“I think most of them would say they don't understand yet”: Mathematicians’ Views on the Benefits of In-class Time

Corinne Mitchell
Virginia Tech

Estrella Johnson
Virginia Tech

Tim Fukawa-Connelly
Temple University

Through analysis of 7 mathematicians’ perspectives teaching proof-based linear algebra, we found that participants wanted to offer benefits to students during class which were analogous to the benefits documented in previous research that mathematicians have when attending and giving mathematics lectures at professional events (e.g., conferences, symposia, or workshops). Findings suggest that mathematicians may see attending mathematics lectures as an authentic mathematical practice in ways that education researchers have not previously attended to.

Keywords: teaching practices, proof-based courses, lecture, active learning

Efforts by the mathematics education research community to improve teaching have led to the development and testing of reform initiatives throughout the undergraduate mathematics curriculum. However, as documented by Fukawa-Connelly et al. (2016) and summarized by Artigue (2016), there has been “very limited influence of our research on university teaching practices. Reading recent publications… I find nearly the same description of standard university practices at undergraduate level as decades ago” (p. 12). One explanation for this is that “standard university practices” may be fulfilling the instructional goals of mathematicians in ways that reform initiatives championed by mathematics education researchers are not. By investigating what mathematicians see as valuable uses of class time, we gain insight that may allow us to propose pedagogical changes in ways that are better aligned with mathematicians’ goals and have broader impacts at the practitioner level. To this end, we report on a collaboration between mathematics education research and mathematicians on the implementation of specific evidence-based active learning strategies (EBALS) in an advanced linear algebra course.

Background and Framing

Surveys on reports of how mathematicians structure their in-class time have consistently shown that lecture is the dominate form of instruction. For instance, Eagan (2016) found that about two-thirds of mathematics teachers use “extensive” lecturing in all or most of their courses, Johnson (2019) stated that “77% of pre-calculus courses, 82.3% of Calculus I courses, and 86.3% of Calculus II courses are taught as either ‘lecturing and answering student questions’ or ‘lecture with some active learning techniques’, and Fukawa-Connelly et al. (2016) found in their study that 85 percent of respondents reported using “lecture” to teach abstract algebra, as opposed to reporting that they use a “non-lecture pedagogy”. Research on instructor’s beliefs about teaching and learning, however, suggest that the predominance of lecture may not simply follow directly from their beliefs. For instance, in comparison to the 85% of respondents who reported that they lecture in their survey of abstract algebra instructors, Johnson et al.’s study (2019) only about 59% of the respondents agreed with the statement “*Lecture is the best way to teach*”, whereas the overwhelm majority of them, 87%, agreed with the statement *“I think students learn better when they do mathematical work (in addition to taking notes and attending to the lecture) in class*”. This raises a natural question: If instructors believe that students learn best when they are doing mathematical work in class, then what do mathematicians see as the benefit of lectures?

A similar question was the focus of a study conducted by Weber and Fukawa-Connelly (2022). Instead of looking at what mathematicians perceived as benefits of attending a lecture for their students, they instead investigated what benefits they perceived for themselves as lecture attendees in a workshop on inner-model theory. As mathematical learners in this setting, the mathematicians did not necessarily expect to learn everything they needed to know from watching a lecture. Rather, they saw key features of the lecture as beneficial to their own independent study of the content. Indeed, the benefits Weber and Fukawa-Connelly (2022) found belonged not only to those attending the workshop lectures, but also to those who provided them. It is natural to wonder whether the same benefits mathematicians see in attending lectures for themselves as learners are also considered by mathematicians as they think about how to spend class-time as teachers.

