Analysis Paralysis: On the Potential Overemphasis of Calculus and Analysis in the Undergraduate and Graduate Mathematics Curricula

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In the United States, students in the K-12 system are often adversely affected by a so-called "race to calculus" that we believe to be caused, in part, by the emphasis on analysis found in the mathematics curriculum at both the undergraduate and graduate levels. This originates partially from a historical need for calculus in industry, a need which is no longer as prevalent as it once was. Here, we examine the curricula at 100 of the top graduate and undergraduate programs in the United States and the prevalence of a variety of topics in these programs, including calculus, analysis, linear algebra, abstract algebra, geometry, and statistics. As expected, analysis was much more prevalent than any other advanced topic. We discuss reasons for this prevalence and how such prevalence should be mitigated in the future.

Keywords: Curriculum Development, Calculus, Analysis

Introduction

It is well known that the structure of the mathematics curriculum at the K-12 level is currently based on a "race to calculus," that is, most of the curriculum is heavily influenced by a need to prepare students to take calculus in college (Bressoud, 2021). There are strong historical reasons for this focus, not the least of which include a need for graduates in physics, engineering, and mathematics during the Cold War (Tucker, 2013). However, today's industrial landscape has shifted and now the main mathematical needs in industry are no longer calculus-based, but rather based on the statistics and linear algebra needed for data analysis (Gravemeijer et al., 2017). As a result, there seems to be a significant societal desire for change in the mathematics curriculum and there are many suggestions about what that curriculum might look like (see, for example, Gravemeijer et al., 2017; Stanford Graduate School of Education, 2021). Yet, change in this regard has been relatively slow.

We have sought to better understand the influences that prevent meaningful changes in the mathematics curriculum from happening. One observation we made was the similarity in the undergraduate mathematics curricula we experienced at the various universities we have attended or taught at. Indeed, there were remarkable parallels in the initial courses that undergraduate mathematics majors were required to take at these institutions. These courses also significantly overlapped with the initial mathematics courses that STEM majors were required to take. These courses seem to typically include a single-variable and multi-variable calculus sequence, a course in linear algebra, and a course in ordinary differential equations. This is not too surprising as the calculus sequence for STEM majors is well understood to be the impetus for the race to calculus at the K-12 level (Bressoud, 2021). While it is clear why physics and engineering majors start with calculus, we sought to understand why mathematics programs also begin there when mathematics includes topics like modern algebra, geometry, topology, and discrete mathematics, each of which provide equally valid starting points.

Further similarities in the curricula with which we were familiar provided a possible explanation. We noted that these curricula seemed to continue to emphasize calculus and analysis throughout, typically requiring a considerably larger number of calculus- or analysis-based courses than courses in other topics in mathematics. This seemed to continue even into graduate school where the only course that we had personally observed almost always being required was a graduate level course in real analysis that typically covered measure theory and the Lebesgue integral. This suggested that the race to calculus might continue past K-12 education into the undergraduate and even graduate levels.

To verify whether our personal observations are typical in United States undergraduate and graduate mathematics curricula, we acquired and analyzed data on various mathematics programs throughout the nation. We also sought to better understand the motivations for such similarities if they were found.

As shown in this paper, we found that this emphasis on calculus and analysis in the undergraduate and graduate mathematics curricula is common throughout the schools that we considered. In the remainder of this paper, we explore the historical motivations for why this emphasis on calculus and analysis exists, provide an analysis of the data we collected which shows this emphasis is currently present throughout the United States, and discuss what this indicates about how we can address the race to calculus and modernize the mathematics curriculum at all levels.

Literature Review

Calculus has not always enjoyed its current privileged position within the college curriculum (Tucker, 2013). The teaching of mathematics at the post-secondary level in the United States has evolved greatly throughout the history of the United States. In the early days of the country, American colleges focused mostly on Euclidean geometry and algebra. Mathematics was "studied as a classical training of the mind rather than as the language of science and engineering" (Tucker, 2013, p. 1). Mathematics almost disappeared from college curricula in the later 1800s and early 1900s as colleges moved away from focusing on a classical education to a focus on practical education. Mathematics avoided falling into the same obscurity as other classical subjects, such as Greek and Latin, through the needs of engineers and similar technical professions that required mathematical training in their fields (Tucker, 2013).

