

A Model of Epistemological Justification for Teaching and Learning Linear Algebra Proofs

Avery Madden
The University of Oklahoma

Sepideh Stewart
The University of Oklahoma

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Research on proofs has long been a topic of interest for the mathematics education community (see Melhuish et al., 2022). Despite this, linear algebra has not received substantial research attention in proof and topics in second courses (Stewart et al., 2019, 2025). Some of the conversations between studies in proof based linear algebra have been on how students should think about the proofs. For example, Stewart and Thomas (2019) observed that a group of students preferred to reason with familiar objects, and some desired to use symbolic or embodied examples to aid their thinking. Meanwhile, Harel (2019) argued that building too strong a geometric reasoning for students may cause difficulties in proving in abstract or unfamiliar cases. Regardless of the approach, there is evidence that a primary goal of instructors is for students to have a strong ability to prove (e.g., Rensaa et al., 2020; Johnson et al., 2025). Furthermore, some studies offer insight into how students perceive proofs in linear algebra. A common theme that has emerged is that students struggle with the interplay between concepts in linear algebra proofs (e.g., Stewart & Tran, 2022; Harel, 2024b; Stewart & Thomas, 2019). There is developing research into combating these challenges for students and fostering a strong proving ability. For example, Meyer et al (2024), examined an instructor's textbook where he promoted student-created proof and exploration. Altogether, more research on examining the instructor's decisions on promoting critical thinking about proofs in linear algebra is needed.

Capturing this idea of thinking critically about proving is something that needs to be defined. The aim of this study is to build a model explaining how students think about the formation of their knowledge and proofs in linear algebra. Harel (2024a) outlines a theoretical framework for analyzing student understanding in relation to both proofs and broad mathematical knowledge. He introduces the notion of epistemological justification, “a way of thinking that manifests itself through perturbation-resolution cycles revolving around the question why and how was a piece of mathematical knowledge conceived?” (Harel, 2024a, p. 1488). Epistemological justification has specific criteria for its existence, those are the intellectual need condition, the elicitation condition, and the awareness condition. Intellectual need itself comprises five different categories according to Harel, those being: certainty, casualty, computation, communication, and structure. Further, there are two types of observed epistemological justification, discerned and embedded (Harel, 2024a). He poses some open questions to continue building on the framework of epistemological justification. One of particular interest is “What instructional practices promote the habit of seeking epistemological justification among students?” (Harel, 2024a, p. 1499). To that end, we hope to apply Harel’s (2024a) theory of epistemological justification to students’ thinking about proof and identifying what instructor decisions may have promoted this thinking.

The research questions guiding this study are: What are some instances of students' epistemological justification, and what instructor decisions promote them? This proposed case study aims to bring attention to a gap in linear algebra education research on how instructors interact with students during the proving process. This model will be used in a second course on linear algebra.

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