Analytic Framing, Methods, and Data

A key premise of the NSF funded *Collaborating with Mathematician's to Enhance Teaching (COMET)* project, of which this work is subsidiary, was that the reasons mathematicians have for rejecting or accepting EBALS provide insight into what they see as beneficial or valuable. Similar to Herbst and Chazan (2011), we aim to understand stable systems of practices by understanding how those systems react to perturbations. By proposing changes to mathematicians' face-to-face instructional practice, we created discussions focused on their considerations when planning class, revealing the reasoning behind and justification of their use of class-time. Here we analyze these reasonings and justifications to answer the research question*: When mathematicians are considering what is a valuable use of class time, do they attend the same benefits that they attribute to the mathematics lectures that they present and attend (e.g., at conference presentations, symposia, and workshops)?*

The data was collected over the course of two semesters at a large American university. Over the course of six semi-weekly meetings throughout each semester, three mathematics education researchers met with a group of mathematicians teaching a proof-based linear algebra course. The first group had four instructors and the other three. In this paper, the year one instructors are assigned pseudonyms A1, A2, and so on, and the year two instructors are assigned B5, B6, and so on. The goal of such meetings was to strategize about incorporating EBALS in the linear algebra course. The course presented an axiomatic treatment of abstract vector spaces and linear transformation on those spaces and covered topics such as eigenvectors and eigenvalues, the Jordan Canonical Form (JCF), its associated theorems, and inner product spaces. Prior to the start of regular meetings, the mathematicians were also interviewed about their current teaching practice. Each of the interviews and collaborative planning meetings were initially transcribed using Otter.ai. First, we conducted a thematic analysis (Braun & Clarke, 2006) to understand the mathematicians' considerations about how to spend their class time. Then, we analyzed how the themes we found relate to conventions of lecture in a professional context.

After editing the AI generated transcripts for accuracy, the first author made an initial pass through the data, identifying any excerpts where participants discussed the purpose of in-class time, either by describing what they wanted to do during class or by explaining why they would not want to do a particular activity. These excerpts were logged in a spreadsheet where the excerpt was described and initial comments about the content of the excerpt were made. Then, these initial descriptions became the focus as we sought commonalities between comments and then arranged them into larger themes. These themes became a codebook that was used to code each transcript during the second pass through the data. After collecting all instances of each theme into a single document, the first and second author met to resolve any disagreements.

Finally, we compared these inductive themes to the benefits of lecture identified in Weber and Fukawa-Connelly (2022). Specifically, we sought analogues to the benefits *exposing participants to interesting results, providing a big picture of the domain, highlighting results and techniques that are important,* and *illustrating how the lecturer thinks about a topic.* That is, we hypothesized that these mathematicians were attending to benefits analogous to those identified by Weber and Fukawa-Connelly when they considered how to spend class time, and then looked for confirming and disconfirming evidence in our thematically coded data.

Findings

When discussing their plans for class, these mathematicians did not expect that the students would develop deep understanding of the material based solely on what happened during class time. Instead, they often indicated that they believed that students needed to study the material outside of class to understand it thoroughly[1]. Six of the seven mathematicians made comments relating to this idea. When asked how students best learn in the course, A1 said,

> "By doing mathematics. . . **Watching me doing it a million times, I don't think has much of an effect and seems like a different skill**. You know, **they could watch me prove a theorem and be really amazed at it**, how clever that is, and so on. **And they will never know how to do that themselves**."

We note the distinction made by A1 between the skill of studying independently and the skill of watching a lecture. Students could be amazed by watching a lecture, but that is not enough for students to learn the mathematics themselves. A2 and A3 echoed this sentiment, with A2 noting that "30 seconds or one minute is not enough" to absorb complex ideas during class, and A3 saying that, if you asked students what they understood at the end of class for an *exit-ticket*, "I think most of them would say they don't understand yet". It makes sense that if you believe that significant time and effort are needed for students to develop a deep understanding of the content, then developing such an understanding would not be a reasonable goal for the few hours a week spent in class.

