Examining Online Mathematics Instructional Videos Through a Narrative Lens: The Role of Mathematical Questions

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The increasing use of online videos in mathematics education has led to a need to understand what makes them captivating. This study applied Dietiker's mathematical story framework to online mathematics instructional videos (OMIVs), specifically analyzing several videos from a popular YouTube channel. Using a qualitative case study approach, we identified the central role of questions from a narrative lens and observed four open question types and three types of their closure. Findings also show how different question types may relate to the plot's characteristics and enrich the plot. This study is an initial exploration of mathematical stories in the context of OMIVs, providing new perspectives that would hopefully inspire further research and practical application in math education.

Keywords: mathematical story, online videos, complex Fourier series, plot, questions

The COVID-19 pandemic accelerated digital learning, making instructional videos more common in mathematics education (Alabdulaziz, 2021). Online mathematics instructional videos (henceforth called OMIVs) support engagement and understanding, and educators increasingly recognize their pedagogical value (Esparza Puga & Aguilar, 2023). Yet their quality varies, raising questions about why some videos are more engaging to the viewer (Jones & Cuthrell, 2011; Kolthof, 2021). A striking example is the YouTube channel 3Blue1Brown, whose creator Grant Sanderson argued that stories play an important role in engaging mathematical learners, with his widely spread videos deeply supporting his argument (TEDx Talks, 2020). Story, more broadly, has long been used as a pedagogical tool in mathematics for its capacity to guide emotions and foster meaning (Zazkis & Liljedahl, 2009). Building on this idea, Dietiker's (2015) mathematical story framework conceptualized mathematics lessons as unfolding events and has applied the framework to the context of lessons (e.g., Richman et al., 2019). This study adapts Dietiker's framework to analyze selected OMIVs, exploring how storytelling shapes math questions and their resolution to inform the design of more engaging instructional content.

Literature Review

Educators increasingly recognize the vital role of technology in mathematics education, as digital technologies continue to shape the current state and future of teaching (Borba et al., 2016; Darragh, 2021). With internet access and applications now available to much of the world's population, digital resources have become embedded in teaching and learning. More recently, the COVID-19 pandemic accelerated this trend, making online learning and digital products central to instruction (Ilmadi et al., 2020). The accessibility of digital devices enables learning beyond the classroom, and research highlights how such tools can enhance student attention, active engagement, and independent learning (Alabdulaziz, 2021; Fakhrunisa & Prabawanto, 2021). Moreover, compared to in-person teaching, online platforms reduce barriers of time and space and foster new forms of interaction among students and teachers (Ilmadi et al., 2020).

Whether lecture recordings or designed instructional videos, these have become a regular part of students' online mathematics learning. Researchers have examined the factors contributing to their effectiveness, such as video length, production quality, and instructor presence, which could influence students' engagement (Acuña-Soto et al., 2018; Larkin, 2017). Equally important are pedagogical considerations: videos support students' conceptual understanding and sustained attention when they align with curricular goals, follow explicit learning objectives, and present sound mathematical explanations (Cardoso et al., 2014; Johanes & Lagerstrom, 2016). Visualization, such as animations, graphs, and text, can enhance comprehension when meaningfully aligned with learning goals (Ismail et al., 2017). In addition, recent research also highlighted that utilizing technology to transition between different realizations of mathematical objects could contribute to audience engagement in videos (Radmehr & Turgut, 2024).

While prior studies have explored technical and pedagogical aspects of OMIVs, to our knowledge, there has been less focus on their narrative and aesthetic dimensions. Researchers believe that narrative is central to mathematics learning, as it may bridge mathematics and students' experience, fostering cognitive and emotional engagement by grounding abstract mathematical ideas in familiar contexts (e.g., Toor & Mgombelo, 2015). Dietiker's (2015) mathematical story framework, developed to analyze curricula and lessons, offers a useful lens for evaluating such narrative structures. This study extends that work by applying the framework to OMIVs, aiming to illuminate how storytelling could play a role in their design and impact.

