A Social Network Analysis of Race/Ethnicity and Gender in Inquiry-Oriented Linear Algebra

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This study investigates mechanisms of equity in active learning by examining student networks in an Inquiry-Oriented Linear Algebra (IOLA) classroom through social network analysis (SNA). Drawing on the sociopolitical perspective, particularly the dimension of power in equity, the research analyzes how race/ethnicity and gender shape students' peer nominations. Data were collected from an IOLA class of 64 students at a Southeastern U.S. university, using discussion board "shout-outs" and replies to track social networks. Findings reveal strong patterns of racial and gender homophily and highlight that women of color occupied disproportionately central positions within the network. While these positions suggest potential empowerment, they may also reflect additional labor on the part of women of color to sustain visibility and influence. The study underscores the need to frame equity in active learning not as inherent but as contingent on power dynamics, solidarity, and segregation within mathematics learning communities.

Keywords: Inquiry-oriented instruction, equity, social network analysis, women of color

Equity in postsecondary Science, Technology, Engineering, and Mathematics (STEM) education remains an urgent issue due to systemic barriers. Interventions often focus on "helping" or "fixing" students, yet such approaches rarely address the structural issues of racism, sexism, and microaggressions that persist in STEM classrooms (McGee, 2020). Mathematics is widely recognized as a gatekeeper in STEM (Battey et al., 2022; Leyva, 2021), where narratives of innate ability and deficit views of minoritized students contribute to inequity. These conditions call for research that investigates how equity is enacted or constrained.

Active learning has emerged as a reform strategy by shifting the focus from lecture to participation, collaboration, and inquiry. Evidence shows that active learning can narrow achievement gaps for historically underrepresented students in STEM (Theobald et al., 2020). However, equity is not automatically guaranteed in active learning. This highlights the need to examine not just whether active learning works but also how equity operates within it.

In light of this necessity, Social Network Analysis (SNA) offers a promising quantitative methodology. By mapping classroom relationships through nominations and interactions, SNA makes visible the patterns of influence that shape student participation (Grunspan et al., 2016). When tied with the critical perspective and the quantitative approach on equity, SNA allows researchers to interrogate how race, gender, and power intersect in active learning classrooms. This study fills the research gap in the intersection between equity, active learning, and SNA.

This study focuses in particular on women of color. Black and Latina women remain significantly underrepresented in STEM fields and often face the "double-bind" of racism and sexism (Ong et al., 2011). Asian women occupy a paradoxical position: while Asian students are frequently stereotyped as overrepresented, Asian women encounter gendered racism that renders them invisible or devalued in STEM spaces (Castro & Collins, 2021). These complexities make the experiences of women of color crucial to understanding equity in mathematics classrooms.

Accordingly, this study investigates "What is the student social network in an inquiry-oriented linear algebra class, and how do race and gender relate to these networks?"

Literature Review and Theoretical Perspective

Equity in Active Learning: Not Automatically Equitable

Active learning refers to a pedagogical approach that promotes students' active involvement. Active learning commonly entails student-centered learning, in-class activities, small group discussions, and challenges to lecture-based and instructor-focused teaching (Freeman et al., 2014; Theobald et al., 2020).

Several studies state that active learning can aid equitable learning. The meta-analysis result by Theobald et al. (2020) shows that active learning courses in undergraduate STEM have lower achievement gaps on historically marginalized groups of students than do traditional lecturing STEM courses. Similarly, Laursen et al. (2014) found that Inquiry-Based Learning (IBL) eliminates a gender gap, based on their quasi-experimental design that compares student outcomes across four universities with over a hundred IBL and non-IBL courses.

However, equity is not inherent in active learning. According to Theobald et al. (2020), one-third of active learning courses in the meta-analysis data had wider achievement gaps in active learning versus passive learning. Also, Johnson et al. (2020) reported a gender gap in inquiry-oriented abstract algebra, with men outperforming women. Additionally, undergraduate mathematics classrooms that implemented small-group work and whole-class discussions involve inequitable experiences reported by women and/or racial minority groups of students, such as microaggressions or sexist and racist events (Ernest et al., 2019; Leyva et al., 2022).

The Sociopolitical Perspective: How to Frame Equity

Given the issues of underrepresentation of historically marginalized groups in STEM education, sociopolitical concepts and theories highlight the critical perspective on underrepresentation and marginalization in mathematics education (Gutiérrez, 2013; Gutiérrez, 2009; Adiredja & Andrews-Larson, 2017). From a sociopolitical perspective, mathematics and mathematics education are political because they involve issues of power and have the potential to challenge or reproduce existing deficit narratives about underrepresented groups of students (Gutiérrez, 2017; Adiredja, 2019).

