Students' Initial Goals, Beliefs and Conceptions of Proof in an Introduction to Proof Course

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The shift to proof-based mathematics in university programs often takes place in an introduction to proof (ITP) course. Literature exists describing the nature of these courses and some of the difficulties students experience in the course, but we know little about students' goal, beliefs, and conceptions of proof at the beginning of an ITP course (David & Zazkis, 2020; Bleiler-Baxter & Pair, 2017). Seven students were interviewed during the first two weeks of their ITP course. These same students are participating in a longitudinal study examining how ITP students' goals, beliefs, and engagement patterns evolve during an ITP course. Students were uniformly capable of providing a description of proof compatible with those commonly used in ITP courses, though students' ability to construct proof was varied. Students could identify components of proof writing that were different from other writing (e.g. the use of the word "or"). Students' goal orientations were seen to be malleable at the beginning of the semester, with most students adopting a learning goal orientation (Dweck & Leggett, 1988).

Keywords: introduction to proof, goal orientation, conception of proof

Mathematical proof is sometimes introduced in secondary school in the US, but frequently the first thorough introduction to proof that US students in some STEM programs receive occurs at the university level (Herbst, 2002). To combat the difficulty many students experience adjusting to proof-centered mathematics, many universities designate an introduction to proof (ITP) course dedicated to the comprehension and construction of mathematical proofs (Knuth, 2002; Moore, 1994). While research has been done examining the nature of ITP courses (David & Zazkis, 2020) and examining how students experience particular components of proof (e.g. Bleiler-Baxter & Pair, 2017; Hoffman, et al., 2024), there is not research that addresses students' goals and beliefs at the beginning of an ITP course. This study aims to shed light on students' goals, beliefs about mathematical learning, and conceptions of proof before extensive formal instruction in an ITP course. This study is situated within a larger project studying how students' goals, beliefs, and engagement habits evolve during an ITP semester and an intermediate proofs course immediately thereafter. The data presented in this report provides insights into the students' initial goals and beliefs that will allow us to see how these goals and beliefs change (or not) and affect engagement patterns. The following research question is addressed here: What are ITP students' initial goals, beliefs about mathematics, conceptions of mathematical proof?

Literature Review & Theoretical Framework

Mathematics education researchers have struggled to produce a definition of proof that draws an indisputable line between proof and non-proof (Stylianides, et al., 2017). This problem is sometimes called the "hard boundary problem" (Czocher & Weber, 2020). For example, there is debate about whether non-syntactic proof, such as computer-generated proofs, picture proofs, or probabilistic proofs qualify as proof (Aberdein, 2009). A definition of proof should provide justification for the inclusion or exclusion of such arguments. Experts lack of an explicit definition for proof and, consequently, there is a lack of clarity about proof for new participants in the community.

While the norms of proof writing, and consequently the definition of proof, held by distinct communities may differ, the values that underly those norms are likely similar (Dawkins & Weber, 2017). For example, most communities considering mathematical proof would likely agree that proof should increase mathematical understanding. A proof submitted for publication may be held to the norm that routine calculations or obvious justifications be omitted from the proof; however, in an ITP course a student may be required to show such calculations and justifications. It is critical to understand the norms established within the community from which a proof originates. Instructors of the same course at the same university may even have differing goals and beliefs about norms that should be adopted by students. We suspect that while practical norms differ between communities, the underlying values of what it means to prove something are relatively consistent across mathematical communities. For example, a value held by most communities considering mathematical proof is that a proof should have explanatory power. What constitutes the best way to explain differs between communities. When presented with arguments presented during a classroom discussion about why the sum of two odd numbers is even, a group of pre-service teachers valued a well-explained empirical example over a valid deductive argument (Morris, 2007). Many mathematicians would likely find a deductive argument more helpful in explaining that mathematical claim. The two communities adopted different norms to uphold the same underlying value that proofs should explain. Even subtle changes to setting such as having a different instructor or sitting next to a different classmate can influence the norms a student chooses to adopt in proof construction.

The purpose of ITP courses is to teach logic rules, proof structures, and set theory (David & Zazkis, 2020). ITP courses in the US have been classified into three categories: standard only, standard + topic, and topic-based. The data for this study was gathered in a standard + topic. Standard + topic courses (40% of ITP courses surveyed) teach skills necessary for proof comprehension and production such as understanding symbolic logic, quantifiers, set theory, and truth tables. They also include an exploration of a new mathematical topic, in this case the topic was abstract algebra.

