with three figures: a 3D projection to the PC1-PC2 plane, a 2D view with perpendicular drops, and a PC1-PC2 scatter plot to anticipate PC1 direction, mark orthogonal drops, decompose vectors ($v = v_{\parallel} + v_{\perp}$), and compute projections using the dot product. The sequence threads one idea across image ↔vector ↔symbol, with pre/post concept checks and figure annotations scored by a 0-3 rubric. Targeted prompts address recurring confusions (e.g. “along” vs. “onto,” PCA vs. regression, and centering.)

At the preliminary design and pre-pilot stage, this approach is expected to bridge gaps and strengthen students’ conceptual understanding of orthogonal projection by coordinating geometric, vector, and symbolic reasoning. It aims to reduce misconception, common errors, deepen explanations, and provide instructors with a portable toolkit-conjecture maps, rubrics, and repair prompts to support representational coordination. Pilot results will inform further refinement and broader instructional adoption.

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