Designing Activities About Definition Values Through Collaboration and Attention to Professional Obligations

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Because many mathematics instructors rely on lectures as their primary mode of instruction, change efforts should engage with and build on instructors' existing values and practices rather than attempt to replace them. We report on the first year of our project, where we take a collaborative approach to improve instruction surrounding definitions through activity design and implementation. Constructing narrative cases, we explored how our collaboration aided participants in navigating alignments and tensions between their professional obligations and their activity. We found our collaboration succeeded in guiding our participants, but it required careful examination of reasons as to why instructors may not institute instructional change.

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Many mathematics faculty continue to rely on forms of lecture that prioritize efficiency and content coverage, both in Calculus (Kirin et al., 2017; Vroom et al., 2022) and proof-based courses (Johnson et al., 2018). Dawkins and Weber (2023) argue that such instructional choices may persist because they fulfill instructors' perceived commitments, such as ensuring mathematical precision, preparing students for advanced coursework, and managing time constraints. This suggests a need for instructional change efforts that engage with and build on instructors' existing values and practices, rather than seeking to replace them. In this study, we focus on one context where values play a central role – the teaching of mathematical definitions. We collaborated with mathematics faculty to design classroom activities that communicate the nature of mathematical definitions, hypothesizing that attending to instructors' values could support them in adopting more interactive teaching strategies while remaining aligned with their professional obligations. This focus is particularly important given evidence that mathematicians and students understand definitions differently (Edwards & Ward, 2004; Fernández-León et al., 2021; Martín-Molina et al., 2020).

Theoretical Grounding

We draw on Herbst and Chazan's (2011) theory of *practical rationality*, assuming that instructors' practices are rational within the context of their available resources and perceived professional obligations. Herbst and Chazan theorize four such obligations that instructors weigh as they make instructional decisions: (1) disciplinary, to represent mathematics authentically; (2) individual, to attend to students' needs; (3) interpersonal, to manage classroom interactions as social spaces; and (4) institutional, to meet the expectations of the schooling system. For instance, Johnson et al. (2025) exemplify such decision-making. The mathematicians they studied were hesitant to adopt active learning approaches that might limit content coverage because they wanted to avoid upsetting the department chair if their section deviated from others (institutional obligation), aimed to adequately prepare students for graduate school (individual obligation), and wanted to have time to cover all the essential topics according to their disciplinary expertise (disciplinary obligation).

Our collaborative work aimed to foreground the instructors' disciplinary obligations, as we suspected these would provide a compelling rationale for implementing an instructional innovation. In particular, through collaboration, faculty developed activities to treat mathematical definitions in ways aligned with the mathematics community's values. By *activity*, we mean any part of an instructor's lesson that solicits some artifact of how students think about a value (e.g., a discussion, individual exercise, exit ticket). We conceptualize values, mirroring Dawkins and Weber (2017), as a community's shared goals or orientations (*values*). Rupnow and Randazzo (2023) identified two values that guide how mathematicians treat definitions: the *communication value*, emphasizing clear and unambiguous expression, and the *freedom value*, emphasizing flexibility and creativity when using or creating mathematical definitions.

As the instructors planned and enacted activities, we noticed that these activities often aligned with, but at times came into tension with, their professional obligations. For instance, a think–pair–share activity that asks students to consider multiple, conceptually distinct definitions may align with instructors' disciplinary obligation to highlight the freedom value while also supporting their individual obligation to engage students in meaningful mathematics. At the same time, such an activity might create tension with their institutional obligation to follow the textbook's chosen definition and to cover material efficiently. Recognizing and navigating these alignments and tensions proved central to the design process. Thus, we view the development of activities that either aligned with or productively responded to instructors' obligations as an advantageous goal for the activities to be realized and sustainable in the instructors' classes. In this study, we investigate: *In what ways do instructors' professional obligations align or come into tension with activity design and implementation around mathematical definitions? How can collaboration help surface and respond to these alignments and tensions?*

Methods

Our data comes from the *Supporting Mathematical Instruction around Definitions through Values-Centered Collaboration* project. We focus on the first year of the project during which participants engaged in a series of focus groups (FGs) (four total meetings) and completed surveys before or between meetings (five total surveys). We also recorded any activities that the instructors implemented in their class. We recruited ten instructors in Fall 2024 and six instructors in Spring 2025 across two PhD-granting Midwestern universities. Four FGs were held each semester.

