Subjective Combinatorics: A Case of Set Partition Problems

Niusha Modabbernia
Simon Fraser University

Research in combinatorics education highlights the importance of examining how students think about counting problems to gain deeper insight into their conceptualizations. Such an examination involves attending to students' interpretations of counting problems, particularly the subjective elements that influence their combinatorial thinking. Subjective combinatorics has recently been introduced to account for these elements in how students interpret counting problems and engage in counting activity. This paper refines the construct and demonstrates its usefulness as a lens for analyzing students' engagement with counting problems through an examination of student work on a set-partition task.

Keywords: combinatorics, conceptual analysis, counting problems, subjective combinatorics.

Enumerative combinatorics, the study of counting problems, is part of many school and undergraduate curricula. Its presence in curricula reflects both its practical relevance in fields such as probability and computer science (e.g., Abrahamson, Janusz, & Wilensky, 2006; Shaughnessy, 1977) and its potential to engage students in critical mathematical thinking (e.g., Kapur, 1970; Martin, 2001; Tucker, 2002). A growing body of research in the mathematics education community has examined how students think about counting problems, identifying common strategies and sources of difficulty (e.g., Batanero, Navarro-Pelayo, & Godino, 1997; English, 1991, 2005; Lockwood, 2013; Lockwood, Swinyard, & Caughman, 2015; Tillema, 2013). Building on this work, it is important to investigate how students interpret counting problems, with particular attention to the subjective elements that influence their combinatorial thinking. To this end, *subjective combinatorics* (Modabbernia, 2025) has been introduced as a lens to examine such subjective interpretations of counting problems, building on ideas developed in the study of *subjective probability* (Chernoff & Zazkis, 2011; Jones, Langrall, & Mooney, 2007). This study situates *subjective combinatorics* within a broader body of mathematics education research and refines the construct by focusing on subjective interpretations of outcomes, illustrating its usefulness through an analysis of student work on a set partition problem.

Brief Background and Contextual Insights

Subjective combinatorics was originally introduced with ideas drawn from subjective probability (Modabbernia, 2025). In this paper, I situate the construct within a broader body of mathematics education research that addresses subjectivity in mathematical reasoning and students' combinatorial thinking. I begin with studies in mathematics education that foreground the interpretive and experiential dimensions of reasoning, followed by research in combinatorics education that has highlighted subjective elements in students' combinatorial thinking.

Mathematical reasoning cannot be separated from students' personal perspectives. As Johnson (1987) observed, "without imagination, we could never reason toward knowledge of reality" (p. ix). This view underscores the centrality of imagination for making sense of experience and highlights how reasoning is inherently interpretive. Speiser and Walter (1996) extend this idea, emphasizing that embodiment and personal experience shape what and how things can be meaningful, and therefore, personal experience is part of how we understand and use our mathematics. Empirical research further illustrates this interpretive dimension. Tourniaire and Pulos (1985), in their review of proportional reasoning, noted that “studies using problems with different contexts yielded unequal success rates, which suggests that context has an important influence on performance” (p.190). Prior experiences also play a critical role. In the study of students’ conceptualizations of calculus, Marrongelle (2004) emphasized that students often drew upon earlier experiences to reason through mathematical tasks. Van den Heuvel-Panhuizen (2005) reinforced the interpretive role of context by demonstrating that students were more willing to generate meaningful responses when problems were situated in contexts they experienced as real. By imagining themselves in such situations, students engaged in sense-making that was directly inspired by the task. At the same time, contexts could also hinder reasoning if they conflicted with students’ personal experiences, as in Mack’s (1993) finding that a child rejected a pizza-sharing problem because she disliked pizza and would have found the problem more sensible if framed in terms of ice cream. Research in probability has also highlighted subjectivity, where interpretations of outcomes are shaped by personal judgment and belief (Jones et al., 2007). As Batanero et al. (2005) noted, “what is random for one person might be nonrandom for another” (p. 24), reflecting uncertainty specific to each individual. Collectively, these perspectives show that students’ interpretations of mathematical problems are shaped by imagination, embodiment, context, and personal experience, providing a foundation for viewing mathematical reasoning as inherently subjective.

