Exploring Features of Graphs Across STEM Fields: An Emerging Theoretical Framework

Teo Paoletti
University of Delaware

Hwa Young Lee
Texas State University

Hamilton Hardison
Texas State University

Osmond Asiamah
University of Delaware

Allison Olshefke-Clarke
University of Delaware

Brandi R. Gaspard
Texas State University

Yuting J. Huang
University of Delaware

Madhavi Vishnubhotla
The New School

Graphs are essential across STEM fields, yet students often lack opportunities to develop graphing competencies. One challenge is that the ways graphs are used vary across disciplines. This study develops a framework for characterizing graphs in undergraduate STEM contexts by examining features such as coordinate systems, reference frames, and the objects of interpretation. Preliminary analyses of STEM resources highlight differences—such as spatial versus quantitative coordinate systems—and similarities that can inform cross-disciplinary instruction. Our goal is to support more coherent approaches to graphing instruction, preparing students to interpret and construct graphs in ways that align with disciplinary practices.

Keywords: Graphs; STEM; Undergraduate Education

Graphical representations are common in STEM fields for modeling, communicating, and analyzing phenomena. Graphs are tools used for understanding and communicating scientific phenomena (Larkin & Simon, 1987; Meira, 1995). Researchers (Knorr-Cetina & Amann, 1990; Tabachneck-Schijf et al., 1997) have shown how scientists and engineers require graphs to complete certain tasks. Relatedly, graphs are ubiquitous in STEM textbooks and practitioner journals (e.g., Lemke, 1998; Roth et al., 1999). Hence, supporting students in developing meanings for graphs is critical for supporting the next generation of scientists.

Despite the importance of graphs, results from research (e.g., Lai et al., 2016; Ruf et al., 2024) and assessments (e.g., Nation's Report Card, 2005) indicate U.S. students are not being provided sufficient opportunities to develop graphing meanings relevant to STEM. One possible explanation for this is that there are fundamental differences in the ways graphs are created, used, or interpreted within and across STEM fields (e.g., Lee & Guajardo, 2023; Paoletti et al., 2020) and that these differences are uncategorized or under-described in STEM education. Our primary goal in this study is to develop frameworks that explain the various ways graphs are used and expected to be interpreted across undergraduate STEM fields. To accomplish this goal, we intend to address the question: *How can we characterize graphs across STEM fields?* In this preliminary report, we characterize a graph based on the types and number of quantities represented, the underlying coordinate systems, and the way information is intended to be represented. By creating such a framework, we intend to develop and support a more coherent, connected, and holistic approach towards supporting students' developing graphing meanings.

Examining Several Graphs Across STEM Fields

We use the six graphs in Figure 1 to exemplify differences in graphs across STEM fields. We acknowledge that interpreting aspects of these graphs requires domain-specific knowledge (i.e., knowledge about the particular quantities in the graph), as such knowledge is important for interpreting graphs (e.g., Altindis et al., 2024; Roth et al., 1999; Roth & Bowen, 2003; Zimmerman et al., 2023). Our goal is not to present a complete understanding of each graph but instead to present some high-level differences across the graphs. Additionally, we highlight similarities across these graphs that could be leveraged in undergraduate STEM courses.