Zone of Proximal Development

Two theoretical frameworks underpin this study: the zone of proximal development (Vygotsky, 1987) and motivation theory (Dweck & Leggett, 1988). The zone of proximal development (ZPD) describes the interplay between intra-psychological and inter-psychological domains that enables the emergence of more advanced cognitive activity (Vygotsky, 1987). The traditional interpretation of ZPD is the interplay between what an individual can do on their own and what they can do with the help of a more knowledgeable other (p. 208-209). Vygotsky also described ZPD as the dynamic interplay between spontaneous concepts and scientific concepts (p. 214-222).

A *spontaneous concept*, often referred to as an everyday concept, develops through everyday experience, usually without the aid of formal instruction (Vygotsky, 1987). Ideas within a spontaneous concept are specific and often lack generalization. For example, a child learning a native language is developing a spontaneous concept because the child knows many specific words but may not be able to generalize about categories such as “noun” or “fruit.” Ideas within a spontaneous concept are specific and lack conscious awareness, volitional control, and generalization. A student entering an ITP course likely has a spontaneous conception of what it means to prove a statement, even if the student has never been introduced to formal proof. His spontaneous conception of proof may rely on the principles of proving things in familiar settings such as justifying an answer to a classmate, explaining a solution to a teacher, or convincing a friend. While these spontaneous conceptions may proport the same purpose of proof, they are likely inadequate to equip students to comprehend or construct valid mathematical proofs. A student's spontaneous conception of proof provides important grounding to their emerging scientific conception of proof. That is, as students learn the nuanced rhetoric of mathematical proof, their spontaneous conception of proof grounds them in the purpose of the practice, which is to explain, justify, convince, etc. A *scientific concept* is one learned through mediated interaction with their formal meanings. Students develop a scientific conception of proof as they engage in formal instruction in an ITP course. This study will focus on students' spontaneous conceptions of proof and how the first 2-4 days of engagement in the ITP course support the emergence of a scientific conception.

Goal Orientation

The goals students set during a mathematics course influence the types of activities students engage in and, consequently, the conceptions of proof they develop. Thus, the proximal and distal goals students set at the beginning of an ITP course are of interest to mathematics education researchers. Dweck and Leggett (1988) proposed the nature of goals can be sorted into two categories: performance goals and learning goals. Individuals who adopt performance goals in a particular context are concerned with maintaining or gaining favorable judgements of ability from external sources. This may lead them to avoid challenging problems, which is typically seen as a maladaptive behavior since many worthwhile pursuits in both academic and non-academic life come only after confronting a challenge. Individuals who adopt learning goals in a particular context are concerned with increasing their ability. Individuals may adopt different types of goals when they are in different settings or learning different subjects. Selection of performance and learning goals are sensitive to outside factors such as task difficulty and task consistency (Steele-Johnson, et al, 2000). In this study we consider students goal orientation in the context of an ITP courses with task difficulty and content consistent with that required by the course. A student's goal orientation is influenced by motivation, personality, willingness to persist through challenges, and personal beliefs about intelligence (Dweck & Leggett, 1988). Students who believe intelligence to be a fixed entity (sometimes referred to as having a fixed mindset) are prone to adopt performance goals; students who view intelligence as malleable (sometimes referred to as having a growth mindset) are prone to adopt learning goals.

Methodology

The university where this data was gathered ran one section of their ITP course in Fall 2025 with 19 students enrolled. Seven students enrolled in the course participated in individual clinical interviews during the first week and a half of the course. Students had attended 2-5 class session at the time of the interview. Interview questions were clustered into three categories. First, students discussed their goals and expectations for the course. Second, students discussed their experience learning mathematics in prior mathematics classes. They described what has helped them be successful in past classes and what they planned to do in the ITP course to achieve success. Third, students were given five number theory claims and asked to determine whether they were true or false and explain what they though a math proof might look like for each claim. Follow up questions about these claims did not include words such as prove, justify, convince, or explain as these words may encourage the student to adopt a particular purpose of proof for the interview (Hanna, 2000). The purpose of this section of the interview was to make inferences about students' spontaneous conception of proof after minimal formal instruction. This section was concluded by asking students to share their current understanding of mathematical proof.