Theoretical Framework

Dietiker (2013, 2015) describes a *mathematical story* as the way mathematical content unfolds across a sequence of changes – that is, the progression of mathematical concepts. Building on this idea, Dietiker introduced four elements of a mathematical story: characters, actions, setting, and plot. *Characters* are mathematical objects, such as shapes, equations, or numbers, that may develop characteristics (e.g., a number being prime) or relationships (e.g., two functions intersecting). Character development occurs through *actions*, which mathematical actors, typically humans, perform. For example, in a video, a narrator may differentiate a function $f(x)$ – here, "differentiate" is the action on the character $f(x)$. The *setting* is the space where characters and their actions take place. Dietiker described it as the representations of the content that situate the story, which help the reader interpret the characters and their actions more clearly; for instance, $y = x$ appears as a line in two dimensions but as a plane in three. The *plot* refers to the aesthetic dimension of how readers encounter the story. Plot shifts occur when information changes, prompting readers to raise or answer questions, experience tension, and feel satisfaction or surprise.

Richman et al. (2019) further developed three characteristics of plot: density, coherence, and rhythm. *Density* concerns the number of open (i.e., unresolved) and closed (i.e., resolved or no longer under consideration) questions; and a plot becomes denser when more questions remain open. *Coherence* refers to how events connect and help learners anticipate or interpret what follows, producing responses such as pleasure or surprise. *Rhythm* is the pattern of opening and closing questions, shaping the story's pace. To interpret the overall plot, Richman et al. (2019) also introduced the idea of a *story arc*, which is the trajectory of a question and all the shifts it undergoes as it moves from being open to being resolved.

The Study

This study employs a qualitative case study to examine how OMIVs can be viewed through a narrative lens. Building on Dietiker and colleagues' work on plot analysis through opening and closing questions (2013, 2015; Richman et al., 2019), this study aims to extend this framework to video contexts. Specifically, we classify and characterize open and closed questions within the mathematical narrative structure in OMIVs, and then examine how these question types connect with characteristics of the mathematical plot and contribute to its development. Guided by this framework, we seek to answer the following research questions:

1. What types of open mathematical questions and their closures appear in OMIVs?
2. How are these question types connected with the characteristics of the mathematical plot?

Data Collection

We collected the data from the YouTube channel 3Blue1Brown (3Blue1Brown, n.d.). As of August 2025, the channel had 7.57 million subscribers, 216 videos, and over 672 million views. Its creator, Grant Sanderson, has been recognized by the American Mathematical Society for his contributions to mathematics education (News from the AMS, n.d.). We selected this channel because of its wide reach and strong reputation in online mathematics education, and we note its pedagogical value has also been recognized in prior research (e.g., Radmehr & Turgut, 2024).

We analyzed four videos from this channel, chosen for their mathematical content, number of views, and the creator's identification of them as more narratively compelling. The videos are: V1 (3Blue1Brown, 2019c) on Complex Fourier Series within a sequence of videos on differential equations; V2 (3Blue1Brwon, 2017) on a Putnam problem and its solution; and V3 (3Blue1Brown, 2019a) and V4 (3Blue1Brown, 2019b) that constitute a two-part presentation that introduces and resolves a mathematical puzzle.

Data Analysis

We have reviewed each video multiple times and divided it into acts, defined by shifts in characters, actions, settings, or relationships (Richman et al., 2019). Acts varied in length depending on changes in focus, from under a minute to several minutes. We identified each mathematical question within each act, including explicit and implicit questions. To do so, we adapted Richman et al.'s (2019) framework by defining *explicit questions* as those posed directly by the narrator, and *implicit questions* as those that emerged from the unfolding plot in ways likely to inspire curiosity in the viewer. For example, "Perhaps you see the pattern here" could implicitly raise the question: "What is the pattern?". Also, following Richman et al. (2019), we grouped similar questions within the same act into a broader question to avoid artificially inflating question density.

For implicit questions, we included the questions we formed when watching the videos and questions we recognized that other audiences might form. Additionally, we assumed that the audience has a foundational understanding of the mathematics involved. Therefore, we focused on questions relevant to a level of knowledge that supports the understanding of the video.

Lastly, we coded the questions into story arcs and connected the story arcs to plot characteristics. Specifically, we explored the plot's density by examining the number of acquired and resolved questions across acts, analyzed its coherence by considering how arcs overlapped or linked, and traced its rhythm through the pacing of suspense and resolution. To contextualize narrative analysis with audience response, we compared the YouTube engagement graphs with plot features, offering insights into potential links between story elements and viewer engagement. The findings are presented below, with a primary focus on V1 due to word constraints.