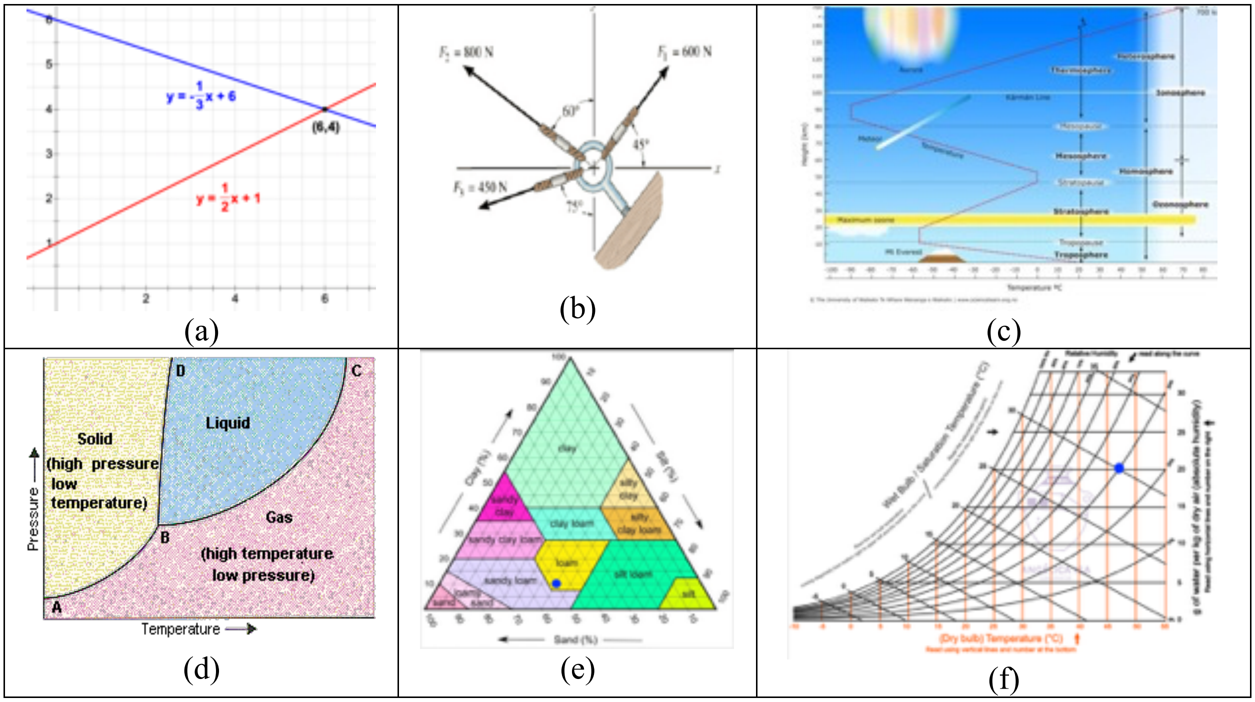

Figure 1: Graphs from a (a) Mathematics, (b) Engineering Statics, (c) Astronomy, (d) Chemistry, (e) Soil Science, and (f) Meteorology.

Figure 1a represents a solution to a system of equations, typical in an algebra class. Figure 1b is a graph from an Engineering Statics textbook (Hibbeler, 2013) used to mathematize the direction and net force exerted on a screw eye by ropes pulling in different directions with different forces. Figure 1c, from a unit on the Atmosphere (University of Waikato Te Whare Wananga o Waikato, 2025), uses the vertical height axis to classify the atmosphere based on altitude. Additionally, the dotted line shows the relationship between height and temperature in different atmospheric regions (e.g., in the troposphere, as height increases, temperature decreases). Figure 1d shows a phase diagram used in Chemistry (Bodner Group, 2025) to represent the effect of pressure and temperature on a hypothetical substance. The graph is divided into distinct areas, representing solid, liquid, and gaseous states of the substance. A person can determine the state given any (pressure, temperature) combination.

Figure 1e shows a soil texture triangle (Hennion, 2025) from the Environmental Science field relating the percent of soil comprised of clay, silt, and sand. Each point totals to 100% of the soil composition. For example, the blue point corresponds to a loam soil that is 10% clay (moving from the point left to the clay axis), 50% sand (moving from the point down-and-to-the-right along the lines to the horizontal sand axis), and 40% silt (moving from the point up-and-right to the diagonal silt axis). Finally, Figure 1f shows a psychometric diagram from a meteorology context (such diagrams are also used in medicine via growth charts for children and in ecology via graphs with isoclines). The graph is intended to show the relationship between temperature, absolute humidity, and relative humidity. For example, the blue point shows that if the temperature is about 47 degrees Celsius (moving from the point down to the horizontal axis) and there are 20 grams of water per kg of dry air (moving from the point right to the vertical axis), then the relative humidity is at 30% (moving up-and-to-the-right along the curve).

At first blush, there are major differences across each of these graphs, given the different domain-specific information they are intending to convey. However, using our preliminary set of frameworks, we can identify commonalities. First, most graphs rely on a Cartesian coordinate system (all except Figure 1e). Second, some graphs are used to represent a physical space or phenomena (Figure 1b, the vertical axis of Figure 1c), whereas other graphs are representing a space distinct from physical space (Figures 1d, e, f). Some graphs are explicitly coordinating two quantities (Figure 1a, b, c, d), whereas others are explicitly coordinating 3 quantities (Figures 1e, f). Further, we note the graphs in Figures 1d and 1e have regions representing distinct states.

