Justification of Exponent and Logarithm Rules From an APOS Perspective

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Investigations have been conducted into how students conceptualize exponents and logarithms, utilizing APOS theory, particularly regarding how students justify exponent rules. The thesis by Williams (2011) presents a list of codes to identify the conceptions that students hold regarding logarithms. However, a gap in the literature remains on how students justify the corresponding logarithm rules. In this poster, I compare the thinking of two students to extend her work.

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This poster draws on data from my dissertation regarding how students in a complex analysis course leverage their knowledge of real-valued exponents and logarithms to reason about the complex-valued analogs. Part of my dissertation aims to describe the conceptions that college students have for real and complex exponents and logarithms, and to compare the similarities and differences in terms of APOS theory (Dubinsky, 1991; Asiala et al., 1997; Dubinsky and McDonald, 2001). In my literature review, I have encountered studies that examine how students justify various logarithm rules, particularly the work of Williams (2011). Part of Williams' (2011) claims is that responses to questions about logarithm rules can be used to characterize their process or object conceptions about logarithms. However, her thesis does not report on responses to such questions. The findings in this poster contribute to a larger goal of extending Williams' (2011) framework on exponents and logarithms from an APOS perspective.

Discussion

There was evidence to indicate that both May and Jim's schemas for exponents and logarithms were unified under a broader schema due to the inverse relationship between the two operations. Another similarity between their two schemas was the combinatorial interpretation of exponential expressions, where a^b would be considered shorthand for the number a multiplied by itself b times when b was a natural number. There was more substantial evidence that their schemas diverged when operationalizing an interpretation of fractional exponents. While it is likely that both May and Jim would use roots when calculating fractional exponents, there was definite evidence that such interpretations were not operationalized in proof contexts.

The differences in proofs indicate some variations in their respective schemas. Jim relied solely on the interpretation of iterated multiplication to justify exponent rules. For example, he attempted to reduce the justification of fractional exponent rules to the case of integer exponent rules. He likely operationalizes definitions when justifying properties of exponents and logarithms. He will likely explain the properties of the complex exponent and logarithm by referring to the definition of the complex exponent and the inverse relationship between the exponent and logarithm. In contrast, May's use of logarithm rules to justify exponent rules suggests that in the complex setting, they are less likely to refer to exponents when justifying properties of the complex logarithm.

These phenomena suggest that a similar conception for exponents does not necessarily imply similar conceptions of logarithms. More broadly, among college students, it is likely that the schema for exponents and the schema for logarithms are unified. However, conceptions relating to exponents are likelier to be homogeneous compared to conceptions for logarithms.

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