These findings, while not situated in combinatorics, emphasize that mathematical reasoning could be subjective. This subjectivity is particularly evident in combinatorics, where many problems are presented through contextual scenarios rather than in purely abstract form. While such presentations enhance accessibility, they also increase the potential for varied interpretations of the problem itself. To consider subjectivity in combinatorics, it is helpful to begin with the way counting problems are stated. A counting problem highlights specific restrictions for representing a set of outcomes and tasks students with determining the cardinality of that set. Prior research in combinatorics education has already drawn attention to interpretive elements in students’ reasoning. At the elementary level, English (1993) found that children’s understanding of the word “different” shaped the way they constructed outcomes. In her study, “children interpret *different* to mean *different in all ways* and thus see the goal of *all different outfits* as an indication to make each new outfit completely different from the previous outfit(s)” (p. 260). Such findings highlight how students’ interpretations of task wording can influence what they count as valid outcomes, underscoring the role of subjectivity in combinatorial reasoning. At the middle-grades level, Fischbein (1987) described intuitions as behaviorally grounded, shaped by prior experiences, and directed toward meeting adaptive demands, even if they sometimes lead to errors. Extending this view to combinatorics, Fischbein and Grossman (1997) showed that students’ reasoning was shaped by the interpretive schemata they brought to a task: “Their reactions are different because their intuitions express different schemata, different programs of interpretation related to their age” (p. 28). These findings highlight how students’ interpretations of combinatorial problems are grounded in prior experience and the schemata that guide their intuitions, reinforcing the role of subjectivity in their combinatorial reasoning. At the high school level, studies further suggest that students’ interpretations of problem statements can strongly influence their combinatorial reasoning. Batanero, et. al. (1997), drawing on Dubois’s (1984) classification, showed that students often did not regard structurally isomorphic problems as equivalent when different objects were involved, instead applying different methods depending on the structures they perceived. They also documented how wording could shift students' reasoning, as in a distribution problem involving four cars and three children where one student reasoned arithmetically: "4/3 = 1; one car for each brother and there is a car left" (p. 191). Such findings illustrate how both the objects embedded in a task and the everyday meanings of words such as "distribute," "divide," or "share" can shape students' subjective interpretations. At the undergraduate level, Antonides and Battista (2022) explained that structuring "is not a form of abstraction, but rather is a process that can be abstracted at various levels and that can build on previous abstractions. Abstraction of structuring leads to mental models of objects, which are integrated sets of abstractions activated to make sense of, reason about, and operate within experiential situations" (p. 798). This perspective aligns with subjectivity, as it highlights how students' reasoning depends on the mental models they construct when interpreting combinatorial tasks.

Taken together, this literature illustrates that students' mathematical reasoning is deeply shaped by subjective dimensions such as imagination, embodiment, context, and personal experience. Within combinatorics specifically, subjectivity emerges through factors including task wording, prior experience, interpretive schemata, and the mental models students construct. These insights highlight subjectivity as a central feature of how students engage with counting problems, providing the foundation for refining subjective combinatorics in the present study. To this end, this study aims to refine the construct of subjective combinatorics and examines its usefulness for analyzing students' engagement with counting problems. In particular, this study attempts to address the following research questions: *What factors contribute to students' subjective interpretations of combinatorics problems? What factors contribute to students' generation of restrictions when solving and evaluating combinatorics problems that are susceptible to subjective consideration?*

Theoretical Framework: Subjective Combinatorics

In an earlier study (Modabbernia, 2025), *subjective combinatorics* was introduced as a construct, viewing combinatorics as shaped by personal judgment and information about outcomes. This perspective highlights how individuals may approach the same counting problem with different assumptions, leading to varied yet reasonable interpretations of sets of outcomes. For instance, in a problem asking for the number of possible outfits from two shirts, five pairs of pants, and three hats, some students may consider an outfit incomplete without a hat, while others may view a shirt and pants as sufficient. In such cases, differences in interpretation lead to different sets of outcomes, and thus to different final answers. In this example, the restrictions could be made explicit by defining an outfit as one hat, one shirt, and one pair of pants, giving the problem a determinate answer. More broadly, while some counting problems are fully specified, others remain open to interpretation; this study focuses on the latter. Such openness can even give rise to more than one plausible set of outcomes, depending on how students interpret what the problem is asking. I refer to this interpretive flexibility as a *multiplicity of plausible sets of outcomes*. This idea supports the broader construct of subjective combinatorics, in which different yet reasonable interpretations lead students to envision different sets of outcomes to count. This interpretive flexibility is evident in a classroom example (not part of the study data), where a student approached the problem: *"In how many ways can a committee of 5 people be selected from a group of 12, if two specific people must be included?"* The student treated the version without the condition as an ordered selection (using P(12, 5)) and the version with the condition as unordered (using C(10, 3)). This shift may reflect the effect of the condition in prompting a different interpretation of what needs to be counted, rather than a mistake. It illustrates how conditions of the task itself can cue a particular structure of the set of outcomes.