In response to this declining emphasis on mathematics, in 1915 the Mathematical Association of America (MAA) began making suggestions on what should be included in undergraduate mathematics curriculum, with calculus as the introductory focus (Tucker, 2013). Additionally, colleges began introducing academic majors and general education requirements, further bolstering the place of mathematics within the college curriculum. During this period, mathematics was a career-oriented major focused on supporting engineers, actuaries, and teachers. However, mathematics gained its modern prominence in the wake of World War II as the national demand for engineers and mathematicians exploded (Duren, 1967; Tucker, 2013).

In 1957, the Soviet Union launched Sputnik, the first manmade satellite into space. This event is widely regarded as the ignition of the space race between the Soviet Union and the United States during the middle and latter parts of the twentieth century (Tucker, 2013). The space race led to an increased demand and respect for collegiately trained mathematicians and other science professionals from math-heavy fields. At the heart of all these suddenly respected fields was calculus, a mathematical subject seen "as a foundation for the modern world and one of science's greatest intellectual achievement[s]" (Tucker, 2013, p. 10).

The newfound demand for mathematically trained college students led to the formation of the Committee on the Undergraduate Program in Mathematics (CUPM), a group tasked by the MAA to set forth standards for what mathematics should be studied by college students seeking degrees in mathematically-based fields of study (Duren, 1967). In 1963, the CUPM published an influential report on the suggested curriculum for undergraduate students who one day wished to become research mathematicians (CUPM, 1963; Duren, 1967; Tucker, 2013). This report pushed for a heavier emphasis of calculus and analysis in the undergraduate curriculum as preparation for learning analysis at the graduate level, (CUPM, 1963). A 1965 report by the CUPM further emphasized the importance of analysis at the graduate level:

> The Panel has consulted many outstanding graduate mathematics departments; a solid grounding in algebra and analysis is what they most want from incoming students. If time and resources permit, it is, of course, desirable to introduce the pregraduate student to a broader range of material, but not at the expense of a depth in algebra and analysis. (CUPM, 1965, p. 450)

Even more influential was the 1972 CUPM report, which did reduce the number of recommended mathematics courses, but led directly to the traditional requirements of calculus 1 and 2, linear algebra, and multivariable calculus seen in many colleges today (Tucker, 2013). This report also acknowledged the preeminent place of analysis in their recommended program and questioned whether such preeminence was appropriate:

> A closely related question is whether the "core" of pure mathematics that all departments should offer is now the same as it was presumed to be a few years ago. As new fields develop, some older fields seem less relevant, and today some mathematicians even question the assumption that calculus is the basic component of all college mathematics. (CUPM, 1972, p. 36).

Despite this questioning of calculus as the basic component of all college mathematics, calculus is still often used as a "gatekeeper" or "filter" for STEM majors, including mathematics (Bressoud, 2021; Bressoud et al. 2015; Tucker, 2013). Many studies have explored how to improve calculus instruction, most notably the 2015 MAA national study of college calculus (Bressoud et al., 2015). These studies have often focused on instructional methods and improving student understanding, with little questioning of calculus as the basis for all college mathematics (see, for example, Berry & Nyman, 2003; Ellis, 2013; Epstein, 2013; Gundlach & Jones, 2015; Jones, 2013, 2016; Jones & Watson, 2018; Mesa et al., 2015; Musgrave & Carlson, 2017; Tallman et al., 2021). However, based on the previously quoted statement from the 1972 CUPM report, mathematics majors should be moving away from calculus or at least be emphasizing analysis and advanced calculus less than in previous years.

