Beyond a Surface-Level Understanding: Developing a Conceptual Assessment of the Derivative

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Background and Literature

Many examinations in mathematics given to students often focus on the students being able to perform procedures such as calculating derivatives. Even when questions are not designed around students invoking specific procedures, problems are often routine enough that students can perform well in the examinations without necessarily having a good grasp of the concepts involved in why their responses are valid. (Tallman et al., 2016, 2021). In contrast, a *concept assessment* or *concept inventory* is an examination which is dedicated to determining how well students understand the concepts associated with a given subject area. Several concept inventories exist for branches of science and mathematics (Hestenes, Wells, & Swackhammer, 1992; Carlson, Oehrtman, & Engelke, 2010), and for calculus the first such concept assessment was Epstein's Calculus Concept Inventory (Epstein, 2013). However, this assessment has been criticized by being not research-based as well as not holding up to tests of validity (Gleason, 2019), leading to additional projects aiming to create improved concept assessments for calculus (Hall et al., 2023; Lee et al., 2025).

Research Aims

This research is part of a larger project, tentatively called Developing CALC, which aims to create a set of smaller, modular concept assessments focused on specific areas within calculus. The first section of the project focused on developing an assessment for definite integrals (Hall et al., 2023), whereas this portion focuses on developing a similar assessment for derivatives. This research is based around much of the literature surrounding student understanding of derivatives, and what makes such an understanding productive (e.g. Zandieh, 2000; Carlson, 2002; Jones, 2018; Reed et al., 2023). Our research question is: "What kinds of questions about derivatives assess whether students have a productive, conceptual understanding of the derivative as opposed to merely surface-level or procedural understandings?"

This second portion of the project is currently in its preliminary stages, with the current goal being the development of the questions themselves. The questions being developed are based on tasks found in previous research, questions in common calculus textbooks, as well as our own teaching experiences. Our aim is for these questions to require students to have good conceptual knowledge of the corresponding topic to be answered correctly, and then the questions will later be given to students to determine if the questions do in fact assess the students' conceptual understanding. The primary goal of this poster will be to serve as a space of discussion, where experienced researchers can critique the tasks we have chosen, come with their ideas for what kind of questions could best serve our aims, which additional topics may be important to ask in an assessment such as this, or to otherwise give feedback on the project at this early stage.

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