Context Dependence of Student Responses and Explanations on Dot Product Magnitude Tasks

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Preliminary findings from a study exploring student understanding of vector products are presented. We discuss multiple-choice and free-response data from 118 students who completed four dot product magnitude tasks across two semesters of introductory physics instruction. Each semester had a physics-free dot product along with a physics context relevant to the content of each course (mechanical work and electric flux). Performance ranged from 28% to 59% and students do not always answer consistently between pairs of tasks. Coding of explanations revealed four main computational strategies used by students along with two more conceptual reasoning categories. The most common strategy was students using a trigonometric definition of the dot product with varied results depending on the context.

Keywords: vector products, student thinking, physics

Vector quantities are ubiquitous in physics, appearing very early in an introductory physics course and continuing to be used extensively throughout an undergraduate physics curriculum. Since mathematics curricula do not usually cover vectors until the third semester of calculus, it is necessary to explore the use of vectors in a physics setting. A body of work in physics education has explored student understanding of vectors (e.g., Knight, 1995; Nguyen & Meltzer, 2003; Flores et al., 2004; Gire & Price, 2014). Our research questions for this report are: How does context impact student approaches to vector product tasks? What aspects of physics contexts influence student reasoning? While this work is part of a longitudinal study of how student understanding of vectors evolves over time, only introductory-level data is presented here.

Preliminary Nature of the Report

We frame this work from a mathematical modeling perspective, looking at different steps in a modeling process that at its broadest includes connecting the physical scenario and the associated mathematical entities as well as performing mathematical operations (e.g., Redish & Kuo, 2015; Uhden et al., 2012; Zbiek & Conner, 2006). We are interested in the role of context on student understanding of vector products and on how students reason about the relevant mathematical operations. A more fine-grained analysis of student reasoning and modeling with our data has not been completed. In addition to framing via modeling, we believe that our findings are well suited for analysis via dual-process theories of reasoning (Evans, 2006; Heckler, 2011; Kryjevskaia et al., 2014; Spiers et al. 2021), in particular the impact of visual and contextual features on student reasoning. A broad question for readers: What other perspectives and/or frameworks might provide additional insight into student reasoning and especially the consistency of their reasoning across multiple tasks?

Background

The dot product (or scalar product) of two vectors, $\vec{A} \cdot \vec{B}$, can be determined in two ways, either as $AB\cos\theta$, where θ is the angle between the two vectors when placed tail-to-tail, or as the sum of the corresponding components, $A_xB_x + A_yB_y + A_zB_z$.

Work on student understanding of vectors has explored both computational and conceptual issues. On the computation side, Mikula & Heckler (2013) investigated students' ability to calculate vector components and found that angle placement impacted performance. Heckler & Scaife (2013) compared different notations and saw better performance on algebraic problems than problems using arrow notation. Van Deventer (2008) and Carli et al. (2020) developed isomorphic mathematics and physics questions and observed context dependence of student strategies. Carli et al. reported that the vector product tasks were most challenging among vector concepts in both the math and physics contexts with a maximum performance of 41%. Barniol & Zavala (2014) developed the Test of Understanding of Vectors, a multiple-choice assessment on procedures and properties of vectors without physics contexts. On a more conceptual level, Zavala & Barniol (2013) asked students about the best interpretation of the dot product in three different contexts. They found no significant difference between the results for the two physics contexts (work and flux) but did see a significant difference between the "no context" question and the physics contexts, with better performance in the physics contexts.

Several of the dot product items in the literature have answer choices that are a mix of scalars and vectors, which could conflate evidence of student understanding of magnitude and direction. To disentangle calculation errors from reasoning about the vector or scalar nature of the results, our tasks had separate questions about the magnitude and direction of each product. In matched data on the dot product direction tasks we found no significant shift between correct and incorrect answers, but there was a significant shift from the common incorrect answer to other choices depending on the context and aligned with the physics (Molinari & Thompson, 2025). While the studies above demonstrate that context influences responses, not all these studies have matched data, and we are interested in not only how entire populations complete different tasks but also how individuals respond in different contexts.

