Mathematicians' Views on Intuitive Concepts in Abstract Algebra

Jessi Lajos
Utah State University

Holly Zolt
Middle Tennessee State University

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Developing an intuitive feel for mathematical objects (e.g., concepts, problems, principles, languages/representations, etc.) (Godino et al., 2022) and thinking intuitively is considered an important skill as students transition into more advanced university coursework (Burton, 1999; Davis et al., 2012). While developing an intuitive feel for the mathematics is important, many studies have documented that abstract algebra, a foundational undergraduate course for advancing math majors and pre-service teachers, continues to be difficult for students to grasp (e.g., Melhuish & Fasteen, 2016; Melhuish et al., 2020). Given these continued difficulties, a more clarified understanding of how to better support students in developing an intuitive feel for key abstract algebra concepts is needed. However, it seems that experts' views on what constitutes an intuitive or non-intuitive concept, the methods they used as learners to develop an intuitive feel for key abstract algebra concepts, and how they integrate such methods into their teaching practices has yet to be explored and compiled for specific undergraduate course contexts. Considering these gaps, we address the following research questions: 1) What criteria do mathematicians (who are also instructors of abstract algebra) use to characterize a concept as intuitive/non-intuitive? 2) Is the "intuitive" quality of an abstract algebra concept fixed, or can it change? If it can change, what methods or events do mathematicians credit with helping them shift an abstract algebra concept(s) from having a non-intuitive to intuitive quality?

While the goal of this work is to explicate what mathematicians mean by "intuitive", we are informed by prior work on intuition. Currently, there are various understandings of what it means for something to be intuitive. In light of this, we chose to draw on Semadeni's (2007) theorization of "deep intuition" (p. 3) as it relates to concepts and intuiting methods (e.g. Nathan et al., 2021).

The data comes from a completed pilot study aimed at investigating mathematicians' views of "intuitive'' versus "non-intuitive" qualifiers in connection with various abstract algebra concepts. These concepts were drawn from Melhuish's (2019) Delphi study that ranked abstract algebra concepts for difficulty and importance (e.g., first isomorphism theorem, quotient groups, group properties, etc.). We conducted hour long interviews with 7 mathematicians from various universities across the United States selected through convenience sampling. We note that all participants regularly teach abstract algebra at their respective universities. Mathematicians were initially asked a series of questions regarding their beliefs on what it means to them for a concept to be "intuitive". They then engaged in card sorting activities (Rugg & McGeorge, 2005) which further illuminated the criteria used to decide if a concept is intuitive. To analyze our data, we used Thematic Analysis (Braun & Clark, 2006).

Based on a compilation of our interview data, we found a multitude of themes or criteria that mathematicians used to characterize a concept as intuitive. One such criteria was being able to work with the concept in practice. We also found that they used varied approaches to develop an intuitive feel such as practice and teaching a course. We expand further on these results and provide future directions for research.

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