We also note curriculum developers likely intend these graphs to serve different purposes. For instance, some graphs are likely intended to help students understand or visualize some aspect of a phenomenon (Figure 1b, 1c). Several graphs could be intended to make inferences about a single moment represented by a point in the graph (e.g., Figures 1a, c, d, e, f) or could be used to make inferences about how two quantities change together in context (e.g., the red line in Figure 1c, following curves in Figure 1f). Hence, while the graphs may seem extremely different at first glance, there are features of these graphs that can be categorized both by using existing theory and by developing new theory. After providing a brief overview of relevant research, we describe our preliminary analysis. Throughout, we return to these six graphs to highlight ways we are leveraging theory to characterize similar and distinct features across these six graphs.

Background, Analysis, and Theoretical Constructs Explaining Aspects of Graphs

Researchers have shown that students of all ages experience persistent difficulties as they engage in creating and interpreting graphs in their STEM courses (e.g., Glazer, 2011; Lai et al., 2016; Potgieter et al., 2008; Ruf et al., 2024). We make two observations from this body of research. First, many of these studies have identified alternative conceptions of graphing and provided insights into varying difficulties students might experience in producing graphs, resulting largely in deficit accounts of students' mathematics (e.g., Leinhardt et al., 1990; Mevarech & Kramarsky, 1997; Moritz, 2003; Ruf et al., 2024). Second, most of this research examined students' interpretations of pre-constructed graphs or students' graph constructions on pre-constructed axes; these researchers did not pay particular attention to the ways students understood the underlying coordinate system (CS) and/or the quantities represented by the CS. Our ongoing research exploring students' developing meanings for graphs is novel relative to this work, as we have begun to create theoretical constructs to help explain nuances in students' conceptions for CSs and the reference frames (RFs) they intend to represent. These constructs can be divided into categories based on (a) the types of CS, (b) the number and types of RFs, and (c) the object(s) students are expected to interpret within the graph space.

Preliminary Analysis

To explore our research question, we have begun preliminary analysis of several STEM textbooks and resources. To analyze the graphs, we conducted a content analysis, which "takes texts and analyses, reduces and interrogates them into summary form through the use of both pre-existing categories and emergent themes in order to generate or test a theory" (Cohen et al. 2007, p. 476). Consistent with this methodology, we used pre-established frameworks, as described above, to categorize, code, and analyze graphical representations in STEM textbooks and resources. Following the main steps of content analysis, we defined a sample of texts, the unit of analysis, and the categories and codes for analysis. We then coded and categorized data. We acknowledge that codes were attributed to each graphical representation based on *our* interpretations of the author's intention of the graphical representation.

Coordinate Systems

A CS is a representational space an individual uses to systematically coordinate quantities (Thompson, 2011) to organize a phenomenon. This space typically involves coordinating two (Figure 1a–d) or more (Figure 1e–f) quantities. Across STEM fields, there are a variety of commonly used CSs, such as Cartesian (Figure 1a, c, d), polar, and logarithmic CSs.

Depending on the purpose the coordination serves, we have distinguished two types of CSs, *spatial* and *quantitative* (Lee, 2017; Lee et al., 2020). A *spatial CS* is a system of measurements through which an individual organizes or coordinates spatial elements in conceived or experienced space. Figure 1b is an example of a 2D spatial CS intended to mathematize the physical space of the screw eye, and Figure 1c contains an example of a 1D spatial CS if we only attend to the different categorizations of atmosphere by height. In contrast, a student may abstract quantities from a phenomenon and coordinate them in some new representational space, different from the space in which the quantities were originally conceived. We call this new structure, used to coordinate abstracted quantities, a *quantitative CS*. Figure 1d–f are all examples of quantitative CSs, because each represents quantities in a space that is different from the physical space in which the phenomenon is occurring.

We note there are often graphs in math classes that do not clearly fall under either of the two types (e.g., Figure 1a), as there are no obvious situational quantities involved. However, we have found the distinction between quantitative and spatial coordinate systems useful in characterizing the types of graphs used in middle school textbooks (Lee & Guajardo, 2023) and STEM textbooks and journals (Paoletti, et al., 2020). Further, this distinction proved explanatory in describing different ways students construct and interpret graphs (Paoletti et al., 2025). We note we have also explored other types of CSs in our work, including polar CSs (Moore, Paoletti, et al. 2013; 2014). Hence, we conjecture these distinctions are likely to be important in exploring different uses of CSs in undergraduate STEM education.