Methods

Online surveys were administered after relevant instruction in calculus-based introductory physics in both semesters of the same academic year at a medium-sized R1 institution. Each survey asked separate questions about the magnitude and direction of a vector product (dot or cross) in a "math" context with no physical scenario and a physics context. The physical dot product contexts were *mechanical work* in the first semester (PHY1) and *electric flux* in the second semester (PHY2). *Work*, a common application context in calculus, is the dot product of force and displacement; *electric flux* is the dot product of the electric field and an area.

Each problem statement included a diagram for the situation (Figure 1) and values for the magnitudes of the vectors and angles in each diagram. The question posed was, "What is the magnitude of the dot product $\vec{A} \cdot \vec{B}$ / $W_{BR} = \vec{T}_{BR} \cdot \Delta\vec{r}$, the work done on the block by the rope / $\Phi_E = \vec{E} \cdot \vec{A}$, the electric flux through the area?" Multiple-choice options were numerical values.

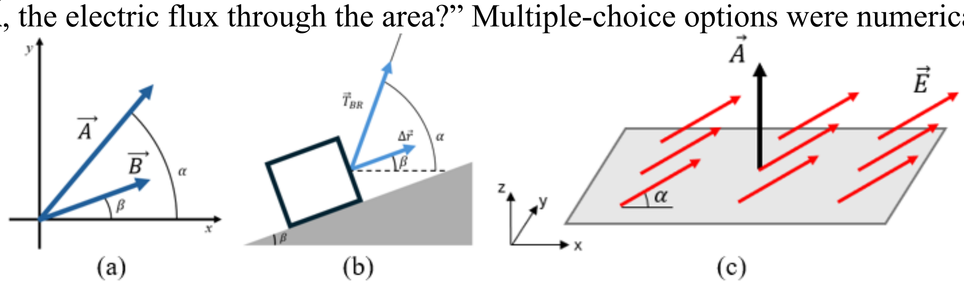

Figure 1. Diagrams provided for (a) math, (b) work, and (c) electric flux tasks.

All three contexts had enough information for students to find the angle between the two vectors ($\alpha - \beta$ for *math* and *work*, and $90 - \alpha$ for *flux*). The *flux* scenario was not a perfectly isomorphic scenario, instead presented in a form more aligned with problems given in a physics class. Since *math* and *work* had two given angles there were more distractors in those contexts than the *flux* question to allow for options involving the cosine of each given angle.

There were 191 dot product survey responses in PHY1 and 181 in PHY2, with 118 students completing both surveys serving as longitudinal data points via matched responses. We also conducted 4 free-response, think-aloud interviews at the end of PHY2 for additional insights.

For both the magnitude and direction of each dot product task, after participants submitted their multiple-choice answers, the next page of the survey re-presented the questions, reminded students which choice they selected, and prompted them to briefly explain their reasoning for each answer. The final code book (Table 1) is the result of an iterative process in which explanations were coded – independently from the multiple-choice selections – to identify the mathematical approach students used to reach their answer. The first four rows are the main computational strategies used by students; projection/alignment and limiting cases (#5 and #6) were codes that appeared both in conjunction with the main strategies and stand-alone. Codes are independent of correctness: within each main strategy, sub-codes were used to distinguish approaches leading to correct or incorrect conclusions, such as using an inappropriate angle or function. Inter-rater reliability achieved Krippendorff's α coefficients of agreement of .92, .87, and .96 for the *math*, *work*, and *flux* contexts, respectively.

Table 1. Table of dot product task reasoning codes and descriptions. Starred categories (*) could be sub-codes of the first four codes.

Code	Description
1. Trigonometric definition	Multiplying the magnitudes of both vectors by a trig function (predominantly sine or cosine) of a correct or incorrect angle
2. Sum of component products	Decomposing, or attempting to decompose, the vectors and taking the sum of the products of the corresponding components
3. Scalar multiplication	Multiplying the magnitudes of the two vectors
4. Addition	Using strategies of either scalar or vector addition, such as tip-to-tail
5. Projection/alignment*	Discussing vector projections or how aligned the two are
6. Limiting cases*	Using maximum and/or minimum values to support their selection
7. Restating	Stating the answer selected is what they calculated without elaboration
8. Other	Does not fit into any of the above categories (also blank, unclear)

Results & Discussion

Multiple-Choice Results

The overall performance on the four dot product magnitude tasks ranged from 28% to 59%. In the *PHY1 math* context, 52% of students selected the correct response, compared to 59% in the *work* context, 42% in the *PHY2 math* context, and 28% in the *flux* context, as shown in

Figure 2. Distractors of similar types are combined for presentation. "Trig errors" includes the answers that could be reached by using sine of the angle between the two vectors or cosine of any given angle. "Addition" includes both the scalar and vector addition choices.

