The Nature of a Community of Practice: Social Expectations at an Informal Summer Mathematics Program

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We present results from a pilot study conducted to inform possible directions for further investigation focused on a specific community of practice: an informal summer mathematics program for mathematically talented students. While the program is attended by high school students, they take undergraduate level proof-based courses and engage in research at a level comparable to a research experience for undergraduates. Journals and interviews were used to investigate aspects of the program using Wenger's Communities of Practice theoretical framework, which describes social characteristics of a community of practice in terms of mutual engagement, joint enterprise, and shared repertoire. Suggestions for feedback from the reader and audience are included.

Keywords: Communities of Practice, Informal Educational Context, Undergraduate Research

A plethora of communities exist around mathematics, both educational and professional, and it is worthwhile as researchers to describe the diversity of these communities. Wenger (1999) describes the importance of Communities of Practice (CoP): "Communities of practice are the basic building blocks of a social learning system because they are the social "containers" of the competences that make up such a system." (p. 4). The CoP framework is a useful analytic tool to study how knowledge is created and competency is assigned within a community. From a situated learning perspective, Lave and Wenger (1991) describe legitimate peripheral participation as a tool to study learning within CoPs. The purpose of this work is to explore the way knowledge is generated within an informal summer mathematics program (ISMP) by describing essential aspects of the community and observing the legitimate peripheral participation of its members. The purpose of this paper specifically is to overview the early stages of the project including the initial round of data collection and preliminary observations about the community of practice.

Components of a Community of Practice

Wenger (1998) establishes that CoPs combine three elements: mutual engagement, a joint enterprise, and a shared repertoire. The first component, mutual engagement, refers to when members interact and establish norms for how they will engage in their practice. Mutual engagement creates relationships among members of a CoP. However, Wenger notes that engagement may not always be positive. It is possible for conflict or tension to characterize the engagement of a community. The second component, a joint enterprise, refers to agreement among members of the community about what they're doing, what their goals are, and they hold each other accountable to those expectations. Wenger describes how the joint enterprise is established, "It is defined by the participants in the very process of pursing it." (Wenger, 1998, p. 77). Finally, shared repertoire refers to resources that are shared with all members such as specific vocabulary, artifacts, and experiences. Wenger points out that a shared repertoire reflects a history of mutual engagement and remains inherently ambiguous meaning it can be reused or applied in novel ways. Biza et al. (2014) describe transforming the CoP framework into *Communities of Inquiry* (CoI) (Goodchild et al., 2013), which they explain to be, "… a CoP in

which inquiry is a fundamental way of being in practice." (p. 164). In a CoI, inquiry manifests itself in all three components of a CoP. Members engage in inquiry-based processes, their goals are aligned with better understanding of concepts, and they share inquiry-based tasks and approaches (Biza et al., 2014).

A range of empirical work has been done using the CoP theoretical framework to study undergraduate participation in mathematics and related fields. Solomon (2007) reports on a study examining undergraduate students' relationship to mathematics within the context of specific communities. Solomon observed a lack of students who identify as Legitimate Peripheral Participants and attributed this scarcity to the dominant discourses present in the community. The continuation of the project in this paper may provide further insight into barriers to legitimate peripheral participation undergraduate students face.

The following sections describe an ISMP using the CoP theoretical framework by giving examples of a joint enterprise, mutual engagement, and shared repertoire within the program. Specifically, we address the following research questions:

1. What social values or expectations do the students hold for themselves and their classmates regarding the way in which the ISMP community engages with mathematics?
2. What is the nature of mutual engagement, joint enterprise, and shared repertoire in the context of informal mathematics communities?

Methods

Context

The data used in this analysis was collected from students participating in a six-week ISMP in the southwestern United States. The program invites students who demonstrate excellence in mathematics to participate for up to three summers during their high school years. Students take the following undergraduate level mathematics courses taught by faculty at the host institution: number theory and Python programming (first year), abstract algebra and real analysis (second year), and topology and combinatorics (third year). Second- and third-year students also undertake research projects supervised by faculty at the host institution. Students spend their afternoons and weekends engaging in a variety of recreational activities and field trips.