Specifically, Gutiérrez (2009) suggests framing equity in four dimensions: *achievement*, *access*, *identity*, and *power,* and this study focuses on the ***power*** dimension, one of the dimensions of the critical perspective on equity, beyond gap-gazing. Because of the political attributes of mathematics and mathematics education, power can be measured across various components of mathematics learning, such as voices in the classroom—who gets to talk, who decides the curriculum, and opportunities for students to engage in mathematics learning (Gutiérrez, 2009). Given the need to investigate the mechanisms of equity in active learning, this study suggests a focus on students' power in such settings.

The Social Capital Perspective: How to Interpret SNA

The social capital perspective is adopted in this study to interpret SNA and students' power measures. According to Knaub et al. (2018), social ties are resources such as knowledge, material, and social actors like humans, and there is no free will, so social actors strategically and purposefully seek out others and generate ties. Thus, knowing people is more than just knowing people; it is a matter of privileges and marginalization among students. Particularly, students from historically marginalized groups may have fewer social ties than they want, or may not have them, because they strategically access social ties as coping strategies to engage more actively in class.

Methods

This study was conducted in a 4-credit-hour Linear Algebra course, which used the IOLA setting; 64 students were enrolled, taught in Fall 2023 at Gauss University (the pseudonym), a 4-year public Southeastern predominantly White institution in the United States. The instructor is a tenured White cisgender woman at Gauss University, an experienced IOLA instructor who understands the nature of inquiry-oriented instruction and equity in mathematics education.

Except for two students under 18 and one who opted out, 61 students consented to the use of their written work for research purposes. Then, among 61 students, 38 consented to self-identify their race/ethnicity and gender on the consent form. These 38 students are referred to as "network participants" in this study. Figure 1 shows the race/ethnicity and gender of the network participants. Note that there is no Black woman among the network participants, even though there seem to be some Black women in the classroom, from the classroom observation.

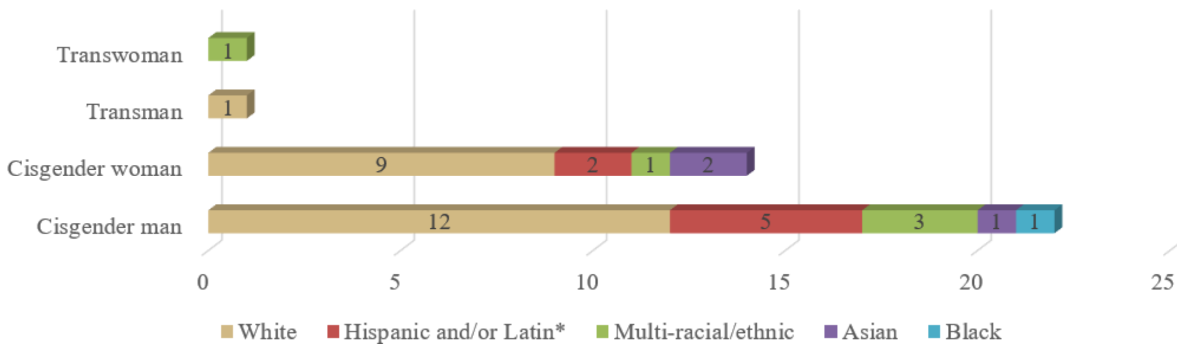

Figure 1. Demographic Information of the Network Participants
Note that students "of color" refer to "non-White," that is, Hispanic and/or Latin, multi-racial/ethnic, Asian, and Black. Latin is an inclusive term intended to recognize the presence and identities of people across the gender spectrum, including gender-nonbinary people of Latin American descent.*

The collected data sources are, first, discussion boards, where students submit online as part of their graded assignments; a survey that asks about students' prior experiences in mathematics; and self-reports of race/ethnicity and gender. The author also attended the IOLA class to observe and take field notes regarding notable patterns of social interactions.

In this study, "nomination" refers to the student nomination assignments on the discussion boards by either "shout-out" or "reply." "Shout-out" means that students nominate other students' names for those most influential in their learning, corresponding to IOLA content topics. Also, students had the option to "reply" to classmates who were helpful with their understanding, and in that case, those replies were considered the same as a "shout-out." There were 14 discussion boards assigned in the semester; 11 of them asked students to "shout-out," and three did not require a "shout-out" but allowed "reply".

