Readers' Movement of Attention Through a Calculus Text

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Mathematics textbooks demand specialized forms of literacy, yet little research has examined how readers actually move through them. Drawing on the concepts of agency and didactical disciplinary literacy, this paper explores how readers allocate attention across textbook components and the reasons they give for doing so. We also propose an "attention continuum" in which shifting patterns of engagement represent agentive choices. These findings highlight how textbook reading intertwines literacy practices, disciplinary positioning, and agency, offering new insight into mathematics learning.

Keywords: Mathematics Textbooks, Agency, Didactical Disciplinary Literacy

Textbooks have long been a staple of college-level mathematics classes. Mathematics textbooks—dense combinations of technical prose, symbolic notation, graphs and figures, and examples—require students to engage with them in particular ways. Textbook authors offer instructions to students about how to read their textbooks, with exhortations to read closely and to attend to the various components of the textbook in particular ways and in a certain order (e.g., Stewart, 2016). Similarly, university learning centers advise students on reading strategies such as skimming and close reading for academic texts in general and scientific textbooks in particular (e.g., University of North Carolina, 2025). Yet, the mathematics education research community has paid relatively little attention to how students read mathematics textbooks.

Following Wiesner et al. (2020), we argue that literacy is central to disciplinary practice, and that textbooks are different from other disciplinary texts and deserve particular attention. Learning from reading a textbook depends both on the reader's background knowledge and the literacy practices they employ, including how they allocate their attention across the various components of the text. Moreover, these reading activities are situated within the disciplinary community and in relation to the reader's positioning within it.

To investigate these ideas, we present results from a study of undergraduate mathematics students and non-mathematics professors as they read an excerpt from a calculus textbook. Using a stimulated recall approach, assisted by eye-tracking software, we investigate how and why readers move their attention through the text. Specifically, we investigate the questions

- How do readers of a calculus textbook excerpt move their attention through the reading?
- What are the stated reasons and motivations for the choices that readers make about their movement through the text?

Literature Review

The concept of *disciplinary literacy* recognizes that disciplinary experts engage with texts in their field in ways that draw not only on their content knowledge, but also on their knowledge of the disciplinary community norms and concepts of knowledge and inquiry, and the reader's positioning within the community (Moje, 2007; Shanahan et al., 2011). *Didactical disciplinary literacy* extends this idea to didactical texts (texts whose principal purpose is didactical, such as textbooks); it encompasses "the habits of mind of the individual immersed in didactical practice and on the literacies of the discipline in which the text and reader are embedded" (Wiesner et al., 2020).

Agency emerges from an "individual's perception and enaction of control and choice" (Weinberg et al., 2024). The concept is a fundamental piece of disciplinary literacy (Moje, 2008) and has been highlighted as essential in both math education and literacy communities (Fulmer et al., 2022; Vaughn, 2014). We adopt the perspective put forward in Fulmer et al. (2022), who reconcile these perspectives by viewing agency as the "interplay of autonomy, identity, power, and knowledge" (p. 632). Agency in mathematics is naturally in tension with the structure and permanency of the discipline (e.g. Pickering, 1995); textbooks—"inherently hegemonic" (Fulmer et al., 2022, p. 633)—offer a particular challenge to the personal agency of the reader (Vaughn, 2014). In this context, we argue that the way a reader directs their attention through the reading offers an important opportunity to act agentively. In particular, as Weinberg et al. (2024), note, "actions that run counter to the status quo are evidence of agentive moments" (p. 669).

Theoretical Framework

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Methods

Participants

We selected participants to capture a wide range of disciplinary and didactical disciplinary literacy practices. To recruit participants, we sent invitations to all math majors and minors who had not yet completed numerical analysis (where Newton's method is a standard topic); all students who had completed first-semester calculus with a grade of C- or better in the previous semester; and all (non-math) STEM faculty at our institution. All interested individuals who were available during interview times were included in the study. A total of 15 individuals particicpated in the study, five from each of the invited groups. We subsequently refer to participants by their group (MM for math major, NMM for non-math major, and NMF for non-math faculty) and a number.

