Mathematical Modeling Knowledge and Affect for Teaching: A Holistic Framework for the Competencies Required to Teach Mathematical Modeling

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In this theoretical paper, we build on prior frameworks on teacher knowledge and affect to propose a holistic framework for the competencies required for teaching modeling—a framework that integrates teacher competencies from both the cognitive and affective domains. We illustrate elements of the framework through the case of one secondary teacher's work from a modeling-focused professional development program. Finally, we describe the implications of our framework for researchers and educational stakeholders.

Keywords: mathematical modeling, teacher knowledge, affect, modeling competencies

Mathematical modeling (hereafter referred to as modeling) is the process of using mathematics to describe and make predictions about real-world situations. As modeling has increased in popularity across K-16 education, the modeling education field has developed numerous programs to help educators effectively teach modeling. For example, over the last two decades, research programs have designed modeling-focused professional development programs for teachers (e.g., Jung & Brady, 2016), investigated the competencies required to teach modeling (e.g., Borromeo Ferri, 2018; Greefrath et al., 2022; Wess et al., 2021), and introduced mathematical modeling in teacher-education programs (e.g., Anhalt et al., 2018). However, this research has predominantly focused on the knowledge required for teachers to teach modeling from a cognitive point of view and often emphasizes a pedagogical perspective (over a subject matter perspective). Yet, to develop a framework that provides a "complete picture" of the competencies required to teach modeling, there is a need to include both cognitive and affective perspectives (Thompson, 1992).

Chamberlin (2019) stated that modeling, cognition, and affect are intertwined with one another, and he proposed a theoretical framework that connects these three constructs. According to his framework, there is an interrelationship between cognition, affect, and modeling where each element influences one another and are in a continuous positive feedback loop with one another. That is, according to Chamberlin's (2019) framework, to successfully model, one needs to engage in successful cognitive and affective processes. Similarly, we contend that to *teach* modeling successfully, a teacher should acquire both cognitive competencies (i.e., knowledge for teaching) and positive affect (i.e., affective competencies required for teaching). In this theoretical paper, we propose a framework for the competencies needed for teaching modeling that combines both knowledge (cognition) and affective perspectives for *teaching* modeling. In light of this goal, we first provide the relevant background that supports the development of the framework. Next, we provide illustrative vignettes of the framework components using data from a secondary teacher, Olivia (pseudonym). Finally, we highlight the implications of our proposed framework for the modeling teacher education community.

Background

The Knowledge Domain

Several frameworks have long shaped research on teacher knowledge and competencies for teaching mathematics. For example, Shulman (1986) categorized teaching mathematics content knowledge into subject matter knowledge (SMK) and pedagogical content knowledge (PCK). Building on Shulman's (1986) work, Ball et al. (2008) presented a Mathematical Knowledge for Teaching (MKT) framework by unpacking SMK and PCK into more fine-grained domains. These include common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and students (KCS), horizon content knowledge (HCK), and knowledge of content and teaching (KCT). CCK refers to the general mathematical knowledge, not specialized or unique to teachers. SCK pertains to technical knowledge unique to teaching (e.g., looking for patterns in students' errors). KCS represents teachers' understanding of how students think and learn and the ability to anticipate what students find confusing or engaging. Teachers' awareness of how mathematical topics connect across the curriculum and discipline is associated with their HCK. Finally, KCT combines mathematical knowledge with teaching practices like sequencing content and identifying suitable instructional methods (Ball et al., pp. 399–403).

Although Ball et al.'s (2008) framework offers important foundations for mathematical teaching, few frameworks have described the additional demands required for teaching modeling. For example, Borromeo Ferri and Blum (2010) proposed a framework for the competencies required for teaching modeling. Their framework proposed that teachers of modeling should acquire skills along four dimensions: (i) theoretical (i.e., knowledge about modeling cycles, and modeling), (ii) task (i.e., knowledge about modeling tasks), (iii) instructional (i.e., knowledge about implementing modeling tasks), and (iv) diagnostic. Building on this framework, Wess et al. (2021) presented modeling-specific PCK by outlining four facets of knowledge: (i) knowledge interventions (i.e., ability to apply adaptive strategies and to assess their effect on learners' solution process), (ii) knowledge about modeling processes (i.e., competencies of diagnosing students' modeling phases), (iii) knowledge about modeling tasks (i.e., the skills in understanding modeling task characteristics and analyzing and developing them), and (iv) knowledge about modeling aims and perspectives (i.e., knowledge of modeling cycles and their suitability for various purposes in classrooms).