SNA was conducted with "Gephi," an open-source graph and network analysis software. The first step of SNA is converting student nominations to matrices. Each matrix represents one discussion board, and students in the network are represented as numbers in the row and column matrix. For instance, for 64 students, each is represented as S01, S02, …, S64, and one discussion board is represented as a 64×64 square matrix. For one discussion board per day, entries in the matrix are either 0 or 1, where 0 indicates no nomination between two students and 1 indicates a nomination. Then, by summing the matrices across days, it shows accumulated networks throughout the semester and allows visualization of the entire network in Gephi.

Then, the number of nominations is analyzed using mean values and standard deviations, box-and-whisker plots of degrees in race/ethnicity and gender, and the Mann-Whitney U-test. Hypothesis tests were also conducted in "DATAtab," an online statistics tester using artificial intelligence. Also, ***homophily*** —the measure of patterns of who relates to whom in terms of similarity in attributes—race/ethnicity and gender —was tested using chi-square.

This study measures four types of centralities to find who is the most central in the network. The first measurement on the social network is ***degree centrality***, which identifies the most nominated students, as it represents the total number of connections. Next, ***Betweenness centrality*** measures how often students bridge paths between two other students. So, a student with higher betweenness centrality is a broker in student groups. ***Closeness centrality*** measures how students connect to others in the entire network, unlike degree centrality, which focuses on direct connections. Therefore, if a student has a high closeness centrality, that student is indirectly connected to other students through nominations, potentially influencing the whole class. Lastly, the ***eigenvector centrality*** measures how much students are networked to well-connected neighbors. So, if a student has a high eigenvector centrality, then that student has access to well-connected student neighbors.

Findings

This part addresses the social networks in an IOLA class and how race/ethnicity and gender relate to those networks generated from student nominations. The social network diagram (See Figure 2) illustrates how students nominate one another.

Who Received and Sent More Nominations? Descriptive Results of Mean Values and Standard Deviation

Note that this part uses the binary groups in race/ethnicity and gender. The racial/ethnic groups are non-White and White, and the gender groups are women, both cisgender and trans, and men, both cisgender and trans. On average, IOLA students received nominations from 3.86 (SD = 2.26) students 7.64 (SD = 5.32) times and sent nominations to 3.86 (SD = 2.66) different students 7.83 (SD = 4.13) times. Regarding gender, on average, women received 10 nominations from around 5 people, which is about 1 more than men did. Women and men sent nominations to around four people, but women sent nominations around ten times, more than men did. This comparison between non-White and White students indicates that non-White students received nominations from around one more person. Still, both groups received slightly more than nine nominations. White students sent nominations to 0.36 different people. However, according to the Mann-Whitney U-test, there is no statistically significant difference between the Non-White and White groups, and between women and men, in how often or how many different people they receive and send nominations to.

Does the Network Have Homophily? Chi-Square Test Results in Race/Ethnicity and Gender

From here, "target" refers to nodes that send nominations, and "source" refers to nodes that receive nominations. Figure 3 shows the Sankey diagrams of the homophily in race/ethnicity and gender. A Chi-square test was performed between the Source Race and the Target Race. All expected cell frequencies were greater than 5; thus, the assumptions of the Chi-square test were met. There is a statistically significant relationship between the Source Race and the Target Race ($\chi^2(1) = 42.39$, $p = <.001$, Cramér's V = 0.4). The calculated p-value of <.001 was lower than the defined significance level of 5%. Hence, the Chi-square test was significant. This means that the tendency for White students to nominate White students and non-White students to nominate non-

White students is statistically significant. Also, according to the Chi-square test, a statistically significant relationship exists between the Source Gender and the Target Gender. ($\chi^2(1) = 61.78$, p = <.001, Cramér's V = 0.48) The calculated p-value of <.001 was lower than the defined significance level of 5%. Therefore, the Chi-square test was significant. This means that the tendency for men to nominate men and for women to nominate women is statistically significant.