Materials

Participants in the study read a four-page excerpt from a commonly used calculus textbook (Stewart, 2016) on Newton's method. The excerpt included introductory prose that attempted to motivate the need for Newton's method; a derivation of a general formula and description of the iterative process and associated limit; and three examples intermixed with text that elaborated on the examples. There were also seven figures that illustrated the ideas presented in the text. This organization of the excerpt reflects a common template in mathematics didactical texts; it informed the interviewer's questioning during the stimulated recall phase and served as a map for tracing readers' navigation through the text during data analysis.

The excerpt was chosen because it relied on knowledge that is generally taught in a first-semester calculus course, but Newton's method itself is often not taught in introductory semester calculus. Thus, we predicted that the topic itself would be novel to most participants but that they would possess the necessary background knowledge to engage mathematically with the reading.

Data Collection

Participants were each interviewed for a 75-minute session, conducted by the third author, a professor in the mathematics department. Following Schindler and Lilienthal (2019), the interview incorporated a stimulated recall of participants' reading. Participants were first asked to read the textbook excerpt for the purpose of learning Newton's method, reading an electronic version of the excerpt with an eye-tracker recording their gaze. After reading, the participant and interviewer jointly viewed a playback of their fixations. The participant was asked to narrate their fixation patterns and describe what they were thinking as they looked. In particular, if the participant's gaze was fixed on a particular component of the text for an extended period of time, if the participant skipped or skimmed over a component, or if the participant moved back to an earlier portion of the excerpt from a later portion, the interviewer prompted them to pause and describe their thinking and motivation.

Analytical Methods

We used a process of open and axial coding (Strauss & Corbin, 1994) to generate codes. Each author reviewed the interview transcripts and identified turns where the participant addressed how their attention moved throughout the text. Using both the turn itself and adjacent turns, the authors individually characterized each turn into an initial code, then worked collectively to refine the codes. This resulted in a collection of behavior codes and reason codes for each behavior code. The authors then individually coded the flagged turns (for both behavior and reason) and collaboratively reconciled differences.

Results

We identified five categories of behaviors–reading straight through, skimming, skipping, close reading, and reading order– and a classification of reasons the participants provided for each type of behavior. Between six and 17 turns were identified for each participant. Additionally, each participant was coded for at least two of the main behaviors we analyzed–skimming, increased attention, and reading order–and eight of the 15 participants were coded for all three.

Reading Straight Through

A turn was coded as *reading straight through* when the reader indicated that they had read the text in the order presented with no indication of more or less attention than expected. For

example, NMM1 stated, “I was locking in, reading straight through.” These moments were not elicited or probed by the interview protocol and were infrequent (five out of 146 turns were assigned this code.) In each case, the reader did not provide any justification for their behavior of reading straight through.

Skimming and Skipping

Turns were coded as *skimming* if the turn indicated rapid reading or reduced attention. For example NMF5 stated, “I just glossed over those formulas.” We classified reasons offered by participants for engaging in skimming as follows: efficiency, familiar, don’t understand, disciplinary reasons, overview, and no reason.

A skimming turn was coded as *efficiency* when the participant explicitly referred to moving through the text at increased speed. In all but one instance, turns that were coded as efficiency also received a secondary reason code. For example, NMM2 stated, “I think I probably read through this one quicker because I kind of understand more how the numbers plugged in.” This was coded as efficiency and familiar; in general, we identified skimming as *familiar* when the reader expressed that they already understood the material sufficiently. Other readers skimmed a portion of the text when they struggled to make sense of it, coded as *don’t understand*. For example, NMM3 explained their choice to move quickly through a segment of text: “I couldn’t figure it out.”

We used the code *disciplinary reasons* to identify turns where the reader justified skimming in terms of what they valued mathematically or based on disciplinary norms. For example, MM5 skimmed the first part of the reading–motivating Newton’s method–so that she could move on to the derivation of the method, explaining that she found the motivation section less important than the derivation section.

Skimming was classified as *overview* when participants indicated they were trying to get an overview or preview of the reading. For example, NMF5 stated, “Well, I was just trying to figure out, like, normally you don't know—in a textbook—you don't know what the best place to start is. So you're looking for a heading, or like, now we're going to learn about this and I guess that's at the top, I'm just seeing right now, so Newton's method.”

