Keeping Time: Utilizing Non-Normative Axes in Graphing Tasks to Support Covariational Reasoning

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Covariational reasoning—reasoning about the relationship between simultaneously varying quantities—can support students' sense-making about dynamic real-world or abstract mathematical phenomena. Covariational reasoning has been shown to be challenging for undergraduates (Carlson et al., 2002), preservice mathematics teachers (Moore et al., 2019), and in-service mathematics teachers (Thompson et al., 2017). However, there is also evidence that students as young as middle school can engage in covariational reasoning (Ellis et al., 2015; Paoletti et al., 2023). Tasks in which students are prompted to construct a graph representing the relationship between covarying quantities in an animation of motion can support students in reasoning covariationally (Johnson et al., 2020; Paoletti et al., 2023). However, researchers must pay careful attention to how particular features of the task support or constrain students' opportunities to engage in such reasoning (Stevens et al., 2017).

Covariational reasoning entails constructing quantities (i.e., identifying attributes of a situation that are measurable; Thompson, 1994), imagining those quantities changing in some way (Castillo-Garsow, 2012), and then coordinating those changes (Thompson & Carlson, 2017). When graphing tasks prompt students to coordinate changes in a non-temporal quantity with changes in the quantity of time, students often reason variationally by keeping elapsing time as implicit and something they sense passing by as the other quantity changes (Janvier, 1998; Johnson et al., 2020; Paoletti, 2015; Stevens et al., 2017). As a result, researchers have recommended avoiding using time as a quantity in graphing tasks that are intended to promote covariational reasoning (Paoletti et al., 2022; Stevens et al., 2017). However, many quantitatively rich dynamic situations are time-dependent, particularly in courses that are preparing students for future work in science and engineering fields. In those cases, adhering to the design recommendation would create other curricular tensions.

In this poster, I address the question: *How do students reason about the relationship between time and distance in a dynamic situation when engaged with a graphing task that utilizes non-normative axes?* I use data collected from a series of task-based interviews (Goldin, 2000) with undergraduate Calculus 1 students. Each student participated in two 90-minute interviews which consisted of a series of animation-based graphing tasks designed to elicit covariational reasoning. I coded for the level of covariational reasoning (Thompson & Carlson, 2017) that I inferred students drew on as they engaged with the graphing tasks. I compared students' covariational reasoning on distance-time graphing tasks that utilized normative and non-normative axes (i.e., time on the vertical axis). I found that students engaged in much more sophisticated levels of covariational reasoning when engaged with the non-normative graphing tasks. This finding indicates that designing graphing tasks with non-normative axes is a strategy for supporting students in reasoning covariationally even in contexts where time is one of the quantities. This has implications for curriculum design and research on covariational reasoning in the context of graphing tasks.

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