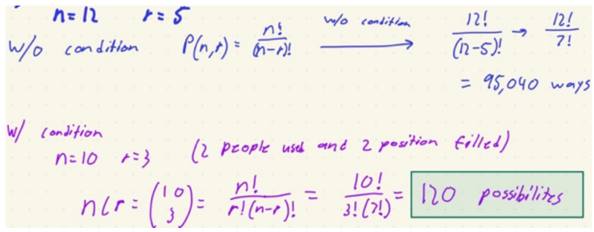

Figure 1. An example drawn from classroom experience (not study data) showing a student navigating different sets of outcomes.

This interpretive variation highlights the role of subjective interpretation not just in the process of counting, but in defining the set of outcomes itself. By situating subjective combinatorics within a broader body of research and highlighting the multiplicity of plausible sets of outcomes, this study refines the construct and extends its usefulness for analyzing students' thinking. Building on this refinement, I now define this construct more precisely.

For problems that allow *multiple plausible sets of outcomes*, *subjective combinatorics* views combinatorics as a matter of belief, shaped by personal judgment and information about outcomes. These are influenced by (1) *student-related factors* such as imagination, embodiment, personal experience, interpretive schemata, and the mental models they construct, and (2) *problem-related factors* such as the context in which the problem is posed and the wording of its conditions. Through this lens, differences in students' reasoning may reflect the interplay of these influences, giving rise to diverse yet reasonable framings of what counts as an outcome.

Method

At the time of data collection, 35 participants were enrolled in a secondary mathematics methods course within a professional development program in mathematics education. All held degrees in mathematics or science, though their academic backgrounds varied. The course, typically completed in the final semester of teacher certification studies, is designed to strengthen mathematical knowledge, foster investigative skills, and explore pedagogical approaches to secondary mathematics, with an emphasis on topics underrepresented in school curricula.

The topic of counting problems was addressed as the final area of exploration. In class sessions, the participants revisited the multiplication and addition principles from their secondary school studies and examined combinations, permutations, the binomial theorem, and combinations with and without repetition. They engaged with a range of counting problems, discussed potential sources of error, and concluded the unit by completing the following task.

> You work with a class on counting problems. One of the problems asks students to solve the *Committees Problem*: There are 16 students. How many ways can we organize them in 4 committees of 4 people?
> Three students suggest three different answers.
>
> Alex's answer: $\binom{16}{4}\binom{12}{4}\binom{8}{4}\binom{4}{4}$
>
> Ben's answer: $\frac{\binom{16}{4}\binom{12}{4}\binom{8}{4}\binom{4}{4}}{4!}$
>
> Nikki's answer: $\binom{16}{4}4!\,\binom{12}{4}4!\,\binom{8}{4}4!\,\binom{4}{4}4!$
>
> 1) Which answer do you agree with? Explain your reasoning. If you disagree with all, please provide your own answer and justify your reasoning. 2) What do you think was implicitly assumed in each answer? 3) Why do you think the students made these assumptions? The suggested length for responding to each question is about 150 words.

Figure 2.The Task

The *Committees problem* was adapted from a standard combinatorics problem that allows varied interpretations, particularly around grouping. The task encouraged interpretations grounded in students' assumptions about what is being counted. One key source of subjectivity lies in whether the committees are labeled or unlabeled. Additionally, students may or may not attend to the fact that committees are typically unordered collections of individuals, which affects whether orderings within each group are considered meaningful. The three suggested responses reflect different assumptions about what is being counted. Alex's answer treats the committees as labeled and distinct, assuming the order of group formation matters. Ben's response views them as unlabeled. The inclusion of 4! in the denominator suggests that arrangements of the same four groups represent the same outcome. Nikki's response builds on Alex's by assuming that the order of students within each committee matters, multiplying each binomial coefficient by 4! and treating each group as an ordered list. These responses illustrate how combinatorial reasoning can be shaped by assumptions about grouping and distinguishability. The data consisted of participants' written responses. Instances reflected subjective interpretations served as the unit of analysis. Having repeatedly read the data, I looked for common themes by highlighting noteworthy explanations.

