

Examining Undergraduates' Conceptions of Series Convergence via Script Writing

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Students typically encounter the definition of series convergence as the limit of a sequence of partial sums in second-semester calculus. However, students struggle coordinating series and partial sums (Martin, 2013; Martínez-Planell et al., 2012), which Martínez-Planell et al. attribute to a weak limit schema. This supports claims by McDonald et al. (2000) that students require a “a rich sequence schema ... to fully understand series as the limit of a sequence of partial sums” (p. 98) and that students encounter difficulties in applying their understanding of sequences to subsequent mathematical concepts. Other research on student understanding of series includes how students think about series convergence (e.g., Eckman & Roh, 2022; Martínez-Planell et al., 2012) and how students interpret or develop series notation (e.g., Eckman & Roh, 2024a, 2024b; Larsen et al., 2024). Researchers recognize the challenges students encounter in learning series and the need for meaningful tasks that enhance student learning about series (e.g., Lindaman & Gay, 2012; Torres Manzanarez & Álvarez, 2025; Vroom & Alzaga Elizondo, 2025).

This exploratory study centers on examining students' understanding of series convergence using both non-scripting and scripting-based tasks to gain insight into the affordances of script writing for assessing understandings in this context. Script writing requires student engagement in imagined role-play by writing a script that extends a prompt that begins a dialogue between a teacher and student or between two students. In 2018, Zazkis issued a call-to-action for researchers to use script writing to explore or assess mathematical understandings. This poster addresses the following research question from a larger study: What is the nature of possible conflict factors with regard to students' concept definition and concept image of series convergence that arise in student responses to non-scripting-based tasks and a scripting task?

In a departmentally-coordinated second-semester calculus course, over a 15-week semester, 105 students across ten different sections granted researchers access to their second midterm exams and scores. Participants ($n=18$) scoring in the third and fourth quartiles consented to engage in two interview sessions. They completed two interview sessions which included: (a) a non-scripting task-based interview, (b) a script writing training activity, (c) a scripting task, and (d) a semi-structured interview about their script. Half engaged in the non-scripting task in the first interview session and the scripting task in the second interview session. The order was reversed for the other participants. Qualitative data analysis entailed coding interview responses using a priori codes on student conceptions of series identified in the research literature and using the theoretical lens of concept image and concept definition (Tall & Vinner, 1981). In addition, emergent codes from the data were identified.

Aligning with Martin (2013) and Martínez-Planell et al. (2012), participants encountered difficulties relating the sum of a series and the limit of the series' sequence of partial sums. However, the scripting task revealed aspects of participants' understanding of series in relation to partial sums even for participants who were unsuccessful demonstrating this on the non-scripting tasks. This exploratory work underscores the potential viability of scripting tasks as a tool for investigating student understanding of series convergence. Furthermore, this study shows promise for future work in developing and using new scripting tasks on series in second-semester calculus courses and tracking their effectiveness for improving student learning.

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