Using Expectancy-Value Theory to Explore Motivations in Inquiry-Oriented Linear Algebra

Lauren Janda
Arizona State University

This study examines undergraduate student motivation in an inquiry-oriented linear algebra course using expectancy-value theory (EVT). While EVT has more frequently been applied in K–12 settings, its use in undergraduate mathematics remains limited. Through interviews with four students, this study explores how components of EVT—expectation for success, attainment value, intrinsic value, utility value, and cost—relate to students' engagement. Findings show that the expectancy-value theory can be helpful in making sense of the motivations of undergraduate students. Students were motivated in different ways, with many citing their key influences as collaboration, real-world applications, and classroom expectations that promote a novel definition of success. Inquiry-oriented instruction appeared to support positive motivational outcomes by normalizing struggle and promoting group-based problem solving. This suggests EVT is a useful framework for understanding motivations in undergraduate mathematics, particularly in the context of inquiry-oriented instruction.

Keywords: motivation, expectancy-value theory, inquiry-oriented instruction

Introduction and Literature Review

Students' motivations have been a topic of mathematics education research for decades, with a particular focus in the last 20 years (Schukajlow et al., 2023). Several studies have been conducted looking at the motivations of primary or secondary school students (e.g., Middleton et al., 2023; Fielding-Wells et al., 2017; Eccles et al., 1993). Middleton and collaborators (2023) analyzed different aspects of mathematics engagement in high school students and found that, related to motivations, social support in class improved student interest in the situation as well as their beliefs in their competence. Of particular relevance to this study, Fielding-Wells and collaborators (2017) explored motivations in an inquiry-based mathematics classroom of 9-10-year-olds using the expectancy-value theoretical framework. They found that this framework was helpful to make sense of student motivation in an inquiry-based classroom, and aspects of this instructional design could be positively influential on student motivation (Fielding-Wells et al., 2017). However, less research has been done studying the motivations of undergraduate students, with or without the use of the expectancy-value theory (e.g., Weber et al., 2020).

One possible explanation for this is that motivations towards mathematics are developed early in life and maintain relative stability throughout (Middleton & Spanias, 1999). However, motivating undergraduate students should not be dismissed. Eccles and collaborators (1993) found that beliefs of confidence lower with age, which may reflect increasing pessimism with age. As a result, it is worthwhile to analyze why older students have lower competence beliefs and what can be done to support their motivation. Additionally, according to Middleton and Spanias (1999), different instructional designs can alter students' motivations, so there may be merit in examining the ways that instructors at an undergraduate level can affect students' motivations in their math courses. As a necessary first step to that, the results of this contributed report will work to identify common motivations that students are expressing from a particular population.

Specifically, I am interested in looking at the motivations of students in an inquiry-oriented linear algebra course, because I hypothesize that this instructional design may have positive effects on undergraduate student motivation. It has been noted that inquiry-oriented instruction (IOI) is closed related to inquiry-based learning (IBL; Laursen & Rasmussen, 2019), which has been found to have positive student outcomes in many regards: cognitive, affective, and collaborative benefits (Laursen et al., 2014). Because Fielding-Wells and collaborators (2017) found aspects of an IBL environment to have positive outcomes, and IBL has been found to benefit student outcomes (Laursen et al., 2014), I hypothesize that IOI may have positive effects on undergraduate motivations. This paper aims to explore the motivations of undergraduate students in an inquiry-oriented linear algebra course using the expectancy-value theory of achievement motivation.

Theoretical Framework

The expectancy-value theory of achievement motivation, also commonly referred to as just the expectancy-value theory (EVT), is a well-established theory in the field of psychology (Wigfield et al., 2021). This theory was initially developed by inquiring into why women are less likely to pursue STEM-related careers than men (Eccles, 1983). Findings from that large-scale, longitudinal study emphasized that the few sex differences that did emerge indicated that females had a less positive perception of their math ability along with a lower value of the subject than males (Eccles, 1983). EVT declares that individuals make decisions in achievement settings based on (a) their expectations of success on the task and (b) how much they value the task (Wigfield & Eccles, 2000). Mathematics courses are considered achievement settings, because there are opportunities to attain specific goals, such as demonstrating understanding of the content on assessments or passing the course. We will be discussing the constructs here in the order they are listed in Figure 1, which is the order they are presented in Wigfield and Eccles (2000).

