Characterizing Instructor Questions Intended to Support Quantitative Reasoning

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Meanings for the definite integral are most productive when rooted in quantitative reasoning. However, little is known about how instruction may effectively support these meanings. This study examines the discourse actions of one instructor who intended to promote students' quantitative reasoning in a lesson exploring ideas foundational to the definite integral. We draw upon video data of the instructor's conversations with three undergraduate students who were working together during class. We analyzed whether his conversational turns could promote quantitative reasoning and characterized them according to a discourse action typology. Results suggest most of his discourse actions did opportune quantitative reasoning and that different types of discourse actions had the potential to elicit quantitative reasoning in distinct ways. We also identify two additional discourse actions not in the original typology. We conclude with implications for practice and suggestions for future research.

Keywords: quantitative reasoning, discourse, definite integral, instructional practice

A burgeoning body of research suggests quantitative reasoning (QR) is crucial to developing productive meanings for the definite integral (see Jones & Ely, 2023). This work has led to rich descriptions for meanings about the definite integral that are useful across various situations as well as documentation of the strengths and limitations students bring to bear as they develop these meanings (e.g., Oehrtman & Simmons, 2023; Sealey, 2014; Stevens & Jones, 2022). Given most studies investigate the development of students' meanings via teaching experiments or clinical interviews which center only one or two students at a time, additional work is needed to understand how these findings can inform effective instructional practice in classroom settings.

Research suggests larger groups of students have developed these meanings in classroom settings (Ely, 2019; Lehmann, 2024), but little is known about the instructional practices that may have supported students to develop these meanings. It is important to highlight that each class in these studies was taught by that study's author, and we presume that their instructional practices are influenced by a deep background knowledge about this body of research. To help prepare instructors who are less familiar with this literature to effectively facilitate students' developing QR related to the definite integral in whole class contexts, it would be useful to study the practice of instructors who intend to foster QR in this area.

With this goal in mind, we explored an instructor's practice during a lesson on accumulation from rate of change (one approach to developing QR with the definite integral; e.g., Thompson, 1994). This lesson took place within an undergraduate math course for preservice middle school math teachers that has developing quantitative and covariational reasoning as a central aim. Specifically, we address the research questions (RQs): *To what extent does an instructor's conversation with students have potential to elicit QR? In what ways does this occur?*

Conceptual Framework

Instructor discourse is widely accepted as an important facet of math instruction and has been studied extensively from various perspectives (e.g., Morgan, 2014; Ryve, 2011). We adopt Gee's (1996) definition of discourse as "connected stretches of language that makes sense" (p. 127), and we define discourse actions as the comments and questions an instructor verbalizes.

Researchers have proposed a variety of frameworks to characterize the kinds of discourse actions instructors pose at both the K-12 (DeJarnette, 2020) and post-secondary (e.g., Paoletti et al., 2018; Viirman, 2015) levels. Common across these frameworks and other research (e.g., Reiten et al., 2025) is that the types of discourse actions instructors enact are related to their goals for students. Hence, we hypothesize an instructor with a goal of promoting QR may pose certain types of discourse actions that they expect could elicit this type of reasoning from students. In line with Thompson (2011), we consider QR to include the mental construction of quantities, (i.e., measurable attributes in a situation), the relationships between quantities, and the process of solving problems based on quantitative operations.

In this study, we adopt Drageset's (2014) teacher discourse actions framework to identify trends in instructors' discourse actions that have the potential to elicit QR. This typology consists of 13 distinct discourse actions organized into four categories. Redirecting Actions (RAs) include questions or statements intended to redirect current ways of thinking. Progress Actions (PAs) include actions that seek to move problem-solving forward. Focusing Actions (FAs) include actions that draw students' attention to details in the problem. FAs considered Requests for Student Input (RSI) aim to elicit these details from students. In FAs considered Pointing Out Information (POI), the teacher provides these details. For brevity, we present discourse actions that constitute these categories in Table 1. See Drageset (2014) for more detail and examples.