In our review of the literature, we also found no examination of the classes mathematics majors take beyond calculus across multiple institutions. In the 1972 CUPM report, undergraduate mathematics majors are advised to include at least three semesters of analysis, while upper-level modern algebra and geometry are only recommended for one semester. Wu (1999) did discuss creating multiple tracks for mathematics majors, with one track specifically for those aimed at graduate school. Still, there was no discussion of what topics would be most appropriate for mathematics majors bound for graduate school, though he did note that many mathematics major curricula are designed specifically with these students in mind.

Research Question

If the place of calculus as the gateway to college mathematics is accepted, yet many fields of modern mathematics do not have calculus as a core component (CUPM, 1972), it stands to reason that analysis should now be less emphasized, or at least equally emphasized with other mathematical topics. While there are many mathematical topics covered even in undergraduate mathematics courses, three main categories tend to be given priority: analysis, algebra, and geometry/topology (Harvard University Department of Mathematics, 2025; Wu, 1999). This led us to the following research question: Of the three major topics of analysis, algebra, and geometry, is there a difference in the number of courses the top 100 graduate and undergraduate math programs require from these three topics?

Significance of This Study

We hope that this study serves as the catalyst for future discussions and research into the most appropriate collegiate mathematics classes, both for mathematics majors and non-mathematics majors. This study seeks to examine two of the major influences on the mathematics curricula offered at colleges and universities: the mathematics major for students seeking graduate degrees and mathematics graduate degrees.

Methods

We compiled a list of the top 100 graduate programs in math, as ranked by US News and World Reports (US News and World Report, 2022, 2025). An initial analysis of required courses for graduate programs was completed using the 2022 list. After this data was revisited in 2025, we then completed a further analysis of undergraduate programs using the updated list. We did this with the intent of utilizing the corresponding undergraduate programs at these institutions, with the intent of seeing what a student preparing for graduate studies in mathematics is required to take. As such, we removed graduate programs at institutions with no corresponding undergraduate program. In place of such institutions, we included the subsequent institutions to complete our list of 100 universities. In this case, the only such institution was the CUNY Graduate Center, which we replaced with the institution ranked 101st at the time of our survey.

For the purposes of our survey, we grouped courses as being in one of the following categories, which we will define hereafter: Calculus, Analysis, Geometry, Linear Algebra, Abstract Algebra, and Statistics. Graduate school courses were only categorized by Abstract Algebra, Analysis, and Geometry. As the goal was to classify courses which are strictly required for graduation, we only counted a course towards a category if a course covering that category was required to satisfy a graduation requirement. We also did not count course prerequisites for courses taken before a Calculus 1 course, as such courses are commonly tested out of by students in a mathematics major.

For instance, if a student had to choose one course from a set including two analysis courses and an abstract algebra course, this would count towards no category, as the student does not need to take one course which fits strictly one category. On the other hand, if a student must take at least one course chosen from among three analysis courses, this would contribute one course to the required analysis courses for that program. In the case that a course covered multiple subjects, we split the course among the subjects based on the course description and course text. This is done while retaining the total value of one between the split, meaning a course covering geometry and abstract algebra would count as 0.5 abstract algebra, and 0.5 geometry, not one abstract algebra and one geometry.

As different institutions offer courses on different schedules, with the most common options being semesters and quarters, we recorded the number of blocks each institution offers in a standard academic year, excluding the summer block(s). This allows us to normalize all data collected into semester equivalents, so that schools on quarter systems are not represented with disproportionately high course requirements. For each undergraduate institution, if multiple emphases were offered with the major, the one most aligned to graduate study was selected.

The graduate data was reviewed by one researcher as such courses were more clearly aligned to a particular subject area. The undergraduate data was reviewed by five researchers, with each researcher reviewing 30 classes, with 20 being shared by other researchers. Situations in which there were inconsistent course counts were flagged and reconciled jointly. Reviewers counted courses as per the guidelines below, which we utilized as descriptions for each type of course.