After graduating from high school, students may return to join a group of undergraduate and graduate mentors. The mentors are responsible for holding daily problem sessions with eight students to review material from class as well as nightly study groups with four students to work on problem sets. In addition to their academic responsibilities, the mentors, particularly those who were previously students in the program, also uphold the camp culture by organizing activities and honoring traditions.

Data and Participants

Students submit two journal entries reflecting on their ongoing experiences. The first journal entry is after the second week, and the second journal entry is after the fourth week. The journal prompts were written by the program faculty with the purpose of ensuring all students are supported mathematically and socially throughout the summer. For example, some questions inquire about the pacing of the courses, some about the dynamics of their study group, and some about recreational activities. While the prompts were not written specifically for this project, the questions the students are asked to reflect upon are still aligned with the goals of the research.

Journals written by fifteen first-year students, thirteen second-year students, and four third-year students were collected for this project.

In addition to the journals, nine students were chosen to participate in semi-structured interviews. The students were chosen based on availability out of the group of second- and third-year students. This allowed for questions about research, which we know from prior experience to be a treasured component of the experience for returning students. A common interview protocol was used for all participating students; however, for several students, questions were added based on their journal entries to give the students the opportunity to elaborate on their written responses. The interviews were conducted during the fifth and sixth weeks of the program. Each interview lasted approximately 30 minutes. Zoom was used to record the audio and generate transcripts. The interviews were conducted by the first author, who was familiar to the participants as a mentor and staff member at the program. The study was supervised by the second author, who is on the program faculty. It is important to note that the students were aware that their journal entries would be read by the two authors and by other faculty in the program.

Results

In this section we provide examples of themes in the data regarding the students' values and expectations, which point to possible directions for future analysis. The themes reported below were selected when several students shared the same or similar sentiments in response to the journal or interview prompts. The first theme is related to mutual engagement, the second is related to the students' joint enterprise, and the third is related to their shared repertoire.

Mutual Engagement: During the Program, Students Expect to Work Collaboratively.

During the interviews, a conversation contrasting the way students do math at the program versus the way they do math at school often came up. We did not include an interview question specifically aimed at this; this theme emerged in response to different prompts depending on the participant. The majority of participants noted the collaborative nature of mathematics at the summer program compared to school. The following are two illustrative examples.

One student, Eugene, described why he returned for a second summer:

> "I thought like being in a study group with other people and like working on problems together, instead of just like being at home and doing all the problems myself, it's a lot more interesting. And I think it's like a lot more enjoyable, so that's why I came back."

Eugene returned to camp with the expectation that his study group would be collaborative. Later in the interview, Eugene compared his first-year study group to his second-year study group. He described how the first-year group would work together to solve all the problems, use the chalkboards, and bounce ideas off each other. Sounding somewhat disappointed, he shared that his second-year study group was often quiet, and students in the group worked on their own. Eugene's disappointment in his second-year group indicated that his expectation to collaborate with his study group was not met, and their dynamic was abnormal.

Another student, Miles, shared that the summer program allowed him to find friends who shared his interest in math. When asked why it is nice to have friends with the common interest, he shared,

> "…math can definitely get bland at some point if you're working on the same problem for like hours on end. So, like just having another person, even if they're not like helping

doing the same problem as you, just having another person doing something similar and like able to help or talk to about the problem is nice."

Miles came to camp with the expectation that others there would share his interest in math and provide collaborative support that he did not have at home.

The students' shared expectation that they will work together with their peers suggests that collaborating mathematically is a social norm at the summer program. They come to the program expecting that their classmates will engage with them in problem solving. Further investigation into and analysis of the types of socially expected collaboration will help inform the characterization of mutual engagement at the program.

Joint Enterprise: CoP Members are Committed to Supporting Each Other's Learning.

Many of the students wrote in their journals or shared in their interviews about the techniques they used to support their own learning and the learning of others. As above, this was not an explicit question; it came up in a variety of contexts depending on the student.