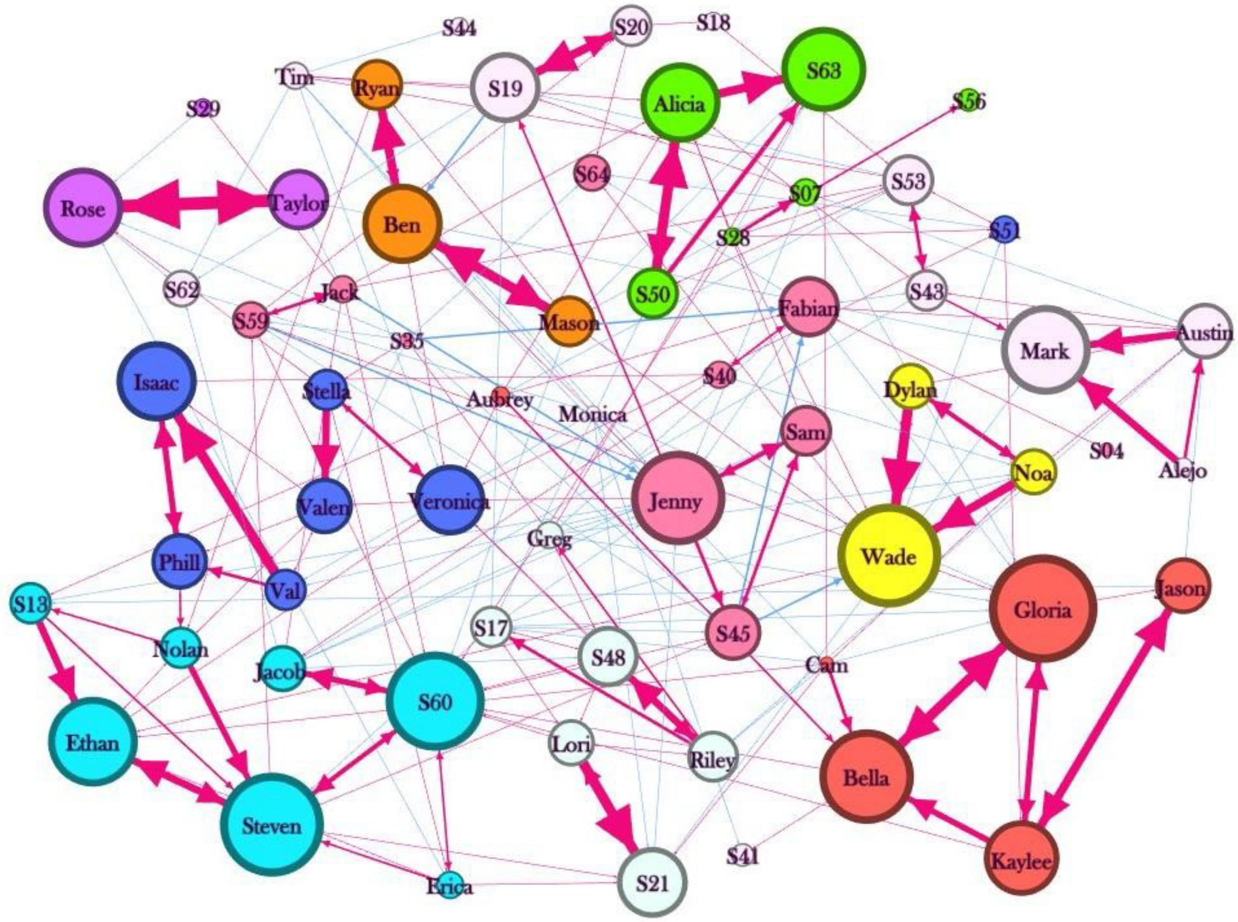

Figure 2. Social Network of the Number of Student Nominations in the IOLA Class at Gauss University.
Note: Circles with pseudonyms are network participants, and circles with codes are students who did not identify their race/ethnicity and gender. Node size represents weighted in-degree—how often the student receives nominations, regardless of redundant people. Node colors represent the small communities of the network determined by nominations. Pink arrows mean nominations by shout-outs, and blue arrows mean nominations by replies.

Who Were the Most Central? Results on Centralities Regarding Race/Ethnicity and Gender

With a comprehensive analysis of the four different centralities, the students of higher centralities include two women of color, who are disproportionately more represented in terms of race/ethnicity and gender in the class. Considering the intersections of the top 6 students (10% of the classroom population) of in-degree, betweenness, closeness, and eigenvector centralities, there are three students among the highest centralities: Jenny, Veronica, and S60. Jenny, a Chinese American cisgender woman, and Veronica, a Hispanic and/or Latina cisgender woman, are women of color. Hence, S60, Jenny, and Veronica not only received nominations from a greater variety of students (in-degree centrality) but also were able to reach other students more closely (closeness centrality), build bridges with different classmates (betweenness centrality), and connect to other well-nominated students (eigenvector centrality).