We began the FGs by introducing instructors to the framework we used for mathematical definition values (Rupnow & Randazzo, 2023) and discussed messages that instructors may send to students about them. We (the researchers and instructors) also discussed examples from our teaching in which we aimed to share definition values with students. Author 2 shared a Real Analysis activity in which the students analyzed an informal definition for increasing sequences and used it to formulate a more formal and unambiguous version. Author 3 shared two activities: one from Abstract Algebra that engaged students in defining "prime," and, later, one in Linear Algebra where students reflected on the name "orthogonal complement" and the conceptual insight it conveys. After these discussions, instructors brainstormed activities that communicated a chosen value (and norms that uphold them) and, in the subsequent meeting, they presented these activities and exchanged feedback. After the instructors implemented their activities in their course, we debriefed and reflected on their implementations. The surveys collected demographic data, perceptions on the role of definitions in mathematics, reflections on the values discussed in FGs, descriptions of their classroom activity, and feedback from their involvement. We drew on these survey responses between FG sessions to revise our initial plan for the following session.

In this study, we highlight two cases of working with two mathematicians, who we refer to as Julian (computational mathematician) and Drew (algebraic combinatorialist). We selected these two cases because we thought both seemed like successful collaborations but in different ways. Based on our experiences with them, we thought that our collaboration with Julian supported alignment in his professional obligations and his activity whereas we thought that our collaboration with Drew supported him in responding to tensions between his obligations and his activity design.

Our data analysis involved constructing narrative cases (Auerbach & Silverstein, 2003) of our collaborations with Julian and Drew. Our narrative writing was guided by the following questions: (a) How, if at all, did [mathematician] feel obligated to share the communication value in his teaching? The freedom value?, (b) How did [mathematician's] activity align and/or come into tension with their professional obligations? (c) How, if at all, did our collaboration with [name] contribute to this alignment or tension? We developed the narratives using an iterative process. First, we re-read faculty's survey responses and answered the guiding questions, interpreting their responses through a professional obligation lens. We then discussed our interpretations until we reached an agreement. Next, we drafted what we noticed related to the guiding questions, referencing any relevant quotes from their surveys. With this first draft, we watched the FG recordings and read the corresponding meeting transcripts to test and refine our emerging claims and enrich the narrative with contextual detail. Once satisfied with the narrative, we engaged in member-checking by inviting the mathematicians to read their narratives and provide feedback, which we subsequently incorporated.

Results

In what follows, we share two abbreviated narratives that highlight our collaborations with Julian and Drew, attending to how their professional obligations aligned or came into tension with activity design, and how collaboration helped surface and respond to these obligations.

Julian's Activity: Aligning Design with Obligations

Julian's Disciplinary Obligations. Julian had an obligation to uphold the communication value in his teaching, particularly through the way he used and presented mathematical definitions in his Differential Equations class (*disciplinary obligation*). Since the course placed little emphasis on proof, he explained that the role of definitions was primarily about "communicating ideas to the students and giving something a name." In this context (and in mathematics more broadly) he explained that definitions serve key communicative purposes: they make concepts "easier to reference and talk about," help ideas "be memorable to others," and clarify "what something is and what it is not." Julian emphasized that modeling clear communication was essential because "we want our students to be clear as well when they're doing their written work- we would like them to try to emulate that."

Julian did not feel as obligated to uphold the freedom value in his teaching as compared to the communication value. While he acknowledged that definitions can also be used in the creation/discovery of concepts, he emphasized this was "mostly done in research" and not typically relevant to his classroom teaching. He explained one tension between sharing this disciplinary value and an obligation to the course to present definitions a certain way to maintain consistency for students (*individual obligation*) - "If we're using a certain textbook… I feel like I have to use whatever the textbook is using." Still, Julian shared that he exercised the freedom value in small ways in his teaching prior to the project. He shared that he sometimes replaces a traditional term with one he thinks will resonate more with students (e.g., "linear independence" as “no redundancy”) with the caveat that he introduces the conventional term as well: “I still feel that I'm required to teach them the usual term as well, because I don't want to send them out into the world with a misunderstanding of, you know, what something is called.” That is, Julian also used traditional concept names to fulfill his obligations to students and to the discipline to promote consistency in concept names.

Activity Design. Because Julian felt obligated to uphold the communication value in his teaching (especially in comparison to the freedom value), we aimed to collaborate with Julian to design an activity that would showcase this value. At the time, he hoped to develop an activity around the Laplace Transform, one of the remaining key definitions in the course. However, he was unsure how to connect this definition to the communication (or freedom) value. In particular, he noted that in the initial teaching case studies shared, we had used the definitions of “prime” and “increasing,” which are intuitive subsets of familiar sets (primes within the natural numbers) or relate to their everyday meaning, whereas the Laplace Transform did not fit that mold. During the following focus group, we supported Julian in several ways to develop his activity so that it would better align with his desire to share the communication value. First, Author 3 shared an example from her Linear Algebra course in which she asked students to reflect on the name “orthogonal complement,” surfacing the communication value. Second, we paired Julian with another instructor who was also designing a Laplace Transform activity, to provide new perspectives during the discussion. Additionally, we encouraged both instructors to reflect on the concept name. We noted that while “Laplace” (a person’s name) may not provide much conceptual insight, the term “transform” points more directly to the heart of the concept, and what Julian described as the important part of the definition. Ultimately, Julian created an activity in which he asked students to discuss: (1) Why is it useful to give the Laplace Transform a name?, (2) Why do you think we call it a “transform”?, and (3) What are the ways that the Laplace Transform <u>transforms</u> something into something else? These collaborative supports helped Julian recognize ways to align his disciplinary obligation to uphold the communication value with the design of his Laplace Transform activity.