Instead, when we asked what was most important to do during class time, responses fell broadly into four categories. For instance, B7 stated:

> "The first thing is that **I give a very organized lecture so they know what is important**. The most important formulas, or theorems, and hopefully some of the arguments. And second, I think **hopefully it'll be a bit inspiring**, so that **they will find it interesting**. For example, in today's class, I did one example about the derivative for polynomials. So they can sort of relate differentiation and integration as linear operators to the matrix representation. So **this is some connection between calculus and analysis to algebra and matrix algebra, as well as how it's related to physics**. So I hope I try to be a little bit inspiring at times."

These mathematicians wanted to spend class time interesting, inspiring, and motivating their students; they wanted to provide an overview of the big ideas in linear algebra, including how they relate to other domains; to highlight important results and techniques; and to illustrate appropriate thinking about the content.

[1]While they expected students to follow along with calculations and proofs of lemmas or simpler theorems, these discussions were often referencing abstract concepts, or theorems which required more involved methods of proof.

Class Time Should Inspire Students

Inspiring students by discussing interesting results during lecture came up frequently during meetings. Furthermore, participants' commentary often focused on motivating the mathematical material presented in the course. Five of seven mathematicians made relevant comments.

A1 referenced wanting to "sell" the material to the students, as well as ways he generated student "buy-in" to the course, to "get [students] hooked" by explaining how a vector is an essential object with applications "not just to linear systems, but to differential equations, to probability and statistics, to game theory, [and] to particle physics . . ." Later, he argues, "the reason you can have all these different applications is that [a vector is] fundamentally an abstract mathematical thing. So that's how I actually put the hook in there. And I think students are buying it so far." Several mathematicians in our study prioritized showing students interesting results from linear algebra that apply to other sciences. We interpret A1's reference to "buy-in" as expressing a desire for students to be willing to engage in the more abstract reasoning required in proof-based linear algebra.

In addition to motivating students, these mathematicians also wanted to motivate the mathematics presented in class. Specifically, they prioritized discussions that developed intellectual need for the material they covered. All of the year 1 collaborators considered motivation when they discussed the lectures on the Jordan Canonical form, complaining that the textbook's treatment was inappropriate for presenting in class. A1 commented, "It's written in a very "mathematician" style: definition, theorem, proof . . . for twenty theorems." A2 echoed this sentiment and explained,

> "It's unfortunate that this book starts with proofs . . . so I also had to adjust this. But **for motivation**, what I told the students is that we always want to find the simplest possible matrix. So that's actually a good way to **explain why we need Jordan canonical form**."

B6, A1, and A2 all noted that motivation was an essential product of lecture that was often missing from the textbook. B6 even contended that this kind of motivation supported students' learning outside of class, saying, "Because students have a textbook that they can read for theorems, I think an important thing is to just motivate things because it's a lot easier for them to learn if they know why you would ever define these things." In short, the mathematicians prioritized motivation in two senses: inspiring students to be curious about the content, and discussing the origins and context of the material to create an intellectual need for learning it.

Class Time Should Provide an Overview of the Content

We found that the mathematicians in our collaborative planning meetings wanted to provide students with an overview of the high-level ideas of linear algebra; they did not intend to spend class time fleshing out every detail exhaustively, because they saw the details as time-consuming and students as unable to digest them in real time. All seven participants made comments in this theme. For example, A3 remarked, "I don't think that's an efficient use of class time to listen to someone prove very slowly this theorem, because I think **you just want to get the main idea**." He did see presenting material in class as communicating to students which content to account for on future assessments:

> "You have to spend time on the proof, because they're responsible for knowing that. You can't just say read the book. **That doesn't mean you have to go through every detail**, you know. I think you have to do the hard direction [of the biconditional], or **you have to introduce the main idea**, right?"

In addition to demonstrating which concepts students are "responsible for knowing", participants frequently commented that class time should be spent showing students how important concepts are related to each other, as well as situating the course content within broader scientific contexts. A1 remarked, “Sometimes it's not fully connected in their minds, these different concepts. So maybe that's something that I could stress more in my lectures, try to make these connections clearer.” We see this attitude as analogous to the mathematicians interviewed in Weber and Fukawa-Connelly (2022) in that, for our participants, emphasizing the high-level ideas and their connections to each other was a benefit of lecture.