Table 1. Drageset's (2014) Teacher Discourse Actions Framework

Discourse Action		Description
RAs	Put Aside	Dismissing a student's response
	Advising a New Strategy	Explicitly suggesting an alternative approach
	Correcting Questions	Asking a question that affirms something *and* suggests a deficiency
PAs	Demonstrating	Explicitly indicating the next step(s) in a process
	Simplifying	Providing hints or scaffolds
	Closed Progress Details	Asking how to move a process forward with a next step in mind
	Open Progress Initiatives	Asking for students' ideas of how to move a process forward
FA-RSI	Enlighten Details	Prompting students to elaborate elements of their explanation
	Justification	Prompting students to justify elements of their explanation
	Apply to Similar	Prompting students to replicate thought process in related situations
	Request Assessment	Prompting students to comment on a classmate's response
FA-POI	Recapping	Restating student's response and emphasizing important detail
	Noticing	Restating student's response and adding information

Methods, Analysis, and Tasks

Data collection occurred during an undergraduate math course for third-year, pre-service middle school math teachers at a large, public university in the Mid-Atlantic United States. Each course session was three hours long, and there were ten students enrolled. The second author, Dr. Paoletti, served as the course instructor. His research interests include QR, and this expertise deeply influences his teaching practice. Indeed, the primary goal of Dr. Paoletti's course was to support QR about core concepts spanning middle school to secondary and even post-secondary mathematics.

This report focuses on the fifth session of the course, which was a lesson on accumulation from rate. During this session, students worked together in pairs or trios to address the task sequence. The data for this study consists of video and audio recordings of the conversations and written work of one trio, Meghan, Amber, and Lydia (pseudonyms), all of whom consented to participate in the study. Relevant to this report, the recordings also captured the conversations the trio had with Dr. Paoletti, who circulated the room to assist and provide feedback to groups.

Analysis

We first isolated segments of the recordings during which Dr. Paoletti was interacting directly with the trio. We transcribed these segments and identified the conversational turns (i.e., a period of speech bounded by another's; Ford et al., 1996) in which Dr. Paoletti responded to student thinking. We coded this set of turns for discourse action type and potential to elicit QR.

To analyze the *extent* to which these turns had the potential to elicit QR, we used a combination of provisional (Saldaña, 2021) and open coding (Braun and Clark, 2006). That is, we began with a codebook that outlined criteria for whether a conversational turn had the potential to elicit QR. When existing criteria was not sufficient, we inductively developed additional criteria. To analyze *how* Dr. Paoletti promoted QR, we deductively coded (Saldaña, 2021) each turn using Drageset's (2014) framework. Throughout this process, Dr. Paoletti shared his "perspectives of practice" (Lampert, 2010) as the course instructor, which provided additional insight to the data. As we coded, we discussed discrepancies to come to a consensus and refine our codebook. This process also led us to generate two, new discourse action codes: Endorse and Counter-Justification. These codes capture elements of Dr. Paoletti' discourse that were not described by the existing framework. We describe these further in the Results. Finally, within each code, we identified patterns in the criteria with which conversational turns had been identified as having the potential to promote QR (or not).

Tasks

To provide additional context for the interactions described in the results, we briefly outline the tasks from this class session. The main purpose of these tasks was to extend the meanings for rate students had begun developing earlier in the course and to provide a foundation for QR with the definite integral. Each of the three tasks in this session leveraged relationships between speed, distance, and time by asking students to make more accurate estimates of distance using various representations of speed and time data. Students began with the *Which Racetrack?* task in which they determined the track a car may have completed based on speed and time data and the total length of each potential track. In *Where's the Car?,* students used instantaneous speed information from a video of a racecar's dashboard to track its location over time. Finally, in *Drag Race,* students used a model of a drag racecar's speed, i.e., $v(t) = 71 \times (1 - (1/3)^t)$, to write an accumulation function for the car's approximate distance from the starting line. See Olshefke-Clark (2025) for more detailed task information.

Results

We address our first RQ by summarizing the frequencies of the different discourse actions Dr. Paoletti employed as he responded to the trio's mathematical thinking and the percentages of turns with potential to elicit QR. We address our second RQ by describing trends in how Dr. Paoletti's different discourse actions had the potential to support QR.

Frequency of QR

Dr. Paoletti contributed 128 turns across 17 small group conversations with the trio. These conversations ranged in length from five seconds to three minutes and totaled approximately 24 minutes of the three-hour lesson. Table 2 summarizes the frequency of Dr. Paoletti' discourse actions organized by category as well as the percentage of turns in each category that we identified as having potential to elicit QR. Overall, the majority (62.1%) of Dr. Paoletti' turns had the potential to elicit QR. Moreover, the potential to elicit QR was substantial across all four of the discourse action categories; three of the four categories had over 55% of its turns tagged as having potential to promote QR, and the fourth had 45%.