Skimming (as well as other behaviors) was coded as *no reason* when a participant did not offer a reason themselves and the interviewer did not prompt them for a reason, or authors could not identify a reason from the participant’s explanation of their behavior.

We coded separately as *skipping* instances where a participant gave an explicit indication that they did not look at a particular portion of text at all. For example, after being asked if they had looked at a figure, NMM2 stated that they had not noticed the figure. In each instance, the participant either did not supply a reason or indicated that they missed the piece or text or were not sure why they had skipped it; these were all coded as *no reason*.

Close Reading

The code *close reading* was applied to turns where the reader’s movement through the text appeared to slow or stop on a particular component of the text. We classified close reading using the reasoning codes understanding, familiar, disciplinary reasons, and no reason (defined as above).

The code *understanding* was applied when participants indicated they were working to make sense of part of the text that they found confusing or that they deemed needed additional time to

understand. For example, NMF1 said: “When you had that notation that's confusing, you look at it more and then—I don't want to go too fast.”

Similar to the skimming behavior, a close reading turn was coded as *familiar* when the participant indicated they were engaging in the behavior because a piece of text was familiar to them or a type of text that they felt comfortable with. This gave them greater access to or interest in the text, motivating their increased attention. For example, MM2 focused on Example 2, explaining the set-up of the problem (finding the sixth root of 2 using Newton’s method) reminded them of work he had done in a previous class.

Complementing the skimming-disciplinary reasons code, the close reading-*disciplinary reasons* code referred to turns where the reader indicated that their increased attention was motivated by what they valued mathematically or based on disciplinary norms. This happened in MM1’s report of close reading of the blue-boxed equation, stating that it was significant in understanding how and why Newton’s Method works.

Reading Order

The code *reading order* was used to categorize turns where the reader did something other than read in the intended order (as perceived by the paper authors). This included reading a piece of the text multiple times, jumping forward in the text (over a portion of the reading), jumping backward in the text, or moving back and forth between two different pieces of the text. We grouped these behaviors under one code as they often involved overlapping behaviors that were difficult to tease apart. For example, when a participant jumped backward in the text, they were likely rereading the earlier section but they may not have explicitly indicated this. Justifications for this behavior were classified as understanding, connections, disciplinary reasons, overview, zoning out, and no reason.

The code *understanding* was applied when participants indicated they were working to make sense of part of the text that they found unclear, to check their understanding, or to build understanding iteratively. For example, MM2 explained their choice to move back to an example earlier in the reading: "I wanted to make sure that like the x_1, that it was more of a choice of what to choose for x_1. You can choose one or something else, yeah I just wanted to make sure."

We coded reading order as *connections* when the reader was attempting to make connections between different pieces of text (such as graphs and text) without indicating they were attempting to resolve a confusion. For example, after starting to read Example 1, MM1 returned to the general equation earlier on the page to coordinate the calculation in the example with the general equation.

Participants also indicated that reading order was motivated by disciplinary norms and values, coded as *disciplinary reasons*. For example, NMF1 explicitly referenced her discipline when explaining her choice to look at the figure captions before looking at any of the text: “It’s how you read a [biology research] paper…. And when I read biology papers—and part of it is the method I was taught, and I like it—is you read the abstract and the figure legends, and then you go back and read the paper.”

Readers also engaged in reading order to get an overview or preview of the reading (coded *overview*). For example, NMF4 explained jumping around the text:“I wanted to see what came next.”

Finally, in three instances of reading order, a participant indicated that they reread a piece of the text because they had “*zoned out*”; this was coded as *zoning out*.

Discussion

This study highlights participants' movement of attention as a realized opportunity for agency. All readers employed a variety of reading behaviors. These movements demonstrated intentional choices about how to move through the excerpt, both through their own explanations and the authors' perceptions of the text ordering. Participant activity was constrained by the text itself and the goal of learning about Newton's Method. However, the participants' explanations for their choices reflected their own goals, understanding, and sense of themselves in relation to the text and interview context. This reflects the notion of agency as the "interplay of autonomy, identity, power, and knowledge" described in Fulmer et al. (2020, p. 632) .

In detailing participants' movement of attention, we have come to conceptualize the behaviors as having fuzzy and overlapping boundaries; and many of the behaviors can be viewed as existing along a continuum, as displayed in Figure 1.