Findings

Classification of Question Types

Types of open question. We observed four types of questions regarding how they were opened in the narrative. The first type is the *main question*, which introduces core mathematical content or ideas and serves as a central focus of the video. Progress on one main question does not necessarily contribute to a previous one. That is, while connections between main questions may exist, each relies on different knowledge. For example, in V1, the first question [Q1] was: "What is a complex Fourier series?". We classify this as a main question because it introduces one central idea without relying on earlier ones. Another example from V1 is: "How did the Complex Fourier Series create animation?". Although it seems to build on the first, we also treat it as a main question since its answer requires new knowledge rather than directly resolving the earlier one. Main questions may overlap, but each sustains its own distinct arc.

The second type is the *sub-question*, which relates to a previous question and helps make progress on it without altering its original conditions. Typically, a sub-question breaks a larger question into smaller, more manageable parts. For example, in V2, one main question is: "What is the probability that the center of the sphere is inside the tetrahedron [formed by four random points on the surface of this sphere]?". A related question is: "Which […] tetrahedra contain the sphere's center?". Because this question contributes to resolving the main question while preserving its conditions, we classified it as a sub-question.

The third type is the *helping question*. Like sub-questions, helping questions relate to a prior question, but they differ in that they modify the conditions to simplify the problem and enable progress. For example, in V2, for the same main question above, one helping question is: "What is the average size of the section that the fourth point could land if the other three points are fixed?". Fixing three points changes the conditions, making the problem more accessible. Helping questions, therefore, often serve as a bridge to solving the main question by reducing its complexity.

The last type is the *reformulated question*, which rephrases an earlier one with different wording or structure but preserves the same core idea. For instance, in V1, one question is: "Why don't we use 'vectors' instead of a 'complex number' in a complex plane?". Later in the video, the narrator revisits the same idea: "You'll notice I'm being a little loose with language, using the words vector and complex number somewhat interchangeably", which essentially asks, "Why are vectors and complex numbers used interchangeably?". Since both questions address the same underlying idea, we interpreted the latter as a reformulation of the former.

Types of question closures. We also observed three types of question closures. The first type is *normal closure*, which occurs when a question receives a clear and definitive answer, leaving no ambiguity. If a question has a normal closure, it has been fully addressed in the current context, enabling the audience to fully understand the explanation. Most of the mathematical questions in the videos fall into this category.

The second type is *inquiry closure*, in which the narrator deliberately leaves questions unanswered for further thinking or exploration. Inquiry closure occurs when the narrator poses a question and explicitly invites the audience to reflect or try solving it themselves. The purpose may be to promote active learning, deepen understanding, or develop problem-solving skills. For example, in V1, the narrator asked, "How do we find the coefficients in the step function?" and left the audience to think more about the question.

The third type is *deferred closure*, which occurs when the narrator postpones the full answer to another video or avoids giving it completely because the content is out of scope or irrelevant to the main topic. For example, in V3, the narrator asked: "What is configuration space, or phase space?". He explicitly stated that further explanation would appear in the next video, and indeed, the following video continued with the same puzzle and expanded on this idea. Therefore, we suggest that this question exhibits a deferred closure because its scope extends beyond the current puzzle to broader areas of mathematics.

Question Types and Plot Characteristics

This subsection addresses RQ2 by examining how question types play a role in the plot's density, coherence, and rhythm.