Table 2. Breakdown of Dr. Paoletti's discourse actions

Category	Frequency	Frequency of Quantitative
Redirecting Actions	20	9 (45%)
Progress Actions	38	21 (55.3%)
Focusing Actions- Request for Student Input	48	34 (70.8%)
Focusing Actions- Pointing Out Information	26	18 (69.2%)
TOTAL	**132**	**82 (62.1%)**

To further characterize Dr. Paoletti' responses to the trio's mathematical thinking, Figure 1 organizes turns by code and potential to elicit QR. Notably, all discourse actions except Request Assessment from Other Students had at least one quantitative instance. Moreover, 10 of the 14 discourse actions had over 50% of instances coded as quantitative. In what follows, we describe ways Dr. Paoletti promoted QR by outlining trends in specific discourse actions that had especially high and low proportions of turns with potential to elicit QR.

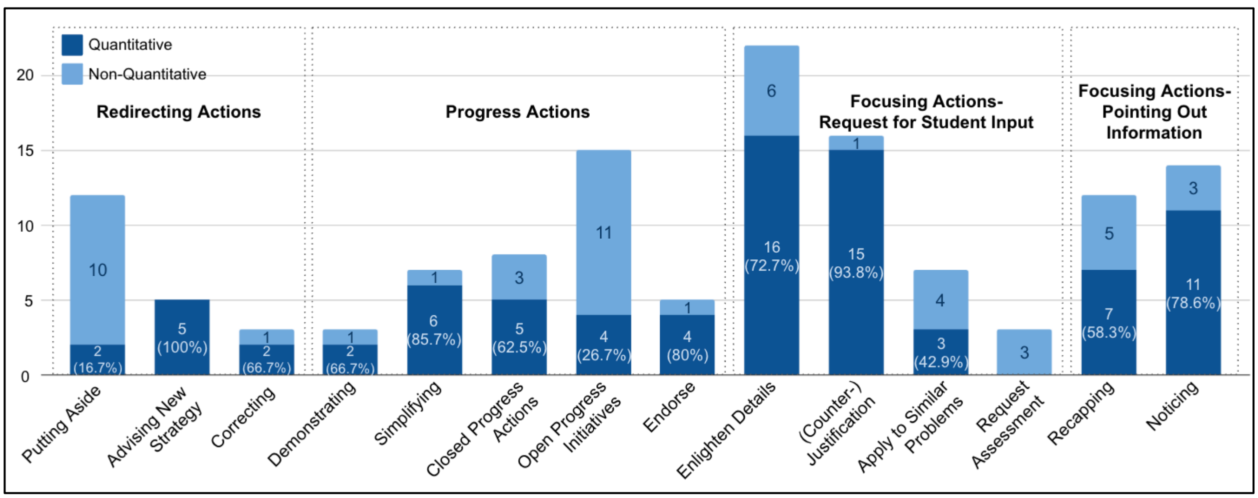

Figure 1: Frequency of (non-)quantitative conversational turns by specific discourse action

Redirecting Actions

RA was the only category with less than a majority (45%) of turns coded as potentially eliciting QR. This was also the smallest category overall, representing only 15.2% of total turns. RA also contained Advising a New Strategy, which was the only type of discourse action type in which 100% of turns had the potential to elicit QR. In these five turns, Dr. Paoletti urged the trio to consider the meaning of their processes in the situation, rather than calculate with no rationale. For example, in certain instances where the trio did not have a justification for their process, he advised them to change their strategy, exhorting, "don't just randomly multiply and divide things!" and "we can't just average things for the sake of averaging things." With these statements, Dr. Paoletti told the trio that their current strategy was insufficient and implied that they should pursue a process that was not "random" or done "just…for the sake of" it. These kinds of statements have the potential to elicit QR because, from Dr. Paoletti's perspective, a more intentional strategy should include considering how process related to the quantities in the situation and the relationships between them.

Progress Actions

PA turns made up over one quarter (28.8%) of total turns, and over half (55.3%) had the potential to elicit QR. Specific actions ranged from having 80% or more (Simplifying and Endorsing) to only about 25% (Open Progress Initiatives) of turns with potential to elicit QR.