Class Time Should Highlight Important Concepts and Techniques

Participants frequently accepted or rejected a suggested EBAL based on whether it concerned content that was important enough to spend class time on, with five of seven making comments in this theme. What warranted importance often varied; content was “important” if it was worth understanding more deeply, introduced a new skill, or if it was seen as either more sophisticated than the prerequisite linear algebra course’s content or as necessary for the preparation of math majors. After accepting a task concerning the JCF, mathematician A1 commented,

> “I think in this case, you made the right choice. The theorem that you chose questions for here, that's actually the main important thing to get the block diagonal structure. That is a really important theorem. And it takes a long time [to prove], because it has several parts, and none of the parts are trivial to do. And then everything else kind of follows from that.”

B5, A2, and B6 often referenced wanting to spend class time on demonstrating new skills, with A2 commenting that he would not “give the full details” because “the main thing is to teach them how to do the calculations” which were new content for the course. We interpret the reference to “the main thing” to be synonymous with “the important thing.” He sees understanding how to find a linear transformation’s JCF as important enough to spend class time on because, in contrast to other calculations, this one is specific to the advanced linear algebra content.

That is, an activity was often considered important if it was appropriately sophisticated for the level of the course. A3 agreed, saying that the use of more abstract vector spaces of functions was “really important if they want to go into a differential equations class.” As well as attending to whether a task was appropriately sophisticated for the 350-level course, participants also weighed importance based on relevance to other courses in the math major, like differential equations. B5 added, “I think it just ties in with so many different courses in the math major that it's worth doing.” Finally, A3 commented, “I think in terms of the applications [math majors] are interested in, the inner product stuff is much more important.”

These participants wanted to show students “important results” that were conceptually rich, worth understanding deeply to use in practice later. The pedagogical context offered more nuance; a concept was also considered important if it was appropriately sophisticated for the level of the course, or if it was relevant to future courses in the math major.

Furthermore, this decision-making apparatus took the form of a biconditional statement. Not only did these mathematicians want to spend class time discussing important concepts, they warned that the amount of time spent on a concept during class *indicated to students* which content was important. When discussing whether he valued having students create their own conjectures during class, A3 commented that many theorems in linear algebra would not be appropriate for such an activity, noting the rank-nullity theorem as an exception. Then, he said, “But I don’t know, I don’t really feel like the rank-nullity theorem is that important. I don’t want to spend that much time on something that I don’t really care about that much.”

Time spent during class on a particular topic corresponded to its importance relative to the course content. Although A3 saw conjecturing as a valuable practice for students, he did not wish to emphasize a theorem that was not important enough to spend that amount of class time on. A2 made similar comments:

> "Especially for some really, really important theorems, for example, the proof that every linear transformation has a Jordan canonical basis. The goal for the students to get these arguments, it's very good. But on the other hand, we cannot use it for all the theorems, because we do not have enough time for this course. We can only do it for the important theorems."

It is notable that these mathematicians sometimes saw active learning as having an undesirable potential to emphasize results or techniques which were not important enough to warrant the time expenditure.

Class Time Should Illustrate How to Think About the Content

The mathematicians in our study wanted to model how to do linear algebra according to the norms and values of the broader mathematical community, beyond showing how the lecturer as an individual approached the content. This included demonstrating appropriate rigor and precision, showing students which ways of thinking may be productive, and giving students the opportunity to have their misconceptions corrected by an expert. All seven mathematicians made comments in this theme.