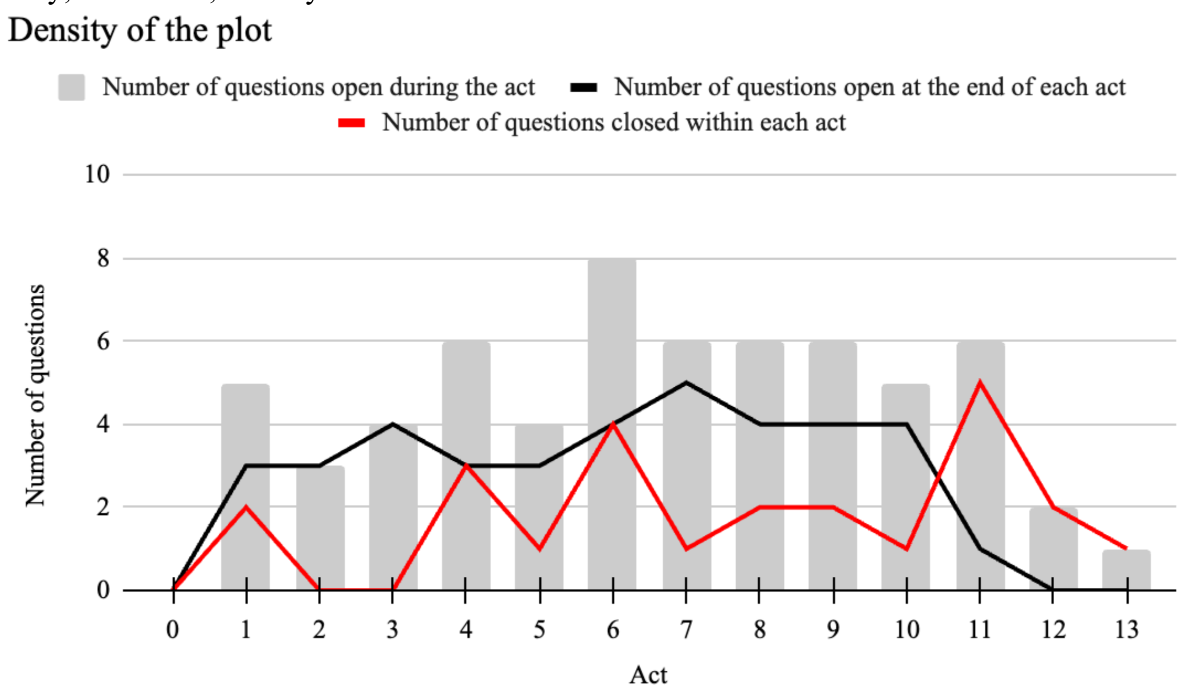

Figure 1: Density of the plot in V1

Questions and density. To recall, the density of the plot refers to how many mathematical questions remain open within a segment of the story (Richman et al., 2019). As Richman et al. (2019) explained, the plot becomes denser if more questions are left open, and a release in density occurs when the number of open questions decreases. Following the approach described in the Data Analysis section, we excluded reformulated questions when considering plot density because they do not introduce new issues, but merely re-express existing ones.

We observed a clear trend when considering how different open questions affect the density. Taking V1 as an illustrative example (see Figure 1), the density trajectory (see black line) first appears as a rising path marked by short periods of small release or sustained in between, then reaching a peak and holding steady for a time before undergoing a sudden release. Additionally, acts that open a new main question tend to generate a higher density for more than one act. The unfolding of complex mathematical concepts appears to be guided by a main question, which establishes the current topic within the narrative and where density accumulates. Supporting questions sustain and elaborate on this main question, causing the density to continue rising, reach a peak, and eventually release. This dynamic is visible in Figure 1, where peaks in density align with acts that open new main questions. For instance, Acts 1, 3, and 7 combine at least one new main question with more than one helping or unresolved sub-questions.

Figure 2: V1 engagement graph

Another perspective concerns the release of density. Figure 2 shows the engagement graph above the progress bar of V1, highlighting the most replayed moment in V1, which corresponds to Act 11. As Figure 1 illustrates, Act 11 involves a major release in density at the end of the act, with several previously open questions resolved at once. In contrast, the ends of previous acts (i.e., 8, 9, 10), show that the density remained relatively high for a while, indicating sustained tension. That is, the closures in Act 11 mark the resolution of a build-up maintained across earlier acts. The coinciding of this release and the peak of audience replays suggests that viewers are especially attentive when the accumulated tension is lifted. We therefore suggest that, rather than being drawn primarily to the opening of new questions, engagement appears concentrated at points where narrative pressure is released.

Questions and coherence. As a reminder, coherence refers to how different parts of the story connect. The classification of opening questions helps explain coherence because each type contributes a distinct way of linking ideas into the plot. In general, we suggest that main questions frame long story arcs, while sub- and helping questions sustain or clarify them, preventing the plot from fragmenting. In other words, sub- and helping questions may maintain coherence by breaking complex problems into manageable steps. For example, in V2, the main question "What is the probability that the tetrahedron formed by the four random points on a sphere's surface contains the sphere's center?" forms a long story arc that is resolved at the end of the video. Yet, it coheres with other sub- and helping questions, which develop supporting concepts and link back to the overarching narrative formed by the main question.