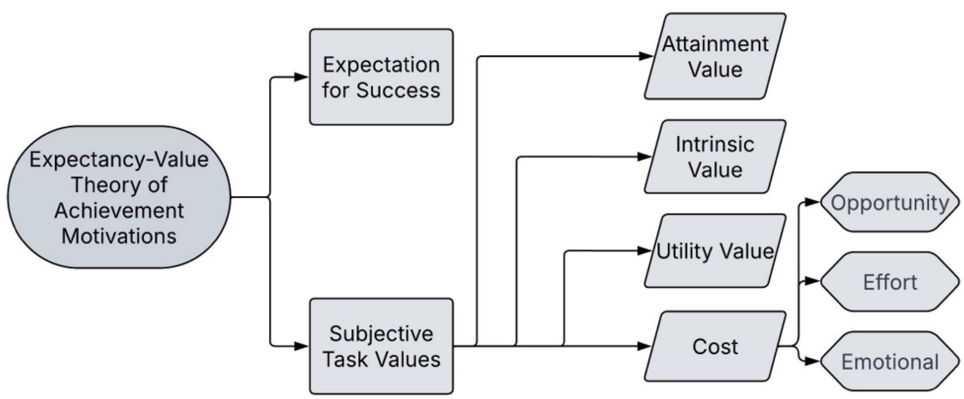

Figure 1. Expectancy-Value Theory of Achievement Motivation Components

The *expectation for success* portion of this theory refers to how much an individual believes that they are able to attain success at a given task (Schukajlow et al., 2023). If a student believes a task is too difficult for them to complete (i.e., they don't think they are smart enough, they don't think they were given enough resources by their instructor, etc.), they will generally be unmotivated to complete that task. Additionally, four types of *subjective task values* are distinguished: attainment value, intrinsic value, utility value, and cost. *Attainment value* is how important doing well on a task is to that individual; this task value is usually tied closely with one's self-concept or identity (Wigfield & Eccles, 2000; Schukajlow et al., 2023). *Intrinsic value* is how interested that individual is in the task or how much they enjoy it (Wigfield & Eccles, 2000). *Utility value* is how much the individual sees the task fitting into their future goals (Wigfield & Eccles, 2000). It is important to note that these task values are subjective, varying by the individual and their experiences. For example, a student who is not particularly interested in mathematics (low intrinsic value) and doesn't care about what grade they get in their math class (low attainment value) can still be motivated to do well in their math course if they view mathematics as something that can help them understand necessary concepts for their job as, say, an electrician (high utility value). *Cost*, which is a factor of decision-making in this theory in line with the subjective task values, is the other end of the scale. Cost may be accounting for how much engaging in the task restricts one's ability to engage in higher valued activities (*opportunity cost*). For example, if I do my homework, that gets in the way of my ability to chat with friends. Cost could also account for how much effort goes into engaging in the activity (*effort cost*), or how much emotional labor goes into the task (*emotional cost*). When people make decisions, they gauge how much it costs and see if that outweighs their values and expectations for success.

I am using this theoretical framework to answer the following research question: *How do students characterize their motivations related to their engagement in an inquiry-oriented linear algebra course?*

Methods

This study investigated the experiences of four undergraduate students from two sections of inquiry-oriented linear algebra at a large public university in the southwest United States. A general day in this course involved the students working on a mathematical task in groups of two to four students, while the instructor and TA circulated the classroom and supported student discussions. The instructor would ask students to share their thought processes with the whole class. The tasks generally had scenarios associated with them and asked students to collaborate with one another and provide reasoning for their answers. An example of the kinds of mathematical tasks provided in this class can be seen in Wawro et al. (2012).

Due to the complexity of students' engagement, multiple data methods were used (as suggested by Middleton et al., 2017): observations, participation surveys (self-report), and interviews. Classroom observations were conducted for the three weeks of class that occurred after the first exam. Additionally, students reported on their participation for one day of each week of the semester by responding to questions in a Google form. One week after the conclusion of the observations, four students participated in an interview with questions asking about their experiences in the class. The interview questions were intended to reveal information about their motivations for engaging with the course material. Note that, for the purpose of this contributed report, only analysis from the interview data will be presented.

