Multiplying Fractions with Sunlight: A Scaling Model

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Physical models are frequently used in elementary mathematics classrooms to support students' understanding of multiplication. These models are discrete, and many represent multiplication as repeated addition, area or an array. Thompson and Saldanha (2003, p. 104) found that "thinking about multiplication as repeated addition leads to severe difficulties in later grades" and Hurst (2015, p. 11) suggested that "multiplication as scaling…is a key aspect of thinking that needs to be developed." McLoughlin and Droujkova (2013) developed a diagrammatic definition of multiplication using parallel lines to scale the length of one segment by the length of a different segment. We developed the SunRule, an interactive device that harnesses parallel rays of the sun to perform multiplication. Beams of light cast shadows from gnomons onto a measuring plane; by varying the tilt and height different multipliers, multiplicands, and products can be generated.

This proposal extends research with K-8 preservice teachers reported in (Dimmel et al., 2021) where participants made sense of multiplication as a scaling, continuous operation. They used informal language to describe the relationship between changes in the multiplier, multiplicand, and product - that they were growing (or shrinking) in similar ways. While not specifically mentioning ratios, the preservice teachers described the shadows as changing in ways that were multiplicative or proportional in nature.

The study will investigate the use of the SunRule specifically to explore fractions, using a new model that zooms in to numbers between 0 and 1. In this version the gnomons and the shadow plane are partitioned into fractions of a whole (typically eighths or twelfths, but it could be any increment). The unit gnomon is scaled down to fractions between 0 and 1 by tilting the device, and the multiplier is scaled between 0 and 1 by changing the height of the gnomon. We ask the question: "How do preservice teachers make sense of multiplying fractions between 0 and 1 when using a dynamic, continuous, scaled model?" One significant change/improvement of the new model is that every rational product falls on the shadow plane, even if it doesn't fall on a specified increment. We also ask: "How do preservice teachers make sense of rational products cast as shadows that do not fall on a ruled increment?"

Figure 1. 3 x 4 = 12

Figure 2. 1/2 x 2/3 = 1/3

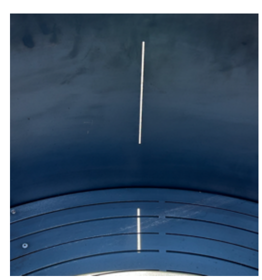

Figure 3. 1/2 x 3/8 = 3/16

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