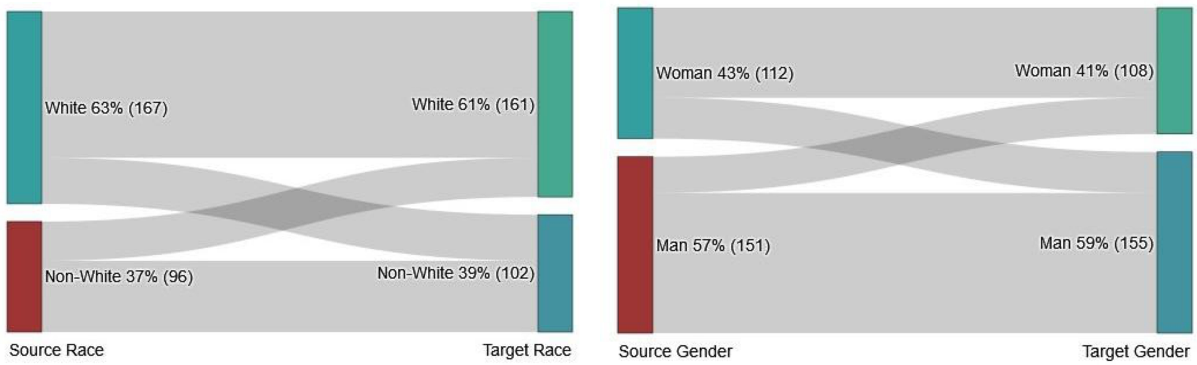

Figure 3. Sankey Diagram in Race/Ethnicity and Gender

Discussion

Limitations of This Study

This study has the following limitations. First, this study could not include Black women as network or focal participants, though Black women were observed attending the Applied Linear Algebra course at Gauss University. So, this study could not document how Black women would have social networks and experience the IOLA setting class. Second, the written works from discussion boards, including student nominations, are collected from only some classrooms because, of 64 students, two are minors and one opts out. Thus, the social network generated by nominations might miss some information. Also, since the network participants are 38 students out of 64, the SNA results related to race/ethnicity and gender might not represent the entire groups of some racial/ethnic or gender in the class.

The IOLA Class Could Imply Equitability

Considering the social capital perspective on the SNA, the results indicate that women of color could construct more empowerment. The rationale for this is that, first, women of color were disproportionately represented among the students of the highest centralities. From a social capital perspective, higher centrality indicates greater social capital, which means greater empowerment. Also, these high centralities mean that women of color could be exposed as influential students in others' learning. This implies that the linear algebra learning community could provide women of color more opportunities to be identified as influential. Additionally, the results showing no significant difference in the number of nominations between men and women, and between White and non-White students, indicate that the IOLA classroom may be where classmates perceive other students as influential. This perception would be reasonably similar across race/ethnicity and gender.

Then, was the IOLA class inherently equitable? Was the IOLA class disproportionally supportive of women of color? The response to this study remains "no." The next part addresses the reason the IOLA class was not necessarily equitable for women of color.

Potential Interpretation of Additional Labor of Women of Color

This study also interprets this potential equity in the IOLA class as resulting from additional ***labor*** by women of color. ***Labor*** refers to how students from historically marginalized groups process and respond to discouraging experiences (Battey et al., 2022). This study found that

Jenny and Veronica had the highest in-degree, betweenness, closeness, and eigenvector centralities in the class, indicating that they may need to put in additional effort to participate in linear algebra. For instance, regarding in-degree centrality, Jenny and Veronica were nominated by the most diverse peer groups in class. In contrast, White students with the highest weighted in-degree centrality were usually nominated by redundant classmates. Additionally, Jenny's and Veronica's higher degrees mean that they had quicker accessibility to the entire class (closeness centrality), made bridges between different classmates (betweenness centrality), and possessed connections to other well-nominated students (eigenvector centrality) and these measures can be interpreted as Jenny and Veronica being exposed to relatively more peers, implying they may need to work on additional labor.