To answer the research question, I first familiarized myself with the data by transcribing the interview data and reading through it. Next, I highlighted excerpts based on whether they addressed the students' motivations to engage in a task. Excerpts were identified as *motivations* if they indicated the reasons why someone engaged with the course content (Middleton & Spanias, 1999). From here, I used the constructs from the expectancy-value theory to classify the various motivations as either *expectation for success, attainment value, intrinsic value, utility value,* and *cost* (Wigfield & Eccles, 2000).

Results

In total, 63 excerpts were identified as containing motivations among the four participants. The classifications for these motivations can be seen in Figure 2. Of note, expectation for success was coded 18 times among three participants. Of the four types of subjective task values, attainment value showed up five times among two participants. Intrinsic value showed up six times among three participants, four of those occurrences were all from one participant. Utility value showed up 22 times among all four participants. Finally, cost was coded 12 times among all participants.

EVT Component	Olivia	Payton	Quincy	Romeo
Expectation for Success	7	1	10	0
Attainment Value	0	1	0	4
Intrinsic Value	1	4	1	0
Utility Value	3	8	8	3
Cost	2	3	6	1

Figure 2. Counts of Student Motivation Excerpts by EVT Component

Expectation for Success

Expectation for success was coded particularly strongly with Olivia and Quincy, in addition to a single time with Payton, with these excerpts generally noting that what counts as 'success' in this class is varied compared to their other classes. For example, Payton compared the differences between the homework expectations and resources provided in this course, stating that, “the homework throws it at you like it expects that you've already learned it, because it just, it presents the problem and you should have the knowledge of the techniques used to solve it, versus the in-class activities are very much presented to us where we're not assumed to have that understanding.” Here, Payton is describing that in-class activities provide more support and resources than the homework activities, because for in-class activities, the students are not expected to know how to solve the new material. Overall, Payton demonstrated a high expectation for success in this course, but had a lower expectation for success on the homework because there was not as much support if he got stuck.

Olivia and Quincy also displayed high expectations for success in this course, but associated those expectations with different components of the course than Payton. For example, instead of comparing the homework’s expectations to the in-class expectations, Olivia compared this course to her experiences in other courses, stating that “the frustrations I feel in those classes are like [...] ‘I plugged in the numbers I'm supposed to and it's not working, so like, what am I doing wrong?’ [...] I feel better when my whole team doesn't understand it, or even if they do understand it, I feel better because they're gonna explain it to me.” Later, she shared that when she does not understand in this class, she feels as though it is “definitely, not as like personal towards me. [...] We don't really know what we're supposed to be doing yet mathematically, so if you don't understand, like the bigger picture, it's okay.” Here, and generally, Olivia understands that there is an expectation for students to struggle initially with concepts in this course, and so even if she does not immediately solve the problem, she still expects to succeed by struggling through the problem with her group. Quincy shares the same kind of opinion, stating that in other classes he gets frustrated and can take it personally when he doesn’t understand, but he said he is able to “kind of just like trust the process with this class.” Each of these students shares that the course expectation of students figuring out the mathematics for themselves in groups helps them to feel less pressure to immediately know how to approach the problem, making the idea of what counts as successful more accessible for students.

Attainment Value

Attainment value showed up for Romeo the most, specifically as he shared about his work ethic and how that shapes his identity. When asked how Romeo pushes through days when he is tired or has low energy levels, he said, “you gotta push yourself to do hard things, you know. That's like my mentality always, you know, just everything's gonna be -- well, there's gonna be challenges, and you're just gonna have to, you know do your best.” In this sense, Romeo would likely have a strong attainment value in any of his courses, as they are all simply “hard things” that he has to push through. Payton shared a bit less explicitly about his work ethic, stating that “whenever I’m trying to learn something new, I always try and get it as much as I can, just through my own understanding” before he uses any other resources such as Google. Generally, attainment value did not show up much in these interviews, which may be because, before conducting the interviews, I was not using the expectancy-value theory to analyze the data, so there were not interview questions aimed at exploring a students’ desire to do well on the in-class activities.