The Affect Domain

Although the interconnection between affect and cognition remains debated, affect is generally considered as consisting of the "non-cognitive aspects of human thought" (Hannula, 2012, p. 137). Research on mathematical affect takes various perspectives (Chamberlin, 2019), but the most prominent of these perspectives is McLeod's (1992) three-dimensional framework for affect. The framework comprises emotions, attitudes, and beliefs, with beliefs representing the most cognitive component and being less amenable to change (see Table 1 for the definitions). Di Martino and Zan (2011) proposed that attitudes function as a bridge between emotions and beliefs. Consistent with this view, DiMartino and Zan (2011) proposed a framework for attitude where emotional disposition, vision of mathematics, and perceived competence were included. In Di Martino's and Zan's (2011) framework, emotional disposition is defined as one's likes/dislikes towards mathematics, vision of mathematics is conceptualized as one's beliefs and values regarding mathematics, and finally perceived competence is conceptualized as "perception of being/not being able to succeed in mathematics (Di Martino & Zan, 2011, p.476).

Table 1. Definitions of emotions, attitudes, and beliefs, directly taken from Phillip (2007, p. 250).

Emotions	Feelings or states of consciousness, distinguished from cognition. Can be positive or negative.
Attitudes	Manners of acting, feeling, or thinking that shows one's disposition or opinion. Can be positive or negative.
Beliefs	Psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less intensively, and are harder to change than attitudes.

While students' mathematical affect has been studied through varied conceptualizations, research on teachers' mathematical affect is still emerging. Phillip (2007) notes that while beliefs are considered part of affect in research, they are treated as a distinct construct in studies on teacher affect. Research on teacher affect has documented that teachers' mathematical attitudes inform their mathematical beliefs and in turn instructional practices (Charalambous, 2015; Wilkins, 2008). At the same time, research has also documented that teachers' attitude towards mathematics plays a role in students' attitude towards mathematics (Mensah et al., 2013), and ultimately students' mathematical performance (Ma & Kishor, 1997). Thus, teachers' affective competence is an integral part of students' mathematical (modeling) learning. The takeaway is therefore two-fold: (1) teachers' positive affect correlates with positive teaching and learning outcomes, and (2) frameworks for modeling (and mathematical) knowledge required for teaching should consider both knowledge and affective domains to provide a holistic understanding of the competencies needed to teach modeling.

Conceptual Framework for the Competencies Required to Teach Modeling

We integrated theories (Prediger et al., 2008) of teacher knowledge (Ball et al., 2008; Borromeo Ferri & Blum, 2010; Wess et al., 2021) and affect (Di Martino & Zan, 2011; McLeod, 1992) to propose a framework for the competencies required for teaching modeling (Figure 1). The framework proposed in this paper consists of two major *domains*: knowledge and affect. At the core of this framework is the knowledge perspective. Surrounding the knowledge domain lies the affective domain (in green), which is influenced by the knowledge domain. The positions of the affect and knowledge domains were inspired by Mandler's (1989) work, highlighting that one's actions (based on their knowledge) result in affective responses.

The knowledge domain is further divided into *dimensions* (i.e., subject matter knowledge for modeling (SMK-M) and pedagogical content knowledge for modeling (PCK-M)) and *components* (theoretical knowledge, curricular knowledge, instructional knowledge, and assessment knowledge). This framework combines modeling knowledge for teaching with modeling specific perspectives of teacher expertise to teach modeling effectively. We adapted Ball et al.'s (2008) categorization of mathematical knowledge for teaching to modeling. SMK-M refers to the understanding of the modeling process and the mathematical content that teachers must possess to guide students through the modeling process. PCK-M refers to the knowledge required to effectively implement modeling in the classroom and diagnose students' modeling. Within SMK-M, we situate *theoretical knowledge*—knowledge about modeling cycles, modeling tasks and their suitability for various purposes. Both horizon modeling knowledge (HMK) and mathematical common content knowledge (CCK) impact teachers' knowledge of mathematics that then influences how they engage with and make sense of modeling, whether it involves applying basic mathematical fluency (CCK) or understanding broader mathematical connections across topics and levels (HMK).

We positioned *curricular knowledge* as the blending (Fauconnier & Turner, 2003) of SMK-M and PCK-M, as it draws on both concepts. It represents a bridge where teachers' knowledge of modeling intersects with their ability to transform that knowledge into meaningful learning experiences through modeling tasks. This overlap justifies its position in the framework. Knowledge about modeling task characteristics encompasses the theoretical perspectives underlying modeling task design. Similarly, *knowledge about content and curriculum* (KCC) and *specialized content knowledge for modeling* (SCK-M) are also situated within curricular knowledge as both require an understanding of mathematics and the ability to situate that in curricular structures. Teachers must know how modeling fits into the broader mathematical curriculum (SCK-M) and align with the content standards (KCC).

Instructional knowledge belongs in PCK-M because it is about transforming content knowledge about modeling into effective teaching practices. *Knowledge about intervention* requires pedagogical decisions on how to guide and support students during modeling tasks. On the other hand, *knowledge about modeling and teaching* focuses on teaching modeling in ways that foster mathematical understanding. Both components go beyond just knowing modeling to how to teach it, both as a vehicle and as content. Finally, the *assessment knowledge* in modeling refers to the teachers' knowledge of modeling, alongside their ability to anticipate and interpret the ways students are likely to approach modeling tasks. This knowledge enables teachers to better diagnose students' understanding of the modeling process as they go beyond grading answers to truly assess students' modeling competencies and learning of modeling.