One expectation set by the faculty at the program is that mentors provide students with written feedback on their proofs (often referred to as "grading" even though no numerical grades are assigned) and assign several proofs per problem set for the student to rewrite and resubmit. One student, James, created his own strategy for responding to feedback. When prompted in the second journal about the feedback he received from his mentor, James described how, rather than just rewriting the proofs on his own, he preferred to think about the feedback he received and have a discussion with his mentor about it during study group. He wrote, "This turns out to be a very productive strategy for me, as rather than 'doing corrections,' we can instead have constructive conversations in person during study group." James' reflection is notable for two reasons. First, his motivation for adopting this new strategy of responding to feedback was his own desire to be more productive and engage with the ideas in a more meaningful way. Second, his mentor willingly adopted the new way to engage James with feedback, going against the faculty's expectations, but honoring the fact that it supported James' learning.

Another example of the joint enterprise is from the perspectives of two students, Kaitlyn and Aaron, who were in a study group together. Kaitlyn wrote in her second journal that their study group was very collaborative, often solving all the problems together on the board. However, she claimed that Aaron was the most "advanced" out of all of them, so his contributions were often diagrams or hints to help the others arrive at the solution on their own. Aaron also wrote about this dynamic in his first journal. He says, "I usually tend to guide the discussion, but when I realize I am being too overbearing, I often decide to let the others figure out the problem without me." Both Kaitlyn and Aaron were asked follow-up questions about this dynamic. Kaitlyn expressed her appreciation of Aaron's approach, and Aaron shared that he used it because he thought it created a better learning opportunity for his group mates. These examples suggest that the students in the program have a shared purpose to engage deeply with mathematical ideas and support one another's endeavors to do so. More investigation into this shared value and its manifestation across contexts will be useful in describing the joint enterprise of the students in the program.

Shared Repertoire: The Program Provides Students With Shared Artifacts and Experiences.

During this program, students participate in intense mathematical exploration together as well as build memories through recreational activities and free time. During their courses and

research, they receive and create shared artifacts including their problem sets, proofs, and past research projects. An illustrative example of a shared artifact comes from the interviews with Eugene and Miles. When asked how their research groups chose their respective topics, they both explained that their groups built upon the research of groups from previous summers. Both referenced a shared document listing past projects, and future directions students could go. Eugene explained that his group was working on extending a result by a previous group, who showed a property is true for primes, to now be true for all natural numbers. The shared artifact in this case is the previous result and proof, which was available to Eugene's research group. Further investigation is warranted into other shared artifacts of the community, as well as the shared stories which live on in the community members' memories.

Discussion and Future Work

The examples above are the result of the early stages of an investigation into the community of practice at the summer mathematics program. From the journals and interviews, there are clear examples of the three aspects of a community of practice: mutual engagement, joint enterprise, and shared repertoire. This investigation is also practically useful for future study design. For example, the summer 2025 data collection provided an estimate of how many participants to expect in the future and what logistic challenges may arise. Guiding feedback on the direction for future data analysis would be appreciated from the readers and audience, as well as feedback on the theoretical framing of CoPs.

Future work will continue to build descriptions of the three aspects of the community of practice for this ISMP, examine how they influence each other, and ultimately lead to a useful way of understanding the way learning takes place in the context of the program. The results from future work will inform disciplinary knowledge about summer programs as well as other non-traditional communities within undergraduate mathematics such as REUs, clubs, and cohorts of mathematics majors. Intended future steps of this work include collecting more journal and interview data as well as observational data during the summer 2026 program and develop a coherent analytic approach to triangulate themes in the journals, interviews, and observations.

References

Biza, I., Jaworski, B., & Hemmi, K. (2014). Communities in university mathematics. *Research in Mathematics Education*, *16*(2), 161-176.

Goodchild, S., Fuglestad, A. B., & Jaworski, B. (2013). Critical alignment in inquiry-based practice in developing mathematics teaching. *Educational Studies in Mathematics*, *84*(3), 393-412.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge university press.

Solomon, Y. (2007). Not belonging? What makes a functional learner identity in undergraduate mathematics? *Studies in Higher Education*, *32*(1), 79-96.

Wenger, E. (1998). Communities of practice: Learning as a social system. *Systems thinker*, *9*(5), 2-3.

Wenger, E. (1999). Communities of practice: Learning, meaning, and identity. Cambridge university press.

Wenger, E. (2012). Communities of practice: the structure of knowledge stewarding. In *Knowledge Horizons* (pp. 205-224). Routledge.