Results

In the following analysis, participants' interpretations of the Committees problem, particularly whether the committees and their members were treated as ordered or unordered, are presented alongside their reasoning. The excerpts illustrate the assumptions participants brought to the task, revealing how they interpreted what needed to be counted. These interpretations played a central role in shaping their combinatorial thinking, influencing how they envisioned the set of outcomes and, in turn, the counting processes they considered appropriate. Among the 35 participants, 9 agreed with Alex's solution, 22 with Ben's, and 2 with Nikki's. One participant supported both Alex and Ben's solutions, and one found all three responses plausible. These interpretations were influenced by a range of factors, including wording, real-world experience, and unconscious implicit ordering—each of which is explored below.

Interpretation 1: The Committees Are Distinct

The role of wording. In counting problems, the language used in the problem statement can strongly shape students' interpretations. Certain words may prompt assumptions about the nature of the outcomes, such as whether elements are distinct or interchangeable, which in turn influences how students conceptualize the set of outcomes. The following excerpt illustrates.

Tara: The committees are likely to have specific roles to fill, so they aren't all the same, if the question wanted the partitioning to be into indistinct groups then it would have used a more general word, like "group" or "team."

Tara's explanation highlights how the specific wording of a counting problem can guide students toward particular interpretations. Her reflection on the difference between terms like "committee," "group," or "team" suggests that certain words may prompt students to interpret the set of outcomes in particular ways. In this case, the word "committee" led Tara to assume that the groups were distinct, revealing how subtle differences in phrasing can shape how students interpret the set of outcomes. Consistent with this interpretation, Tara chose Alex's solution, which reflects the assumption that the committees are distinct.

Real-world experience. Students' interpretations were also shaped by how they connected the counting problem to real-life contexts. The following excerpt illustrates how Tara drew on her everyday experience with committees to interpret the structure of the outcomes.

Tara: A committee generally is made with a specific intent, so creating the 4 committees are likely to have different goals, therefore, it matters which one each student got placed in.

Tara relied on her real-world interpretation of committees, typically tied to distinct roles, to infer that each group served a specific purpose. This led her to treat the committees as distinct and the arrangement as ordered, where assigning students to different committees resulted in different outcomes even if the group compositions remained the same. Tara's reasoning illustrates how real-life associations can shape students' assumptions about what distinctions matter in a counting context and how they conceptualize outcomes. Reflecting this interpretation, Tara selected Alex's solution, which treats all four committees as distinct.

Unconscious implicit ordering. Sequential thinking is a common cognitive pattern in daily life, where actions are broken into manageable steps. In many real-life situations, we approach tasks this way simply because it feels intuitive. When carried into counting problems, this natural tendency can introduce an implicit structure and lead to assumptions about how elements should be selected or grouped. The following excerpt exemplifies how such ordering emerged naturally.

> *Ana*: we want to create 4 groups of 4 students, out of a total of 16. To make it easier, let us do it one at a time: First, we will choose a group of 4 from 16 people, that is, C (16,4). Now, we want another group of 4, but now, out of twelve total people, since four of the previous sixteen have already been chosen. That makes it C (12,4). We have to repeat next step like that for the remaining 8 people to get C (8, 4) and then Finally, we have to create the last group of 4, but now we have only 4 students to choose from. Since the order of choice does not matter, it is rather intuitive that our C (4,4) is equal to 1. Therefore, all we must do is join all those combinations.

Ana's explanation reflects how a step-by-step approach, while intended to simplify the task, can introduce an ordering structure. Although she explicitly stated that the order of choice does not matter, likely referring to the order of students within each group, her sequential construction suggests an implicit assumption of ordering among the groups themselves. Based on this interpretation, Ana supported Alex's solution. This example illustrates how the structure of a counting process can shape the interpretation of the set of outcomes.

Interpretation 2: The Committees Are Indistinct

The role of wording. In some cases, students viewed the committees as indistinct based on how they made sense of the term *committee*. They viewed the term as describing a set of equivalent groupings. This perspective is reflected in the following excerpt.

> *Sarah*: The committees are 4 random groups, so it doesn't matter which group anyone is in.

Sarah's response suggests she treated "committee" as a neutral term, viewing the groups as interchangeable. By referring to them as "random groups," she overlooked any functional distinction, leading her to interpret the committees as indistinct and disregard ordering among them. This interpretation prompted Sarah to select Ben's solution.

Mathematical experience. Some participants approached the problem with expectations shaped by their prior encounters with combinatorics. Their interpretations reflected the distinctions they had experienced in earlier tasks, including how such distinctions were stated. The excerpt below shows how students' mathematical experience can inform their assumptions.