We classified a course as a calculus course if it covered traditional calculus 1-3 material or differential equations, but with students not being graded or tested on proofs or proof-based material. This includes courses on differential calculus, integral calculus, multivariate calculus, ordinary differential equations, and partial differential equations.

We classified a course as an analysis course if it covered real analysis, complex analysis, or measure theory. Additionally, we classified courses which cover traditional calculus 1-3 material or differential equations with students being graded or tested on proofs and proof-based material as analysis courses. Note this is mutually exclusive with a being considered a calculus course.

We classified a course as a geometry course if it broadly covered the subjects of plane geometry, differential geometry, algebraic geometry, algebraic topology, or general topology. We specifically did not classify analysis courses which covered the topology of the real line or the complex plane as topology courses. This exclusion was so that nearly every introductory undergraduate real analysis course did not end up counted partially as a geometry course.

We classified a course as a linear algebra course if it covered the theory of linear operators, either directly by definition, or by the theory of matrices. We made no distinction between computational linear algebra courses, proof-based courses, or courses which are a combination thereof. We did not count algebra courses which covered matrices as an example space as linear algebra courses, so as not to conflate linear algebra courses with algebra courses, similar to the exemption made with topology courses.

We classified a course as an abstract algebra course if it covered the theory of groups, rings, fields, Galois theory, category theory, or any similar algebraic structures through a proof-based approach. As such, we do not count college algebra, intermediate algebra, or other non-proof-based courses towards this subject.

Finally, we classified a course as covering statistics if it covered statistical thinking, statistical methods, probability theory, or other statistics related subjects. This includes mathematical statistics, and measure-theory based statistics. However, measure-theory based courses were cross-listed with Analysis depending on the particular course description and text.

Results

The initial survey of graduate programs found that exactly half of the institutions examined had strict course requirements for their pure mathematics programs, with the other half only requiring advisor approval or satisfactory scores on qualifying exams. These institutions were excluded from analysis, leaving 50 graduate institutions. In our survey studying the undergraduate programs, an overwhelming majority (84) of the institutions examined worked on a semester system. All schools that worked on a quarter system had their number of classes for each subject area multiplied by 2/3 to find the equivalent number of semester classes needed for each subject area.

Descriptive statistics for both data sets after norming the undergraduate data are found in Table 1.

Table 1: Descriptive Statistics for Normed Number of Classes by Subject Area

	Undergraduate Schools				Graduate Schools			
Subject Area	Min	Max	Mean	SD	Min	Max	Mean	SD
Calculus	2	4	3.46	0.561	-	-	-	-
Linear Algebra	0	3	1.18	0.496	-	-	-	-
Statistics	0	3	0.33	0.561	-	-	-	-
Algebra	0	2	1.07	0.575	0	3	1.20	1.030
Geometry	0	2	0.22	0.450	0	3	1.02	1.097
Analysis	0	4	1.73	0.900	1	4	2.22	0.840

After data was collected, SPSS was used to perform a one-way repeated measures analysis of variance (ANOVA), as all variables were repeatedly measured on the same subjects (Field, 2018). As the upper-level classes were deemed the most important ones to compare, the ANOVA was only performed on those variables. Data on the other variables is included to give additional context to these more important data points.

As part of this ANOVA, we tested the following hypotheses:

H_0: There is no difference in the number of algebra, geometry, and analysis classes required by the top 100 graduate and undergraduate math programs in the United States.

H_a: There is a statistically significant difference in the number of algebra, geometry, and analysis classes required by the top 100 undergraduate math programs in the United States.

After the ANOVA was run on the graduate data, the Huynh-Feldt estimate of departure from sphericity was $\epsilon = 0.844$. Using this adjustment, the difference between the number of classes offered in different subject areas was found to be statistically significant, $F(1.687, 24.819) = 53.833, p < 0.001, \omega^2 = 1.413$. This is convincing evidence that there is a difference between the number of required classes in algebra, geometry, and analysis for mathematics majors in the 50 of the top 100 graduate math schools that had stated class requirements.