Intrinsic Value

Intrinsic value usually presented as the participant expressing that they enjoyed a specific activity. For three participants in this study, they shared their interest or enjoyment in a specific task in response to the interview question “What is your favorite task we’ve done so far in class?” The reason for the interest was different for each participant, demonstrating some of the different ways that this code can show up for people. Although Romeo did answer this interview question, he did not express particular interest or engagement in the task, attributing his choice of favorite task to how “clear-cut” the solution was for him, so his response was not coded as intrinsic value. Quincy picked his favorite task based on which task provided the best example. Olivia picked her favorite based on what activity she could visualize best. Payton picked his favorite task based on which task introduced new math content. Additionally, for Payton, he expressed interest in the course content without being prompted. He finished his interview saying, “this is probably one of the more enjoyable math courses that I’ve had in a long while [...] it’s a lot of collaborative work too, so it’s very much more social and fun to do which I enjoy.” Payton also shared that he enjoyed teaching or leading his group. Overall, the intrinsic value code was particularly subjective, showing up in a few different ways depending on the participant and depending on the various things each participant enjoys.

Utility Value

For these participants, utility value usually showed up as engaging in a particular way as a means to an end. Payton expressed that he will sometimes take the back seat in his group’s work and work on the problem on his own for the sake of seeing how his groupmates do it and if they find more efficient solving methods. Specifically, Payton said, “I’d like to see how it’s done to see how it compares to how I do it, cuz I am very set in how I solve a lot of these problems and I definitely have done some like inefficient ways of solving them.” Romeo shared the same reasoning, but instead of taking a back seat in his group, he asks his group a lot of questions to gain the perspective of more efficient solving methods. Romeo said, “So to be honest, I'm the one asking a lot of questions, because you know, everybody has their own process, and I'm trying to understand how they solve it through." For Olivia, she saw the utility value in tasks through the real life applications that these tasks can be used for, when asked to share about her favorite task she said "This whole class feels like just like trying to solve puzzles and so like this was the more helpful puzzle to solve because I'm like, okay, like people do this to actually like... in real life, if that makes sense?" All of Quincy's utility value codes were related to him viewing his engagement with the material as a means to the end of passing quizzes and exams. In one instance, Quincy said "even if I'm focusing for like 20%, I come out of the class ready for the next exam." Overall, utility value showed up as the participants trying to do what helps them learn the material best, as in Payton and Romeo's case, appreciating the real-life applications of the activities, like Olivia's case, or doing what they need to perform on assessments, as in Quincy's case.

Cost

Cost showed up in a few different ways as well. For Payton and Romeo, cost showed up as continuing to work on their board work during whole-class discussions when people are presenting. Payton explained that "my group very often tries to continue working on whatever our brain was still doing." This demonstrated the opportunity cost: fully engaging with the whole class discussion would get in the way of them continuing their work on the board. This opportunity cost also showed up when the participants talked about the hypothetical of people working on an assignment for another class. For example, Quincy said "my kind of reasoning is that maybe, like you got […] something due that day and we just like need to finish that, so […] you kind of set it [the in-class activity] aside because it's like, whatever you're doing at that moment is not gonna matter up until, like the quiz." To Quincy, it is understandable that someone may work on an assignment that they feel is more pressing for another class because whatever is currently being worked on in class will always be less pressing until the assessments.

Another aspect of cost mentioned is the energy levels or 'moods' that the participants were feeling during class. Three of the four participants shared the sentiment that if they are tired, "it's easier to get derailed because I'm not as focused" (excerpt from Payton's interview). Additionally, participants shared that if they are having personal issues, it makes it more challenging for them to engage fully with the in-class activities. I would consider this an effort cost: these students perceive that they do not have the energy to continue engaging because of physical energy levels.

The final way that cost showed up is related to the participant's expectation for success. When Olivia and Quincy felt that even if they fully engaged with the material, they still would not understand it, they would disengage and usually distract themselves with activities on their phones. Quincy said that if he doesn't understand what is happening in class, then:

> I'm just gonna immediately just put my AirPods in like, listen to music, or like, watch a video or like, do something else that, that is well, honestly, that's what I would do now like, even if I have, I told my full attention to this, I'm not gonna understand it. So why, why should I waste my time, or why should I waste my effort..?

These participants did not expect success from engaging with the activity: to them, the cost was too high, and they did not expect to get payoff of understanding.

Discussion

The ways that the expectation for success code presented for these participants was particularly interesting because their perception of what counted as success and the resources available to reach it shifted given the classroom expectations for this course. Generally, being successful in this course became more accessible to these students compared to other courses, because they believed there were ways to be successful in this course outside of completing a problem successfully. I conjecture that this increased accessibility of success may be attributed to the classroom expectations of an inquiry-oriented linear algebra course, but more investigation is required to validate that claim. For example, according to Rasmussen and Kwon (2007) teacher inquiry into student thinking is one of the defining aspects of an inquiry-oriented classroom. One of the participants in this study stated that he felt motivated when the instructor, instead of saying that answers were “bad or wrong,” would say “that’s one way of looking at it.” This student found it motivating that the instructor would interpret students’ perspectives instead of dismissing the student contribution because it was incorrect.