> *Madison*: Because there was no specific indication that the committees are distinct, there was no reason to think that they were. If the question wanted to imply that each committee is distinct then the question would have indicated that in some way.

Madison's reasoning highlights how the absence of explicit wording led to the assumption that no distinction between the committees was intended. Her interpretation was shaped by prior experience with combinatorics tasks and the ways distinctions had been communicated in those settings. In this way, Madison's explanation illustrates how students draw interpretive meaning not just from what is stated in a problem, but also from what is left unsaid.

Interpretation 3: Committee Members Are Ordered

Real-world experience. Some participants considered not only the distinctness of the committees but also whether members within each committee should be treated as ordered. These interpretations often reflected real-world contexts where roles, positions, or functions within a group are assigned. The following excerpt illustrates how a participant invoked a familiar setting, a sports tournament, to justify why the order of members might matter.

> *Simon:* Imagine we have 16 students and we want to make 4 teams of 4 for a soccer tournament … There might be specific positions we can fill on a soccer team, and if there's a tournament bracket so it might matter what team we end up on.

Simon's response reflects how his familiarity with structured team settings informed his assumptions about the task. By relating the scenario to a soccer tournament, he drew on the idea that individuals in a group may have assigned roles or positions. This connection to real-life experiences introduced distinctions that might not have been explicitly required by the problem but felt natural within the imagined context. Simon's interpretation guided him toward Nikki's solution, which reflects the view that both the committees and their members are ordered.

Interpretation 4: Committee Members Are Unordered

Real-world experience. Some participants drew from real-life contexts to justify that the arrangement of members should not matter. Their reasoning emphasized that a group's identity is determined by its members, not their arrangement. The following excerpt illustrates this view.

> *Sam*: It does not matter whether you think about your committee as Sherry, Joseph and Timmy or as Joseph, Sherry, and Timmy. In either case, the same three people are in the committee. It might also be helpful to contrast this with something like a smartphone passcode (where the order of digits matters), and 1234 is not the same as 4321. One of these will unlock your phone, and the other will not.

Sam emphasizes that a group's composition remains unchanged regardless of member order. Comparing committee formation to a passcode, she noted that in one case, reordering does not affect the group, whereas in another, it fundamentally changes the outcome. This contrast shows how real-world reasoning shaped her interpretation of when order matters.

The interpretations discussed in this section reveal that students' combinatorial reasoning is influenced not only by the explicit conditions of the task but also by subjective assumptions tied to language, context, and structural framing. These factors play a central role in how students construct sets of outcomes and determine appropriate counting strategies.

Conclusion

This study refines the construct of *subjective combinatorics* by emphasizing the *multiplicity of plausible sets of outcomes* and clarifying the roles of *student-related factors* (such as imagination, embodiment, personal experience, and mental models) and *problem-related factors* (such as context and wording). These refinements provide a sharper account of how subjectivity shapes combinatorial reasoning, illustrating that different yet reasonable interpretations of sets of outcomes can emerge from the same task. In doing so, the study demonstrates the usefulness of subjective combinatorics as a lens for analyzing students' engagement with counting problems and points toward future research on how subjectivity shapes mathematical reasoning more broadly.

References

Abrahamson, D., Janusz, R. M., & Wilensky, U. (2006). There once was a 9-block. . . —a middle-school design for probability and statistics. *Journal of Statistics Education*, *14*(1), 1-28.

Annin, S. A., & Lai, K. S. (2010). Common errors in counting problems. *Mathematics Teacher*, *103*(6), 402–409.

Antonides, J., & Battista, M. T. (2022). Spatial-temporal-enactive structuring in combinatorial enumeration. *ZDM–Mathematics Education*, *54*(4), 795-807.

Batanero, C., Henry, M., & Parzysz, B. (2005). The Nature of Chance and Probability. In G. A. Jones (Ed.), *Exploring probability in school* (Vol. 40, pp. 15–37). Springer. https://doi.org/10. 1007/0-387-24530-8_2.

Batanero, C., Navarro-Pelayo, V., & Godino, J. D. (1997). Effect of the implicit combinatorial model on combinatorial reasoning in secondary school pupils. *Educational Studies in Mathematics*, *32*(2), 181–199.

Borovcnik, M., & Bentz, H. (1991). Empirical research in understanding probability. In R. Kapadia & M. Borovcnik (Eds.), Chance encounters: *Probability in education* (pp. 73-106). Dorecht, The Netherlands: Kluwer.