Reference Frames

A CS comprises RFs. RFs are mental structures through which an individual gauges the extent of various attributes of objects or phenomena (Lee, 2017; Levinson, 2003). RFs have been central to studies in spatial cognition (e.g., Levinson, 2003; Taylor & Tversky, 1996), children's conception of space (e.g., Piaget & Inhelder, 1967; Piaget et al., 1960), and quantitative reasoning (e.g., Joshua et al., 2015; Lee et al., 2019). RFs relate directly to the reference points and organizational structure (e.g., axes) an individual uses in a CS. For example, if we only focus on the categorization of atmosphere based on height, we can interpret Figure 1c as representing a single RF (height). If we use the dotted-red graph to consider the relationship between height and temperature, we can consider the graph as coordinating two RFs.

It is typical for CSs to represent two RFs (e.g., Figure 1a, b, d). There are various goals curriculum designers may have when tasking students to interpret a graph representing two RFs. They may ask students to interpret the meaning of a single point in a moment (Moore et al., 2025). For example, they may ask students to look at the dotted-red line in Figure 1c and ask them what the average temperature is at an altitude of 15 km. They may also ask students to make inferences about how two quantities change together, such as asking students to interpret the dotted-red line to describe how height and temperature change together in the mesosphere (i.e. as the height increases, the temperature decreases). Reasoning about two quantities changing together is called covariational reasoning (Thompson & Carlson, 2017).

There are also examples across STEM fields of graphs intended to represent three (or more) RFs in quantitative (Figure 1e–f) and spatial (e.g., a 3D analog of Figure 1b) CSs. When students are tasked to coordinate three (or more) quantities, they are asked to engage in multivariational reasoning (Jones 2002; Panorkou & Germia, 2024). Researchers have noted that students as young as middle school and high school (Bagossi, 2024; Panorkou & Germia, 2024) can engage in multivariational reasoning to interpret graphs such as those in Figure 1e. By exploring the prevalence of such graphs in undergraduate STEM courses, we can develop curricular materials that better prepare students to interpret such graphs in STEM fields.

Types of RFs. In addition to attending to the number of RFs represented in a graph, our research has identified novel ways students interpret graphs as representing continuous versus ordered-discrete RFs (Olshefke-Clark et al., 2024, accepted). Whereas students are often tasked with interpreting graphs as representing continuous quantities (typically using covariational or multivariational reasoning), other times we expect students to attend to distinct regions of a graphical space (e.g., Figure 1d, e). In these latter cases, curriculum designers may expect students to attend to these discretized chunks represented within the graph space when interpreting the graph—either regions as a whole, or to individual points within these regions, rather than considering lines representing changing quantities (e.g., the type of soil in Figure 1e or the state of the matter in Figure 1d). However, the extent to which curricula in STEM fields are expecting students to interpret graphs based on points, lines, or regions is an open question.

Implications and Feedback Questions

We anticipate the types of CSs (spatial v. quantitative; Cartesian v. non-Cartesian), the number of RFs, the types of RFs (continuous v. ordered discrete), and the object(s) students are expected to interpret (points, lines, regions) within the graph space will prove explanatory when examining the ways STEM curricula use graphs to represent phenomena. By exploring the prevalence and function of these different features of graphs in STEM curricula, we can develop curricular materials that math teachers can use to better prepare students to interpret (or construct) graphs in their future undergraduate STEM coursework. Further, by comparing the different types of graphs used across STEM fields, we may identify tendencies for certain fields to use certain types of graphs and develop curricular materials specific to those fields to better prepare students to interpret graphs as curricula developers intend.

Although we believe our criteria will successfully characterize many types of graphs we encounter in STEM textbooks, we also expect there will likely be other significant features of graphs of which we are currently unaware. Thus, we have several feedback questions related to our current framework: Are there important features of graphs across STEM fields that our framework may not capture? If so, what are they? Are there other extant frameworks characterizing graphical features that may be useful in this study?

We also conjecture that our framework may be useful in analyzing graphs beyond curriculum. For instance, we are curious if RUME attendees have insights into the question: How might our current (or a revised) framework be used in analyzing graphs used in STEM instruction? Would this framework be useful in analyzing the ways in which instructors expect students to interpret or construct graphs? Hence, we hope to obtain feedback that will inform both the theoretical framework and new avenues we can extend this work into to better explore the use of graphs across undergraduate STEM fields.

Acknowledgments

This material is based upon work supported by the NSF under Grant No. DRL-2200778 and DRL-2142000.