How to Interpret Racial/Ethnic and Gender Homophily: Solidarity and Segregation

There is strong homophily in student nominations regarding race/ethnicity and gender in the social networks in the IOLA class. This study suggests two contrasting interpretations of this homophily: ***solidarity*** and ***segregation***. The first view is to see homophily as the solidarity of minoritized groups of STEM students. That is, the tendency for women to nominate other women and for students of color to nominate other students of color can be interpreted as solidarity within the linear algebra classroom community. The second view is to interpret homophily as segregation between minoritized groups and non-minoritized groups in STEM. More specifically, the tendency of White and/or cisgender male students to nominate each other segregates female, non-cisgender, and/or students of color in a linear algebra learning community. These two perspectives contrast yet possibly coexist. That is, solidarity within minoritized groups of students and segregation between them and non-minoritized groups can occur simultaneously in a mathematics class. For instance, a group of racially diverse cisgender men would be racially inclusive but also might be exclusive from women and women of color, thus segregating them. Also, a group of White women and White non-cisgender students can be interpreted as a community where students stand in solidarity regarding gender inclusivity.

Conclusion and Suggestion for Future Work

This study demonstrates that active learning is not inherently equitable but is shaped by classroom power dynamics. In an IOLA course, women of color held central positions in peer networks, reflecting both empowerment and additional labor. Strong racial and gender homophily suggested overlapping patterns of solidarity and segregation. These findings emphasize the need for intentional design to ensure equity in mathematics. Future research should extend SNA across broader contexts and include groups not represented here, particularly Black women, to better understand how equity is enacted and sustained in active learning.

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References

Adiredja, A. P. (2019). Anti-deficit narratives: Engaging the politics of research on mathematical sense making. *Journal for Research in Mathematics Education*, *50*(4), 401-435.

Adiredja, A. P., & Andrews-Larson, C. (2017). Taking the sociopolitical turn in postsecondary mathematics education research. *International Journal of Research in Undergraduate Mathematics Education*, *3*(3), 444-465.

Battey, D., Amman, K., Leyva, L. A., Hyland, N., & McMichael, E. W. (2022). Racialized and gendered labor in students' responses to precalculus and calculus instruction. *Journal for Research in Mathematics Education*, *53*(2), 94-113.

Castro, A. R., & Collins, C. S. (2021). Asian American women in STEM in the lab with "White men named John." *Science Education, 105*(1), 33–61.

Ernest, J. B., Reinholz, D. L., & Shah, N. (2019). Hidden competence: Women's mathematical participation in public and private classroom spaces. *Educational Studies in Mathematics*, *102*(2), 153-172.

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the national academy of sciences, 111*(23), 8410-8415.

Grunspan, D. Z., Eddy, S. L., Brownell, S. E., Wiggins, B. L., Crowe, A. J., & Goodreau, S. M. (2016). Males Under-Estimate Academic Performance of Their Female Peers in Undergraduate Biology Classrooms. *PLoS ONE, 11*(2), e0148405–e0148405. https://doi.org/10.1371/journal.pone.0148405.

Gutiérrez, R. (2009). Framing equity: Helping students "play the game" and "change the game.". Teaching for Excellence and Equity in Mathematics, 1(1), 4-8.

Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. Journal for research in mathematics education, 44(1), 37-68.

Gutiérrez, R. (2017). Political conocimiento for teaching mathematics. *Building support for scholarly practices in mathematics methods, 11*.

Johnson, E., Andrews-Larson, C., Keene, K., Melhuish, K., Keller, R., & Fortune, N. (2020). Inquiry and gender inequity in the undergraduate mathematics classroom. *Journal for Research in Mathematics Education*, *51*(4), 504-516.

Knaub, A. V., Henderson, C., Rasmussen, C., & Lo, S. M. (2018). Four perspectives for interpreting social networks. In *Researching and enacting change in postsecondary education* (pp. 55-73). Routledge.

Laursen, S. L., Hassi, M. L., Kogan, M., & Weston, T. J. (2014). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study. *Journal for Research in Mathematics Education*, *45*(4), 406-418.

Leyva, L. A. (2021). Black women's counter-stories of resilience and within-group tensions in the white, patriarchal space of mathematics education. *Journal for Research in Mathematics Education*, *52*(2), 117-151.

McGee, E. O. (2020). Interrogating structural racism in STEM higher education. Educational Researcher, 49(9), 633-644.

Ong, M., Wright, C., Espinosa, L., & Orfield, G. (2011). Inside the double bind: A synthesis of empirical research on undergraduate and graduate women of color in science, technology, engineering, and mathematics. *Harvard educational review, 81*(2), 172-209.

Theobald, E. J., Hill, M. J., Tran, E., Agrawal, S., Arroyo, E. N., Behling, S., ... & Freeman, S. (2020). Active learning narrows achievement gaps for underrepresented students in undergraduate science, technology, engineering, and math. *Proceedings of the National Academy of Sciences*, *117*(12), 6476-6483.