Chernoff, E. J., & Zazkis, R. (2011). From personal to conventional probabilities: from sample set to sample space. *Educational Studies in Mathematics*, *77*(1), 15–33.

Dubois, J. G. (1984). Une systématique des configurations combinatoires simples. *Educational Studies in Mathematics, 15*(1), 37–57.

Eizenberg, M. M., & Zaslavsky, O. (2004). Students' verification strategies for combinatorial problems. *Mathematical Thinking and Learning, 6*(1), 15–36.

English, L. D. (1991). Young children's combinatorics strategies. *Educational Studies in Mathematics*, *22*(5), 451– 474.

English, L. D. (1993). Children's strategies for solving two–and three–dimensional combinatorial problems. *Journal for research in Mathematics Education*, *24*(3), 255-273.

English, L. D. (2005). Combinatorics and the development of children's combinatorial reasoning. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (Vol. 40, pp. 121–141). New York: Springer.

Fischbein, E. (1987). *Intuition in Science and Mathematics. An Educational Approach* Reidel, Dordrecht, The Netherlands.

Fischbein, E., & Grossman, A. (1997). Schemata and intuitions in combinatorial reasoning. *Educational studies in Mathematics*, *34*(1), 27-47.

Halani, A. (2012). Students' ways of thinking about enumerative combinatorics solution sets: The odometer category. *the Electronic Proceedings for the Fifteenth Special Interest Group of the MAA on Research on Undergraduate Mathematics Education. Portland, OR: Portland State University*.

Hawkins, A. S., & Kapadia, R. (1984). Children's conceptions of probability–A psychological and pedagogical review. *Educational Studies in Mathematics*, *15*, 349-378.

Johnson, Mark (1987). *The body in the mind*. Chicago: The University of Chicago Press.

Jones, G. A., Langrall, C. W., & Mooney, E. S. (2007). Research in probability: Responding to classroom realties. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning,* (pp. 909-955). New York: Macmillan.

Kapur, J. N. (1970). Combinatorial analysis and school mathematics. *Educational Studies in Mathematics*, *3*(1), 111–127.

Lockwood, E. (2011). Student connections among counting problems: An exploration using actor-oriented transfer. *Educational Studies in Mathematics*, *78*(3), 307.
Lockwood, E. (2013). A model of students' combinatorial thinking. *The Journal of Mathematical Behavior*, *32*(2), 251–265.
Lockwood, E., Swinyard, C. A., & Caughman, J. S. (2015). Patterns, sets of outcomes, and combinatorial justification: Two students' reinvention of counting formulas. *International Journal of Research in Undergraduate Mathematics Education*, *1*(1), 27–62.
Mack, N.K. (1993) Learning rational numbers with understanding: The case of informal knowledge IN: Carpenter, T., Fennema, E. and Romberg, T. (eds.) *Rational numbers: An integration of research*. Hillsdale, New Jersey: Lawrence Erlbaum Associates, 85-105.
Marrongelle, K. A. (2004). How students use physics to reason about calculus tasks. *School Science and Mathematics*, *104*(6), 258-272.
Martin, G. E. (2001). *The Art of Enumerative Combinatorics*. New York: Springer.
Modabbernia, N. (2025). Examining Subjective Factors Shaping Students' Combinatorial Thinking. *Proceedings of the 27th Annual Conference on Research in Undergraduate Mathematics Education*. Alexandria, VA: Virginia Tech University.
Shaughnessy, J. M. (1977). Misconceptions of probability: An experiment with a small-group, activity-based, model building approach to introductory probability at the college level. *Educational Studies in Mathematics*, *8*(3), 295–316.
Speiser, B., & Walter, C. (1996). Second catwalk: Narrative, context, and embodiment. *The Journal of Mathematical Behavior*, *15*(4), 351-371.
Tillema, E. S. (2013). A power meaning of multiplication: Three eighth graders' solutions of cartesian product problems. *The Journal of Mathematical Behavior*, *32*(3), 331–352.
Tourniaire, F., & Pulos, S. (1985). Proportional reasoning: A review of the literature. *Educational studies in mathematics*, *16*(2), 181-204.
Tucker, A. (2002). *Applied Combinatorics* (4th ed.). New York: John Wiley & Sons.
Van Den Heuvel-Panhuizen, M. (2005). The role of contexts in assessment problems in mathematics. *For the learning of mathematics*, *25*(2), 2-23.