Endorsing. During the process of coding Dr. Paoletti' turns, we identified a type of discourse action that was not captured by Drageset's (2014) existing framework. These turns explicitly encouraged students to continue progressing through a particular process by providing confirmatory feedback (i.e., endorsing) their current approach. We considered this discourse action to be a type of PA because they "serve to move the process forward" (Drageset, 2014, p. 298). This type of discourse action included statements like "I think now you're on track. Follow that line of reasoning further and see." Most (80%) of these codes had the potential to promote QR because Dr. Paoletti often endorsed the trio's explanations when they made suggestions for a next step that was motivated by quantities in the situation. In one instance, when the trio were proposing an idea to improve their estimate of the car's distance in *Drag Race,* Meghan shared, "We were talking about going, like, in between the two, like one and a half seconds." Dr. Paoletti replied, "So I think that's a good idea," thus endorsing a line of thinking that referenced situational quantities and encouraging them to progress along this path.

Open Progress Initiatives. Although Dr. Paoletti encouraged the trio's QR with endorsing turns, we found he demonstrated less potential to elicit QR when he asked the trio general questions about how they might progress. Similar to Put Asides, Open Progress Initiatives were often too vague or non-specific to be considered having potential to elicit QR. For example, on multiple occasions during *Where's the Car?* and *Drag Race*, Dr. Paoletti simply asked, "how can we get more accurate?" Though a potential (and perhaps ideal) response to this question could involve considering the relationships between quantities, we do not consider these questions of this nature to be clear in communicating to the trio that they should consider quantities instead of just calculation. Because Open Progress Initiatives are, by definition, open to students' authentic suggestions, it makes sense that these turns may not be dense in potential to promote QR. Dr. Paoletti was soliciting the trio's ideas for moving forward, not explicitly guiding them.

Focusing Actions – Request for Student Input

FA-RSI turns were both the most frequent (36.4% of total turns) and had the highest percentage of quantitative turns (70.8%). Interestingly, FA-RSI contains discourse actions that had both the highest (93.8%; (Counter-)Justification) and lowest (0%; Request Assessment from Other Students) densities of potential to elicit QR.

Request for Student Assessment. No turns that requested the trio to provide feedback on another member's response were identified as having the potential to elicit quantitative reasoning. However, very few (3) turns were coded as requests for student assessment, and it is possible that with more turns of this type, the potential to elicit QR could be observed. As a hypothetical example, if Dr. Paoletti had asked the trio to assess a group members' QR-rich explanation, this could be considered as having the potential to elicit QR.

(Counter-)Justification. This was the most common type of turn that Dr. Paoletti contributed, and all but one had the potential to elicit the trio's QR. Almost all of Dr. Paoletti prompts to justify were phrased as a "why?" question and asked the trio to explain how a process they had completed worked based on the relationships between quantities in the situation. Notably, when coding for Justification, there were multiple instances in which Dr. Paoletti asked the trio to explain why a process didn't work, which is slightly different from Drageset's (2014) definition of Justification as "questions about why the answer found or method used is correct" (p. 295). We consider Counter-Justification to be a special case of Justification because it elicits QR in a similar way but is specifically focused on why an answer or method is not correct.

As an example of how Counter-Justification was intended to promote QR, consider when the trio was explaining to Dr. Paoletti how they combined speed and time data in *Which Racetrack?* to come up with an estimate for distance which they believed was too large:

Meghan: I multiplied these two together for all of them and then added them.

Dr. Paoletti: Why shouldn't that work?

Meghan: Why shouldn't-? Because they're not equal speeds. Like, you're not doing 13 per- Well, 'cause you're multiplying meters by seconds by seconds. That's meters per second per second. And that's not what we're looking for.

Dr. Paoletti responded to Meghan's first explanation (which only referenced arithmetic calculations) with a request for Counter-Justification, asking why her process *would not* make sense. Although her response was not normative, it did explicitly reference quantities in the situation. As such, Dr. Paoletti' Counter-Justification both promoted and successfully elicited QR because it required Megan to interpret her arithmetic calculations in terms of the situation, which helped her consider why her process did not produce an expected result.

Enlighten Details. Almost three-quarters of Enlighten Details turns (72.7%) provided opportunities to engage in QR. The details that Dr. Paoletti asked the trio to enlighten were often prompts to connect their calculation to relevant quantities in the situation. In some cases, he paused the trio in their explanation of their process and prompted them to add units to a number they referenced (e.g., "3.7 what?"). In others, he asked more generally what a number represented in the context of the task (e.g., "what do these numbers represent again?"). In either case, these prompts encouraged them to keep quantities at the forefront of their explanations.