A2 observed that "the nice thing" about class was the chance he got to show students how to express their correct ideas abstractly and rigorously, saying "I point out that okay, yes. Roughly, you're right. But see, this is not precise. The precise way is [this]." It was important for participants that time spent in class not only showed students appropriate rigor, but also focused on material that was sufficiently sophisticated for the course. For example, it was important that examples used vector spaces other than $\mathbb{R}^n$. A3 commented,

> "In things like signal processing, we really use vector spaces of functions that are usually not polynomials, polynomials very rarely come up in in applications. It's usually either periodic functions, or exponentials, or logarithms, things like that. I think this course is really an opportunity to talk a little bit more about those kinds of functions."

B5 and B6 both described tension regarding the use of notation aligned with the textbook that was unconventional in the mathematical community, with B6 saying "I'm really torn between whether to use the book's notation or to use the notation that everyone else uses." A1 was willing to spend time in class on tasks that developed productive ways of reasoning about the content, saying, "there should be a way of making that part of their habit of thinking."

Our participants wanted to spend class time demonstrating what it meant to do math at this more advanced level. We see this benefit as analogous to the benefit Weber & Fukawa-Connelly (2022) found that the mathematicians in their study valued observing the habits of mind of a professional, experienced practitioner of a niche subfield. Because the audience members our participants referred to were students, it is sensible that they felt a responsibility to include *informal content* (see Fukawa-Connelly et al. 2017) like attending to precision and rigor, or correcting misconceptions, where the lecturers in the former study may not have.

Conclusion and Implications

There are many motivations for mathematics education researchers to advocate for the use of EBALS, and many benefits of their use recognized by mathematicians. These motivations and benefits vary based on the specific strategy, and the implementation of these strategies may be more or less compatible with lecture. For instance, Johnson et al. (2025) distinguished between "two types of active learning pedagogy in advanced mathematics: pedagogy to *replace* lecturing and pedagogy to *augment* lecturing" (pg. 147, emphasis in original). They go on to argue that, while there has been comparatively limited research in undergraduate proof-based mathematics courses on the use of EBALS that augment lecture, there has been substantial research in undergraduate mathematics education on initiatives designed to replace lecturing.

In this push to replace lecture, a common sentiment is that class time should be used to engage students in authentic mathematical activity. For instance, in providing a joint characterization of Inquiry Based Mathematics and Inquiry-Oriented Instruction, Laursen and Rasmussen (2019) state that a

> "distinguishing characteristic of inquiry is the nature of students' mathematical work. In inquiry classrooms students reinvent or create mathematics that is new to them. They do so by engaging in mathematical practices similar to those of practicing mathematicians: conjecturing and proving, defining, creating and using algorithms, and modeling" (p. 4).

However, this description of the practices of mathematicians leaves out some of the ways that mathematicians learn about new areas of mathematics, by attending lectures at workshops and studying the foundational texts of that area (Weber & Fukawa-Connelly, 2022). And indeed, we found that our mathematician collaborators attended to the similar benefits mathematicians attending – and giving – professional lectures when evaluating how to use class time.

We see two ways in which this finding can inform the work of mathematics education researchers looking to have a broader impact on undergraduate instructional practice. First, many mathematicians have goals like covering course content and improving students' abilities to write conventional proofs (e.g., Johnson et al., 2013; Johnson et al., 2025; Wu, 1999). Our findings add nuance to these goals. When we attending to the goal of providing students with a "big picture", we can make sense of content coverage as a priority for mathematicians regardless of department pressure (see Johnson et al., 2018). Content coverage may be seen as a prerequisite for situating the high-level ideas and techniques in relation to each other. Notably, active learning pedagogy designed to replace lecturing tends to prioritize depth over breadth in terms of content (Roth McDuffie & Graber, 2003). Additionally, demonstrating proof conventions like abstraction and rigor during class may be important to mathematicians not only because it prepares students to write formal proofs, but because it gives the audience access to the habits of mind of an expert. Modeling a formal proof by providing a lecture may be a more effective way of meeting that goal than active learning strategies that are designed with the intention that "the teacher must help transition the developing mathematics of the classroom, from that of informal student reasoning to that of formal mathematics" (Kuster et al., 2018). While we see the desire for more student-centered instruction as legitimate, it seems that there are places in which that desire may be at odds with some of the goals that mathematicians have for class time.