Additionally, the fact that this was an inquiry-oriented classroom being studied provided another unique aspect: student reinvention (Kuster et al., 2018). Student reinvention means the students are making sense of the mathematical concepts for themselves, instead of adopting another’s reasoning. For an inquiry-oriented classroom, the tasks are designed to support student reinvention by building on student reasoning before formal reasoning is introduced (Kuster et al., 2018). Although these participants never mentioned student reinvention explicitly, Olivia did mention that she thought in group work that “going off of each other's ideas is really cool to see because I'm like, ‘Okay, this is what people were doing when they were inventing this math.’” A potential contributor to this shifted expectation for success for these participants is the instructor inquiry and student reinvention, two aspects unique to inquiry-oriented courses. While the results of this study demonstrate that the expectancy-value theory is suitable to make sense of the motivations of undergraduate students in an inquiry-oriented course, it is worthwhile to consider whether this shifted expectation for success can be most directly attributed to the fact that this is an inquiry-oriented course.

Multiples of these components of the expectancy-value theory of achievement motivation (*expectation for success, attainment value, intrinsic value, utility value,* and *cost*) can play a role in any one instance of an individual’s decision-making. In this study, we saw instances of students making decisions based on how much they expected to succeed at a task, weighed against how much effort they would need to put in to reach that success. As such, classrooms implementing inquiry-oriented instruction may provide unique environments to explore students’ motivations, because the expectations of what is considered success may change for students based on the overall expectations of the class.

References

Eccles, J. S. (1983). Expectancies, values, and academic behaviors. In *Achievement and achievement motives* (pp. 75-146). Freeman.

Eccles, J., Wigfield, A., Harold, R. D., & Blumenfeld, P. (1993). Age and gender differences in children's self-and task perceptions during elementary school. *Child development*, *64*(3), 830-847.

Fielding-Wells, J., O’Brien, M., & Makar, K. (2017). Using expectancy-value theory to explore aspects of motivation and engagement in inquiry-based learning in primary mathematics. *Mathematics Education Research Journal*, *29*(2), 237-254.

Kuster, G., Johnson, E., Keene, K., & Andrews-Larson, C. (2018). Inquiry-oriented instruction: A conceptualization of the instructional principles. *Primus*, *28*(1), 13-30.

Laursen, S. L., Hassi, M. L., Kogan, M., & Weston, T. J. (2014). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study. *Journal for Research in Mathematics Education*, *45*(4), 406-418.

Laursen, S. L., & Rasmussen, C. (2019). I on the prize: Inquiry approaches in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education*, *5*(1), 129-146.

Middleton, J., Jansen, A., & Goldin, G. A. (2017). The complexities of mathematical engagement: Motivation, affect, and social interactions. *Compendium for research in mathematics education*, 667-699.

Middleton, J. A., & Spanias, P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education*, *30*(1), 65-88.

Middleton, J. A., Wiezel, A., Jansen, A., & Smith, E. P. (2023). Tracing mathematics engagement in the first year of high school: Relationships between prior experience, observed support, and task-level emotion and motivation. *ZDM–Mathematics Education*, *55*(2), 427-445.

Rasmussen, C., & Kwon, O. N. (2007). An inquiry-oriented approach to undergraduate mathematics. *The Journal of Mathematical Behavior*, *26*(3), 189-194.

Schukajlow, S., Rakoczy, K., & Pekrun, R. (2023). Emotions and motivation in mathematics education: Where we are today and where we need to go. *ZDM–Mathematics Education*, *55*(2), 249-267.

Weber, K., Lew, K., & Mejía-Ramos, J. P. (2020). Using expectancy value theory to account for individuals' mathematical justifications. *Cognition and Instruction*, *38*(1), 27-56.

Wigfield, A., & Eccles, J. S. (2000). Expectancy–value theory of achievement motivation. *Contemporary educational psychology*, *25*(1), 68-81.

Wigfield, A., Muenks, K., & Eccles, J. S. (2021). Achievement motivation: What we know and where we are going. *Annual Review of Developmental Psychology*, *3*(1), 87-111.