Focusing Actions – Pointing Out Information

Almost 70% of FA-POI turns had the potential to elicit QR, and both types of discourse actions that compose this category had a majority of turns with the potential to elicit QR. Recall that in FA-POI turns, the instructor himself is pointing out important information. It makes sense, given his overarching goals, that Dr. Paoletti would frequently point out information related to quantities and their relationships, thus giving students opportunities to engage in QR.

With Noticing actions, that is, instances where Dr. Paoletti paused the trio's progress to emphasize an important detail in their explanation, he often reiterated and/or expanded upon elements of QR present in their explanation. For example, consider the exchange:

Lydia: Like, the difference between it is 24.

Dr. Paoletti: The difference of what? [Enlighten Details]

Lydia: The difference in speed.

Dr. Paoletti: It's the difference in speed, right? [Noticing]

Dr. Paoletti used a Noticing action to emphasize the quantity that Lydia had added to her response to his Enlighten Details question. As such, this provided the potential to elicit QR by further pointing out that this detail of Lydia's explanation was important. We note that although in some cases, this kind of discourse action may provide implicit endorsement of their approach, we consider this distinct from an Endorsing actions because they are not explicit affirmations.

Dr. Paoletti's way of opportuning QR was similar across instances of Recapping as well, except Recapping actions focused on the conclusion of a process rather than intermediate steps.

Discussion

This report provides an account of the discourse actions posed by one instructor in a whole class setting who had a goal of promoting QR about ideas foundational to the definite integral. Specifically, we explored the extent to and ways in which Dr. Paoletti's conversational turns had the potential to elicit QR for a small group of students in his class. We found that the majority (62.1%) of Dr. Paoletti's discourse actions had the potential to elicit QR, suggesting that his discourse did align with his overall goal. Moreover, we illustrated different ways that Dr. Paoletti presented opportunities for the trio to engage in QR across various types of discourse actions.

Our results suggest there is not a single type of question or statement that instructors need to rely on to elicit students' QR; Dr. Paoletti posed 14 types of discourse actions, and he provided opportunities for the trio to engage in QR with all but one. However, FAs may be a particularly important skill for instructors to hone. We highlight FA-RSI and FA-POI had the highest densities of potential to promote QR, and together these made up the majority of Dr. Paoletti's turns. Dr. Paoletti used FAs to direct the trio's attention to relevant quantities in the situation and pushed them to explicitly include the quantities and the relationships between them in their explanations of calculations and results. Prior research (e.g., DeJarnette et al., 2020) indicates that teachers have particular difficulty posing questions and comments that "mak[e] specific connections or relationships explicit" (p. 9). Practicing incorporating FAs, which can be as simple as asking students a Counter-Justification, "why not?," is one way that instructors can promote QR in class by pushing students to explicitly consider relationships and connections.

Additionally, our identification Counter-justification and Endorsing actions contribute to research on instructor discourse more generally by extending Drageset's (2014) framework. We do not believe these two discourse actions are particular to classes emphasizing QR and conjecture both discourse actions could be leveraged by teachers with other foci.

Implications and Future Directions

It is important to emphasize this study does not draw a link between the types of questions Dr. Paoletti poses and their impact on the trio's QR. Rather, we intended for this study to provide an illustration of how instruction focused on promoting QR may be enacted and to begin a conversation on bridging existing research on students' QR about the definite integral with instructional practice in the undergraduate math education space. Additional work is needed to understand what types of questions and other learning opportunities actually result in development in students' QR related to the definite integral.

Though understanding the questions that an instructor poses to support QR may support other instructors in doing the same, DeJarnette et al. (2020) remarked that "even if teachers learn to pose questions that should elicit meaningful explanation, justification, and mathematical relationships, these questions will not make productive contributions if students do not recognize or respond to the demands" (p. 10). Thus, future research should attend to how these QR-focused discourses are co-constructed with students and consider questions such as: How do students respond to questions intended to elicit QR? How do instructors respond to normative and non-normative student responses? How might students' take up instructors' QR-promoting discourse actions in their discourse with one another? Investigating questions such as these in whole-class settings can provide important insights about how instructors can support students in developing meanings about the definite integral that are productive in their future coursework and careers.

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