References

Altindis, N., Bowe, K. A., Couch, B., Bauer, C. F., & Aikens, M. L. (2024). Exploring the role of disciplinary knowledge in students' covariational reasoning during graphical interpretation. *International Journal of STEM Education*, *11*(1), 32. https://link.springer.com/content/pdf/10.1186/s40594-024-00492-5.pdf

Bagossi, S. (2024). Second-order covariation: enlarging the theoretical framework of covariational reasoning. *Mathematical Thinking and Learning*, 1-22. https://www.tandfonline.com/doi/pdf/10.1080/10986065.2024.2393889

Cohen, L., Manion, L., & Morrison, K. (2007). Research methods in education (6th ed.). Routledge. https://doi.org/10. 4324/9780203029053

Glazer, N. (2011). Challenges with graph interpretation: A review of the literature. *Studies in Science Education, 47*(2), 183–210. https://doi.org/10.1080/03057267.2011.605307

Hennion, D. (2025). *Soil texture triangle: Soil composition*. LearnDirt. Retrieved from: https://learndirt.com/learn/soil-texture-triangle-soil-composition/

Hibbeler. (2013). Engineering mechanics: Statics (13th ed.). Pearson.

Jones, S. R. (2022). Multivariation and students' multivariational reasoning. *The Journal of Mathematical Behavior*, *67*, 100991.

Joshua, S., Musgrave, S., Hatfield, N., & Thompson, P. W. (2015). Conceptualizing and reasoning with frames of reference. In T. Fukawa-Connelly, N. E. Infante, K. Keene & M. Zandieh (Eds.), *Proceedings of the 18th Meeting of the MAA Special Interest Group on Research in Undergraduate Mathematics Education*, (pp. 31-44). Pittsburgh, PA: RUME.

Knorr-Cetina, K., & Amann, K. (1990). Image dissection in natural scientific inquiry. *Science, Technology, & Human Values*, *15*(3), 259-283. https://journals.sagepub.com/doi/pdf/10.1177/016224399001500301

Lai, K., Cabrera, J., Vitale, J.M. et al. Measuring graph comprehension, critique, and construction in Science. *Journal of Science Education and Technology 25,* 665–681 (2016). https://doi-org.udel.idm.oclc.org/10.1007/s10956-016-9621-9

Larkin, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive science*, *11*(1), 65-100. https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1551-6708.1987.tb00863.x

Lee, H. (2017). Students' construction of spatial coordinate systems (Unpublished doctoral dissertation). University of Georgia, Athens, GA.

Lee, H. Y., & Guajardo, L. R. (2023). A content analysis of tasks involving two-dimensional Cartesian graphs in grade 6 – 8 U.S. textbooks. *Investigations in Mathematics Learning*, *15* (3), 222–240.

Lee, H. Y., Hardison, H. L., & Paoletti, T. (2020). Foregrounding the background: Two uses of coordinate systems. *For the Learning of Mathematics*, *40*(2), 32-37.

Lee, H. Y., Moore, K. C., & Tasova, H. I. (2019). Reasoning within quantitative frames of reference: The case of Lydia. *The Journal of Mathematical Behavior*, *53*, 81–95. https://doi.org/10.1016/j.jmathb.2018.06.001

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60*(1), 1-64.

Lemke, J. (1998). Multiplying meaning. *Reading science: Critical and functional perspectives*