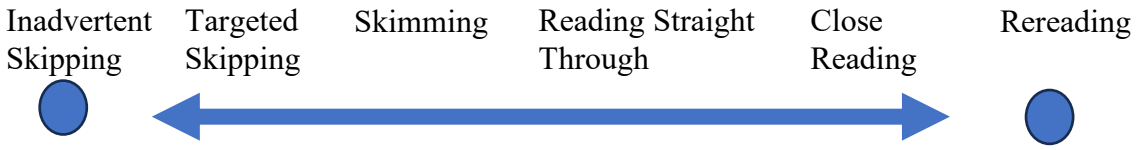

Figure 1. The attention continuum

Here the transition between skimming and close reading represents a dialing up and down of attention, reflecting the speed at which a person moves their attention and how much and what grain size of the text they give attention to. Moreover, skimming itself operates on a spectrum, where high-speed, large-grain size skimming might typically be described as skipping. In particular, we interpret instances when a reader identifies the intellectual/conceptual content of text component and make an active choice to not read it as skimming; this is labeled as "targeted skipping" in Figure 1 to highlight this extreme end of the continuum. We distinguish this from unintentional skipping, where a reader is not aware of the text they skipped. The turns in this study coded as skipping appear to fall into the category of unintentional skipping. The reading order-zoning out may also be a consequence of a lack of attention similar to unintentional skipping. At the other end of the model, rereading–while a different behavior from close reading–arises from similar motivations as close reading and represents a similar dialing up of attention. Not represented in this figure are other kinds of reading order behavior, namely jumping forward and backward. This type of behavior appears to arise, in part, from combining of skimming, reading straight through, and close reading.

The attention continuum contrasts with other research (for example, Phillips and Phillips, 2007) that presents skimming and close reading as in opposition to each other. We argue that viewing these behaviors as a continuum is useful for understanding a reader's movement through the text as a web of choices they make and coordinate throughout the reading. Moreover, this perspective positions all behavior other than reading straight through as inherently agentive, because it, "run[s] counter to the status quo" of reading straight through (Weinberg et al., 2024, p. 669).

The attention continuum can, potentially, be thought of in terms of various forms of attention. Posner and Petersen (1990) point to awareness of stimuli, planning, and goal-directed behaviors. For example, sustained attention, which enables an individual to concentrate on a particular stimulus for an extended time period (e.g., Chen & Wu, 2015; Roda, 2011), could be connected to close reading. Conversely, alternating attention, which supports rapid shifts among

a set of stimuli (Lezak et al., 2015; Parasuraman, 1998) could support behaviors like skimming and targeted skipping.

Participants' stated reasons for attentive behaviors reveal a variety of ways that readers used attentive movement to engage in a learning experience. Moreover, we believe that didactical disciplinary literacy provides a useful lens for understanding these reasons. While some of the readers' moves through the excerpt could be characterized in terms of general literacy strategies (such as skimming to get an overview), their explanations were often tied to the specific didactical structure of mathematics textbooks (for example, coordinating an example and general equation, as part of developing understanding). In some cases, we explicitly connected participants' reasons to disciplinary norms and values (via the disciplinary reasons code). Although this study focused on common themes among the participants, we also found considerable variation between participants in the particular ways that they engaged in behaviors and how these behaviors connected to their reasons. Although the small size and complexity of the dataset makes it difficult to draw clear distinctions between the three groups of participants, we believe the variation between participants suggests that the individual's positioning in the disciplinary community and as a learner may offer a useful perspective on how they move through a text.

The results of this study display a varied and nuanced set of behaviors that readers engaged in while reading a mathematics textbook. Readers' explanations for how they navigated the excerpt appear to be connected to their prior knowledge, their perceptions of the excerpt as a didactical text, their positioning in the discipline, and their understanding of disciplinary norms and values. While this study provides an initial description of reader's attentive movement and associated motivations, it also highlights this as an area for further investigation. In particular, additional research could reveal a richer description of how individuals organize their movement through a text, how this movement is guided by the text itself (as both a disciplinary object and a didactical text), the reader's background knowledge, and the reader's disciplinary positioning and identity.

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