Consistency between tasks shows evidence of the broader challenge. Table shows the proportion of students who answered consistently, correct or incorrect, between four pairs of tasks. Between the *PHY1 math* and *work* contexts, 58% of students selected consistent responses, and only 30% were consistent between the two semesters in the *math* contexts. Each comparison to the *flux* task has less than 30% consistency between response pairs. Across all four tasks, 10% of the 118 students correctly determined the magnitude of the dot product.

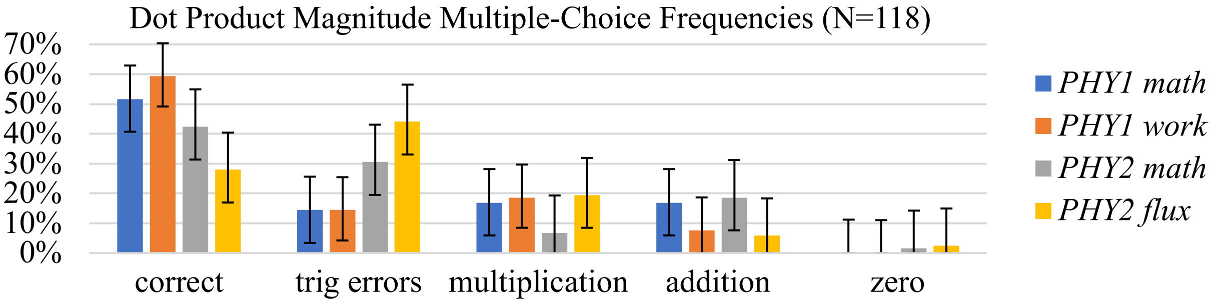

Figure 2. Response frequencies for all four dot product magnitude tasks across the year, for matched students.

Table 2. Multiple-choice response consistency across two tasks by percent of the population, N=118.

Categorization of paired responses	*PHY1 math* vs. *work*	*PHY1 math* vs. *PHY2 math*	*PHY2 math* vs. *flux*	*Work* vs. *flux*
Consistent and correct	44%	22%	16%	19%
Consistent but incorrect	14%	8%	11%	5%
Inconsistent	42%	70%	73%	77%

Explanation Results

To understand why students are selecting the answers they are, we go to the explanation codes. The trig definition approach was the most common strategy explained by students in all four contexts, with about half of the free responses coded as such in each context (shown in Figure 3). Of these students we grouped their multiple-choice responses as correct, trig errors, and other errors (multiplication, addition, and zero). The sum of "restating" and "other" codes (#7 and #8 from Table 1) were more common than the remaining specific approaches, with between 20% and 31% of responses coded as such.

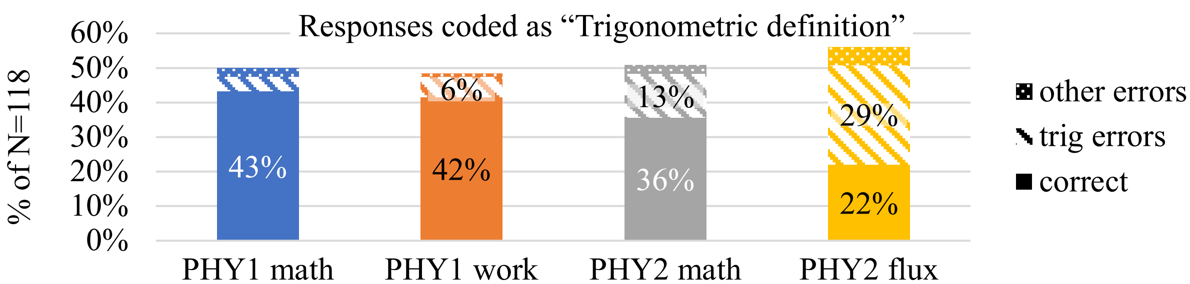

Figure 3. Breakdown of multiple-choice selections of students coded as using trigonometric definition explanations. Percentages not shown are ≤ 5%.