on discourses of science, *87*.
Levinson, S. C. (2003). *Space in language and cognition: Explorations in cognitive diversity* (Vol. 5). Cambridge University Press.
Meira, L. (1995). The microevolution of mathematical representations in children's activity. *Cognition and instruction*, *13*(2), 269-313. https://www.jstor.org/stable/pdf/3233716.pdf
Mevarech, Z. R., & Kramarsky, B. (1997). From verbal descriptions to graphic representations: Stability and change in students' alternative conceptions. *Educational Studies in Mathematics*, *32*(3), 229-263. https://link.springer.com/content/pdf/10.1023/A:1002965907987.pdf
Moore, K. C., Paoletti, T., & Musgrave, S. (2013). Covariational reasoning and invariance among coordinate systems. *The Journal of Mathematical Behavior*, 32(3), 461-473.
Moore, K. C., Paoletti, T., & Musgrave, S. (2014). Complexities in students' construction of the polar coordinate system. *The Journal of Mathematical Behavior*, *36*, 135-149.
Moore, K. C., Paoletti, T., Gantt, A. L., Olshefke-Clark, A., Asiamah, O. A., & Yasuda, S. (2025). *Quantitative meanings for graphs: States, transformations, and gross/quantified covariation.* Paper presented at the forty-seventh annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. State College, PA.
Moritz, J. (2003). Constructing coordinate graphs: Representing corresponding ordered values with variation in two-dimensional space. Mathematics Education Research Journal, 15(3), 226-251.
Nation’s Report Card (2005) Item Maps. Retrieved from: https://www.nationsreportcard.gov/itemmaps/?subj=MAT&grade=8&year=2005.
Olshefke-Clark, A., Paoletti, T., Margolis, C., Lee, H. Y., Hardison, H. L., & Gantt, A. L. (2024). Students’ meanings for coordinate systems: Continuous and ordered-discrete reference frames. In Kosko, K. W., Caniglia, J., Courtney, S. A., Zolfaghari, M., & Morris, G. A., (Eds.) *Proceedings of the forty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Kent State University (pp. 475-484).
Olshefke-Clark, A., Paoletti, T., Margolis, C., Lee, H. Y., Hardison, H. L., & Gantt, A. L. (accepted). *Continuous and ordered-discrete: A distinction in how students construct reference frames*. Journal for Research in Mathematics Education
Panorkou, N., & Germia, E. (2023). Young students' forms of reasoning about multiple quantities. *For the Learning of Mathematics*, *43*(1), 19-23.
Paoletti, T., Asiamah, O. A., Lee, H. Y., Hardison, H. L., Tasova, H. I., Olshefke-Clark, A., Gantt, A. L., Margolis, C., Gaspard, B. R., Zolt, H., & Bui, M. (2026, February). *One student’s differing meanings for points in spatial and quantitative coordinate systems.* Paper presented at the forty-seventh annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. State College, PA.
Paoletti, T., Lee, H.Y., Rahman, Z., Vishnubhotla, M., & Basu, D. (2020) Comparing graphical representations in mathematics, science, and engineering textbooks and practitioner journals. *International Journal of Mathematics Education in Science and Technology*, 1-20. https://doi.org/10.1080/0020739X.2020.1847336
Perie, M., Grigg, W., & Donahue, P. (2005). The Nation's Report Card: Reading, 2005. NCES 2006-451. National Center for Education Statistics.

Piaget, J., & Inhelder, B. (1967). *The child's conception of space* (F. J. Langdon & J. L. Lunzer, Trans.). The Norton Library.

Piaget, J., Inhelder, B., & Szeminska, A. (1960). *The child's conception of geometry.* New York: Basic Books, Inc.

Potgieter, M., Harding, A., & Engelbrecht, J. (2008). Transfer of algebraic and graphical thinking between mathematics and chemistry. *Journal of Research in Science Teaching: The Official Journal of the National Association for Research in Science Teaching, 45*(2), 197-218.

Purdue University Department of Chemistry. (n.d.). *Phase diagrams*. Retrieved from: https://chemed.chem.purdue.edu/genchem/topicreview/bp/ch14/phase.php

Roth, W. M. & Bowen, G. M. (1999). Complexities of graphical representations during lectures: A phenomenological approach. *Learning and Instruction, 9*, 235–255.

Roth, W. M. & Bowen, G. M. (2003). When are graphs worth ten thousand words? An expert-expert study. *Cognition and Instruction, 21*(4), 429-473.

Ruf, V., Thues, D., Malone, S., Kuechemann, S., Becker-Genschow, S., Vogel, M., ... & Kuhn, J. (2024). A systematic review of empirical research on graphing numerical data in K-12 STEM education. *arXiv preprint arXiv:2411.13195*. https://arxiv.org/pdf/2411.13195

Tabachneck-Schijf, H. J., Leonardo, A. M., & Simon, H. A. (1997). CaMeRa: A computational model of multiple representations. *Cognitive science*, *21*(3), 305-350.

Taylor, H. A., & Tversky, B. (1996). Perspective in spatial descriptions. *Journal of Memory and Language, 35*(3), 371-391. https://doi.org/10.1006/jmla.1996.0021

The University of Waikato Te Whare Wananga o Waikato (2021). *Vertical structure of the atmosphere*. Science Learn. https://www.sciencelearn.org.nz/images/268-vertical-structure-of-the-atmosphere

Thompson, P. W. (2011). Quantitative reasoning and mathematical modeling 1, 2. New perspectives and directions for collaborative research in mathematics education, 33.

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421-456). National Council of Teachers of Mathematics.

Zimmerman, C., Olsho, A., Loverude, M., & Brahmia, S. W. (2023). Expert covariational reasoning resources in physics graphing tasks. https://arxiv.org/pdf/2306.00921