Alignment with Professional Obligations. Julian reflected on several ways that implementing the activity fulfilled his professional obligations. He emphasized that the activity elevated a potentially new obligation to himself as an instructor and to his students to make the course more interactive and fun (*interpersonal obligation*). He reflected, “it was surprising, the class kind of erupted in discussion.” He later added: “it was the best part of the class. The rest of the class felt kind of dry and boring compared to that because... I think the students had fun, I had fun.” Julian also explained how his activity supported other professional obligations. He recounted one student’s insight as he explained it fulfilled his obligation to share the communication value (*disciplinary obligation*): “One student said something which I thought was interesting. He said, ‘well, if we didn't give things like this a name it would be like none of us have names and then we would just have to refer to each other, you know, without names, which would be really difficult.’ So I thought that was really a good answer.” The activity simultaneously addressed Julian’s obligation to support students in deepening their understanding of the concept itself (*individual obligation*). He said, “I think the students will remember the Laplace transform and its properties more deeply now - like it'll be something that they've actually thought about. And it may help them in the future when they recall what is the Laplace transform and they'll think, ‘oh, yeah, that's that thing that transforms all sorts of things into something else that's useful.’”

Because the activity aligned with multiple obligations (*disciplinary, interpersonal, and individual*) Julian expressed a commitment to implementing similar activities in future courses. In particular, he noted that such designs would be “super easy to incorporate” when the name of a concept is chosen to convey meaning (e.g., Laplace Transform, complementary solution).

Drew’s Activity: Responding to Tensions between Design and Obligations

Drew’s Disciplinary Obligations. Drew desired to share different aspects of the communication value in his teaching (*disciplinary obligation*), depending on the context and audience. Across all his teaching contexts, he explained that definitions are “to get [students] to understand and reason about a concept.” He distinguished between two kinds of definitions: “the short but less useful version of a definition and then the more intuitive, less precise version,” using the epsilon-delta definition of a limit and its informal counterpart as examples. His choice of which version to teach depended on the course context and the needs of the students (*institutional* and *individual obligations*). In some undergraduate settings, he felt it was important to teach both: “And I try to impart to them that this is a formal thing we have to do it for these reasons, but here’s maybe the useful thing that you do instead that is in itself some kind of definition, just not the usual mathematical kind, perhaps.” He felt formal definitions “need to be supplemented with the informal stuff - to get students to the point where they can appreciate what the formal definition was for.” In this sense, he felt a consistent disciplinary obligation to showcase how more intuitive versions of the definitions communicate conceptual meaning, and in contexts where formal definitions were expected (such as proof-based courses), he also felt obligated to the discipline to connect that meaning to formality.

While Drew felt consistently obligated to share the communication value through his teaching across different levels of courses (albeit in different ways), he sometimes felt obligated to share the freedom value, but mostly in upper-division courses. He shared an instance while teaching Abstract Algebra prior to his involvement on the project when he felt this obligation. The course textbook defined rings as not having an identity element. “And that was sort of very new to me,” he reflected. “I gave them a little five-minute discussion of like, here’s why we’re doing it this way. These are the advantages. Not everyone does it this way. It’s sort of arbitrary what the word can mean.” In this instance, he felt obligated to convey to his students that definitions are choices and that there is freedom in how mathematical objects are defined.

Activity Design. Because of his professional obligation to the discipline to share the communication value, Drew wanted to develop, with our support, a classroom activity that would highlight this value. He especially wanted students to see definitions as a packaged way of expressing the main idea. He expressed interest in this after we shared a teaching scenario in which Author 2 supported her students to notice the idea of “increasing” first, and then supported the students in recognizing how precision supports clear articulation of that concept. Drew remarked, “I thought it was cool, this [activity] got the students to think about the level of precision in language that they're taking in and the way they're thinking compared to the way a textbook reads.”

Drew initially began designing an activity for his current Calculus I course, centered on the definition of Riemann integration. He explained that he wanted to show his students the velocity graph for a car and then ask how far it went. He thought this would get them to “confront their intuition for like distance – time times speed equals distance – just doesn't work here because there's a curve”. He then intended to support students in visually seeing how partitioning the interval could make better approximations, and then connecting this process to the definition by supporting them to see that this process “is a limit,” but just one that “looks very different from the limits we've done throughout the course.” He wanted to further develop this activity to share the communication value in the sense that “this definition is a catchall uniform way to talk about the different ways of doing the same thing” and that it “[wraps] these different choices of how you might do this area approximation into one definition”.