All graduate class categories were then compared using a post-hoc pairwise comparison with a Bonferroni correction. When the two categories that generally had the highest number of required classes, analysis and algebra, were compared, more analysis classes were required ($M = 2.220, SE = 0.119$) than algebra classes ($M = 1.200, SE = 0.146$). This difference, 1.020, 95% CI $[0.699, 1.341]$, was significant $p < 0.001$ with an effect size of $d = 1.118$.

After the ANOVA was run on the undergraduate data, the Huynh-Feldt estimate of departure from sphericity was $\epsilon = 0.874$. Using this adjustment, the difference between the number of classes offered in different subject areas was found to be statistically significant, $F(1.747, 172.978) = 178.087, p < 0.001, \omega^2 = 0.476$. This is convincing evidence that there is a difference between the number of required classes in algebra, geometry, and analysis for mathematics majors in the top 100 undergraduate math schools.

All categories were then compared using a post-hoc pairwise comparison with a Bonferroni correction. When the two categories that generally had the highest number of required classes, analysis and algebra, were compared, more analysis classes were required ($M = 1.7283, SE =$

0.09004) than algebra classes ($M = 1.0700, SE = 0.05750$). This difference, 0.658, 95% CI $[0.461, 0.856]$, was significant $p < 0.001$ with an effect size of $d = 0.812$.

Discussion

From the above analysis, there is a heavy emphasis on calculus and analysis in both the undergraduate and graduate mathematics curricula at most institutions across the United States. Just as the introductory calculus sequence causes a race to calculus at the K-12 level, we believe that the typical requirement of graduate level real analysis is the cause of the emphasis on calculus and analysis in the undergraduate mathematics major programs. Both the introductory calculus sequence and the graduate real analysis course indicate pressure from above against any change in the curriculum. Indeed, one major reason why mathematics programs may find it difficult to move away from requiring the initial calculus sequence is the need to prepare their mathematics majors for that graduate level real analysis course required by the majority of graduate institutions. This suggests that for any meaningful change to happen at the K-12 level, the undergraduate mathematics curriculum first needs to change. But for the undergraduate mathematics curriculum to change, first the requirement of graduate real analysis needs to no longer be the norm.

We hypothesize that the graduate real analysis course plays more of a role in verifying that students are prepared for graduate work than in serving as a prerequisite for graduate mathematics courses or research, as most such courses and research topics simply do not use the content of that real analysis course. This preparation verification can be done with a graduate level course in practically any mathematical topic. Thus, the emphasis on analysis at the graduate level seems more due to convenience than necessity.

If the assumption of basing the curriculum on calculus was questioned in the 1970s because of changing times (CUPM, 1972), how much more should we question that assumption in the 2020s? This raises the question of what should be done with this data. One suggestion is that we replace calculus with data science, including statistics and linear algebra, because data science is more ubiquitous in today's industry (Stanford Graduate School of Education, 2021). However, if we shift to a curriculum that supports data science instead of one that supports calculus, we run into two potential problems. The first is also noted by the CUPM 1972 report, that the landscape of what mathematical topics are needed by society is constantly changing. Data science is the flavor of the day now, just like calculus was 50 years ago, but in another 50 years, it will likely be a new topic in mathematics. However we change the curriculum should be flexible enough to adapt to future changing needs. If the race to calculus should teach us anything, it is that embedding preparation for a specific topic into the curriculum creates a curriculum highly resistant to change. The second issue is that one of the main reasons why a race to calculus is so problematic is not just because calculus is becoming less important, but also because racing to a particular topic encourages a curriculum that focuses on skill preparation and proficiency in algorithms that support that topic rather than a more robust culture of mathematical thinking and exploration (Tallman et al., 2021; Tucker, 2013). Racing to any topic, regardless of how relevant, will continue to have that effect.

In future research, we look to explore ideas of how to develop a graduate, undergraduate, and K-12 curriculum that would be independent of the favored topic of the day, while still providing robust preparation in mathematical thinking that could quickly be adapted to support any mathematical topic at the college level.

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