Due to the angles given in each context (see Figure 1), the *math* and *work* trig errors account for three multiple-choice options while there is only one *flux* trig error answer, corresponding to $EA\cos\alpha = EA\sin 90 - \alpha$. Within these *flux* responses, the majority explained by saying they calculated cosine of "alpha," "30," "theta", or "the angle between them," effectively taking the dot product of two vectors that are an angle α apart instead of $90 - \alpha$. This could be for a variety of reasons, ranging from simply using the angle provided to a conceptual confusion about the area vector (i.e., placing it in the plane of the surface). Most of the trig errors in the *PHY2 math* context were from using sine instead of cosine, which is likely a recency effect due to the coverage of the Lorentz force, which is determined by a cross product, the week prior.

Regardless of approach, only up to 50% of students gave explanations coded in the same category (#1-6 in Table 1) across any task pair. Furthermore, the same approach does not necessarily lead to the same answer. For example, 43% of students who were coded using a trig definition for both *PHY2 math* and *flux* and 32% of the students who were coded using a trig definition for both *work and flux* gave consistent responses in the respective task pairs.

The projection/alignment code was not prominent but was more common in the physics contexts than in the math context. One student in *PHY1 math*, 8 in *work*, none in *PHY2 math*, and 4 in *flux* used these ideas either as their sole explanation or in tandem with the trig definition approach. Additionally, during the *flux* portion of the interviews, follow-up questions about why students used cosine in their calculations or about the significance of the angle elicited responses about the electric flux measuring how much of the field is caught by the area, with analogies like a sailboat in the wind (which was generated by the instructor), whereas the same follow-up questions in the *math* context were met with answers such as "that's what I'm remembering the equation as" and "'cause dot products use cosine, cross products use sine." Students seem to attribute the operation of a dot product with more physical meaning in physics contexts than when the physics is pared away. Our findings agree with Zavala & Barniol's (2013) results where students were more likely to describe the dot product as a projection in physics contexts than with no context. However, the very low count of the projection/alignment code among our explanations suggests that students are not likely to offer this reasoning unprompted.

A final note on student reflection. Three of the four interview participants started doing their calculations for *flux* by using cosine of the angle given, α, then either by catching it themselves or realizing after a request for an explanation that the angle they needed was actually $90-\alpha$. Additionally, across all four dot product magnitude tasks, 11 students said they would change their answer after looking at the problem again or thinking about it more.

Conclusions & Implications

From a modeling perspective, the *math* task drops students into the mathematical world, while the physics tasks provide the physical situation and a mathematical model (a dot product) for the quantity in question that should use the same mathematical operations. However, our data suggest that students perform different operations in the two scenarios and thus may not make consistent mathematical choices across contexts, supporting the idea that physicists tend to load meaning onto symbols while mathematicians generally do not (Redish & Kuo, 2015). In our study, the meaning being loaded by the physics seems to influence the mathematical operations performed, an aspect that may not be accounted for in modeling frameworks.

Visual representations of each context and the visual features to which students attend influence what values students use in their calculations. The fact that the survey responses are susceptible to new content being introduced and the difference in explanations across semesters for the same (*math*) task suggests that models for dot products are still dynamic at that later time.

Further exploration of this work will involve utilizing dual-process theories of reasoning as a framework both for task design and data analysis that may help explain inconsistencies across contexts. Further application of findings to modeling frameworks may illuminate specific conflicts between the frameworks and student reasoning.

The presence of physical situations may cue students to reason differently as they do not often call upon the formal geometric interpretation of the dot product to perform calculations. By the time students see the formalism of vector products in mathematics courses, they have likely already seen these operations in a physics class(es), so mathematics instructors may be able to draw on physics contexts to help solidify understanding of vector operations. But these contexts come with their own conceptual overhead and thus need to be introduced with care.

Acknowledgements

This material is based upon work supported by the National Science Foundation under Grant Nos. PHY-1912087 and DUE-2336912, collaborative projects with Michael Loverude (California State University Fullerton) and Warren Christensen (North Dakota State University). The authors would also like to thank Paul Van Kampen and Trace Harris for their assistance with inter-rater reliability.

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