Tensions with Professional Obligations. However, Drew experienced several tensions between his task design and his professional obligations. First, he explained that he typically avoids rigorous definitions in this course, keeping things “very pictorial and heuristic-based.” His students, who were primarily first-year engineering majors, wouldn’t “appreciate too much rigor” (*individual obligation*). Yet, Drew thought sharing this aspect of the communication value would require linking the visual and intuitive work to a formal definition (*disciplinary obligation*). Without this connection, he felt the activity would not align with the value he was trying to promote. With this connection, he felt he would not fulfill his obligation to his students to provide meaningful coursework for them. Additionally, Drew had an obligation to stay in step with the course’s tight coordination, and he did not feel free to change the pacing or topics (*institutional obligation*). From Drew’s perspective, continuing to develop this activity would not fulfill his obligation to the coordinator and other instructors to offer a uniform experience for their students.

Responding to Tensions. For these reasons, we encouraged Drew to shift his focus to an activity for his upcoming Abstract Algebra class in which he felt (a) a greater sense of obligation to share the formal versions of the definitions with his students (*disciplinary obligation*) and (b) a greater sense of freedom in the content, pacing, and structure of the course (*institutional obligation*). This suggestion seemed to constructively respond to this tension as he shared, “I’m definitely more excited about not shoving something into calculus” and “I was trying to squeeze something into calculus where it maybe doesn’t quite fit.” In this case, collaboration did not eliminate tensions but instead supported Drew in recognizing a more fitting context where his obligations and design could be more aligned.

His abstract algebra activity focused on the concept of group isomorphism. Drew described his plan: “My tentative plan is to give them an informal definition of group isomorphism as two groups being fundamentally the same but looking different.” He would begin with similar things from other math contexts such as congruent geometric shapes, different factorizations of polynomials or integers, or games like tic-tac-toe versus choosing three numbers that sum to 15. He planned to give students pairs of isomorphic finite groups and ask them to explain why they thought the groups were isomorphic. He anticipated that students would come up with the idea of pairing elements, and then he would then ask them to try write a formal definition. Finally, he would present a formal definition and discuss why it takes the form that it does. This activity appeared promising because it might allow Drew to meet several of his teaching obligations. It supports the communication value by illustrating how a formal definition can precisely capture the central idea of isomorphism – that two groups behave the same way – while also supporting the freedom value by allowing students the experience of crafting a definition (*disciplinary obligation*). It also aligned with his obligations to students, especially his desire to support their understanding: “I think it will get them to think more deeply about the concept of isomorphism and experience a line of thinking they don’t usually get to” (*individual obligation*). Drew remained excited and hopeful about implementing the activity in the following semester.

Discussion

Through sustained, instructor-centered collaboration, our project aims to improve undergraduate mathematics instruction. By applying a practical rationality lens and attending to the professional obligations that shape instructors' choices, we note our collaboration with Julian and Drew successfully supported them in navigating the alignments and tensions between their professional obligations and the design of their activities. However, this navigation required careful examination and explication of the factors that could explain why instructors are not willing to implement certain types of instructional change. In our case, this meant consideration of our participants' professional obligations and their course learning goals and contexts. Julian's case was an example of a co-designed activity where his professional obligations were aligned with his course learning goals. Further, Julian's experience with both our collaboration and his activity resulted in a commitment to continued activity implementation post-intervention. In contrast, Drew's initial activity resulted in tensions with his obligations and thus caused reluctance to implement his activity. We renegotiated the design with Drew to situate the activity in a context that aligned with – rather than put in tension – his professional obligations.

Moreover, we echo Johnson et al.'s (2025) note that professional obligations depend on the context of the particular course. Our work points to how this can be the case in two different ways. First, tight course coordination and time concerns deterred deviation from pacing and content coverage. While research on coordination has positioned it as a means to support professional development (e.g., Martinez et al., 2021), our findings show that in some cases, coordination may stifle instructional innovation. Second, as we anticipated when entering the project, the instructors did perceive the values related to definitions to be important to the discipline of mathematics. However, consistent with Rupnow and Randazzo (2023) we note that valuing something within one's discipline does not imply instructors find it necessity to share it in teaching across all levels. This raises questions about how mathematics is portrayed and how mathematicians want to portray mathematics – if instructors commonly wait until students are in upper-division courses to share notions of freedom, students who persist in the math major may expect and desire less freedom and be uncomfortable when given more, just as students who desire to think flexibly and creatively may leave mathematics before that creativity is incorporated.

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