Second, in problematizing lecture-based instruction, we implicitly disavow a disciplinary practice that they themselves turn to when they want to learn (i.e., listening to lectures). This stance disavows a practice that is seen as beneficial by many in the community we are trying to reach. Further, we may be asking them to replace that practice with something that fails to provide them with those same benefits. This information not only allows mathematics education researchers to better understand why so many of our instructional innovations have failed to be adopted at scale, it also provides additional considerations for designers and disseminators of active-learning innovations as they think about how their products do (or can be adapted to) provide benefits more aligned with the goals of practitioners while preserving their own values.

Acknowledgments

This research was supported by National Science Foundation award numbers #2315058, #2315056, #2315057. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

References

Artigue, M. (2016). Mathematics Education Research at University Level: Achievements and Challenges. *First conference of International Network for Didactic Research in University Mathematics*, March 2016, Montpellier, France. Downloaded from: https://hal.archivesouvertes.fr/hal-01337874/. Last downloaded September 4, 2025.

Eagan, K. (2016). *Becoming More Student-Centered? An Examination of Faculty Teaching Practices across STEM and non-STEM Disciplines between 2004 and 2014*: A Report prepared for the Alfred P. Sloan Foundation.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative research in psychology*, *3*(2), 77-101.

Fukawa-Connelly, T., Johnson, E., & Keller, R. (2016). Can math education research improve the teaching of abstract algebra? *Notices of the American Mathematical Society* 63(3).

Fukawa-Connelly, T., Weber, K., & Mejía-Ramos, J. P. (2017). Informal content and student note-taking in advanced mathematics classes. *Journal for Research in Mathematics Education*, 48(5), 567-579.

Herbst, P., & Chazan, D. (2011). Research on practical rationality: Studying the justification of actions in mathematics teaching. *The Mathematics Enthusiast*, 8(3), 405-462.

Johnson, E. (2019). Chapter 6. Undergraduate mathematics instruction: Not as bad as you'd think? S. Laursen & B. Ruedi (Eds.), *Levers for change: An assessment of progress on changing STEM instruction*. American Association for the Advancement of Science. Washington, DC.

Johnson, E., Caughman, J., Fredericks, J., & Gibson, L. (2013). Implementing inquiry-oriented curriculum: From the mathematicians' perspective. *Journal of Mathematical Behavior*, 32(4), 743–760.

Johnson, E., Keller, R., & Fukawa-Connelly, T. (2018). Results from a survey of abstract algebra instructors across the United States: Understanding the choice to (not) lecture. *International Journal of Research in Undergraduate Mathematics Education*, 4(2), 254-285.

Johnson, E., Weber, K., Fukawa-Connelly, T., Mahmoudian, H., & Carbone, L. (2025). Collaborating with mathematicians to use active learning in university mathematics courses: The importance of attending to mathematicians' obligations. *Educational Studies in Mathematics*. Published online Jan 17, 2025

Kuster, G., Johnson, E., Keene, K., & Andrews-Larson, C. (2018). Inquiry-oriented instruction: A conceptualization of the instructional components and practices. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 28(1),* 13-30.

Laursen, S. L., & Rasmussen, C. (2019). I on the prize: Inquiry approaches in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education*, *5*(1), 129-146.

Roth McDuffie, A., & Graeber, A. O. (2003). Institutional norms and policies that influence college mathematics professors in the process of changing to reform-based practices. *School Science and Mathematics*, 103(7), 331–344.

Weber, K., & Fukawa-Connelly, T. (2022). What mathematicians learn from attending other mathematicians' lectures. *Educational Studies in Mathematics*, 112(1), 123-139.
Wu, H. (1999). The joy of lecturing - with a critique of the romantic tradition of education writing. In S. G. Krantz (Ed.), *How to teach mathematics* (pp. 261–271). American Mathematical Society.