Closures also contribute to coherence, but in a more indirect way. We suggest that normal closures provide immediate clarity, possibly helping the audience build confidence to engage with longer arcs. Inquiry and deferred closures, in turn, may sustain coherence across videos by leaving questions open for later explanation and creating anticipation.

Questions and rhythm. To remind, rhythm refers to the pacing of the video, shaped by the pattern of how questions are posed and answered. We found that videos usually begin with more than one overarching main question in the first arc. This can create a slow rhythm, as the audience is generally unable to resolve them immediately in the same or next few acts, and they typically reach closure only in later acts. Also worth noting is that main questions are interspersed throughout the narrative but appear intermittently rather than at regular intervals. Yet most of their occurrence reinforce the video's narrative by either strengthening or shifting the focus through transitions in the main topic. This irregular pattern contributes to changes in pacing. In contrast, sub- and helping questions with normal closures maintain a steady pace by resolving smaller puzzles and advancing the narrative. The introduction of a new main question interrupts this steady rhythm, possibly breaking the audience's expectations and creating a new rhythm that may generate surprise.

For example, in V1, the rhythm begins with a consistent pace in Act 1, where two main questions have a long story arc, suggesting the story begins at a relatively slow pace. A recursive and steady pace follows, driven by shorter sub-questions that the narrators give answers to at a faster pace. However, this pace is then interrupted in Acts 6, 7, and 8, where the main question comes into place and takes over. In Act 6, the main question, "What would happen if the heat equation solutions of the cosine waves were all added up?" initially builds up suspense. Still, the narrator answers this question at the end of Act 6, creating a sense that the next suspense would also be easily solved. However, what follows at the start of Act 7 is, "Why does the added complexity come up?", which cannot be answered within the regular pattern. After that, the pace returns to the previous in Acts 9 and 10, preparing for the changing of rhythm in Act 11, where most of the main questions get their closure. We propose that changing the rhythm, by intermittently shifting the focus at a steadier pace, can encourage students to think more, which may explain the small peaks in Figure 2.

Discussion

This study analyzed four OMIVs from the YouTube channel 3Blue1Brown to examine the role of mathematical questions in shaping mathematical narratives. We expanded Dietiker and colleagues' framework by identifying four types of open questions: main, sub, helping, and reformulated, which emerged at different parts of the story. Main questions introduced new concepts, while sub and helping questions extended them in ways that made the original idea more approachable. Reformulated questions rephrased earlier inquiries to reinforce their existence as unanswered questions. We also identified three types of question closures: normal, inquiry, and deferred. Normal closures resolved questions fully, providing clarity and closure. Inquiry closures left questions deliberately open, inviting independent exploration. Deferred closures postponed answers to the future, maintaining suspense and continuity across episodes.

The analysis of the videos showed that the type of question is related to the characteristics of the mathematical plot. It seems that main questions often bring higher plot density, slow down the development of the plot, enrich the plot, and accordingly may stimulate students' thinking. In contrast, the sub- or helping questions help the plot smoothly transition to the next stage by providing clear answers. The closure of questions, meanwhile, reinforces key mathematical concepts, providing the audience with an immediate sense of clarity and conclusion. Together, these forms of question openings and closures enrich the plot by alternating its pace, increasing and releasing density, and stimulating deeper reflection by creating connections between different parts of the story. This is in line with Richman et al.'s (2019) observation that alternating plot characteristics not only maintains students' interest but also improves the coherence and logic of the mathematics lesson.

Our classification of question types and their roles highlights the importance of a strategically structured narrative for maintaining engagement, as supported by previous research (e.g., Acuña-Soto et al., 2019). As noted above, the different question types shape the story's unfolding, contributing to the interconnected progression of the narrative. The findings suggest that an intentional use of questions in OMIVs aligns with the mathematical storytelling framework and points to the potential for future educators to apply this lens when designing OMIVs – by varying question types and strategically timing their closures.

In conclusion, this research demonstrates that questions play a central role in shaping mathematical narratives and provides a lens for digital content, which may also apply to classroom design. We suggest that by considering the type of opening questions and their closure, educators can design lessons that are logically coherent, as well as emotionally and cognitively engaging. While this study focused on a single YouTube channel and did not include audience feedback, future research could examine different OMIVs, contexts, or lesson formats, and investigate how coherence, rhythm, and density may be designed to shape learners' engagement.

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