To develop a framework for the affective competencies required for teaching modeling, we drew on McLeod's (1992) framework for affect. Specifically, we asked what are some qualities that are required for teachers to have a positive disposition towards teaching modeling (Jacobson & Kilpatrick, 2015)? We identified the following: (1) constructive emotional dispositions towards modeling, (2) constructive orientation (i.e., positive attitudes) towards teaching modeling, (3) positive perceived competence in modeling, and (4) optimistic vision (i.e., positive beliefs) towards modeling. Constructive emotional dispositions towards modeling refer to positive emotions and attitudes that teachers develop to meaningfully engage in the teaching of modeling. Constructive orientation towards teaching modeling refers to attitudes that promote modeling in the classroom environment. Positive perceived competence in modeling refers to teachers' confidence in tackling modeling tasks. Finally, an optimistic vision towards modeling is having a positive outlook on the broader impacts of modeling on the individual, community, and societal level.

Illustrative Vignettes from Olivia

To describe components of our framework, we drew from one secondary teacher's (Olivia) participation in a modeling focused professional development (PD) program. The overarching goal of the PD program was to develop teachers' proficiency in teaching modeling at the secondary level. A total of 24 secondary teachers (grades 6-12) participated in the PD program. As part of the PD program, Olivia engaged in seven modeling tasks (which consisted of solving and reporting the solution through formal and informal reports), submitted seven task reflections regarding her experiences working on the tasks, designed three modeling tasks, implemented one modeling task in her own classrooms (grade 9 and 10), and submitted a final report reflecting on her experience designing and implementing the task. These sources serve as illustrative vignettes for describing the functionality of our framework. Given space constraints, we present a selection of competencies and will elaborate on others in our talk.

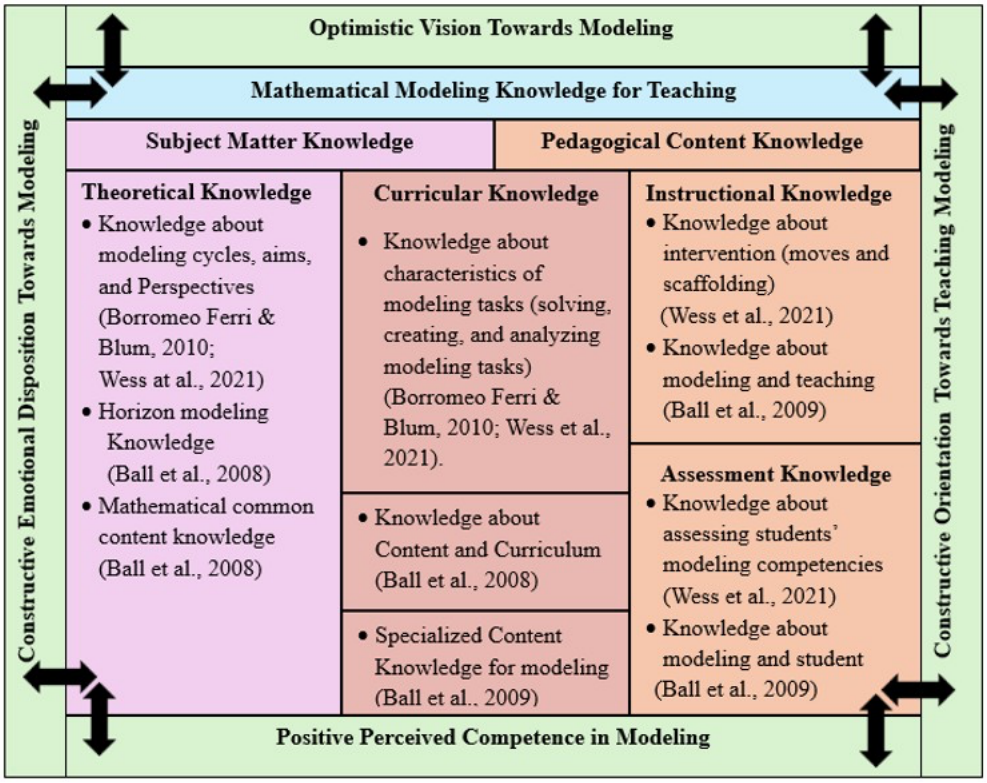

Figure 1. Competencies required for teaching modeling

Knowledge about Characteristics of Modeling Task

Olivia developed the *Eliminating Bottles* task (Table 2), which exhibited robust integration of both modeling task characteristics and socio-ecological awareness. The task exhibited the following task principles: (1) alignment with content standards, (2) learner-centered, (3) model development, (4) model documentation, and (5) reality principle (Aguirre et al., 2019; Galbraith, 2006; Jung et al., 2023; Lesh et al., 2000). First, the task was connected to the state's grades 9-10 content standards, such as surface area and volumes, ensuring that while students address broader social issues of plastic waste prevention, they would also meet formal curriculum standards. The task is learner-centered as the students were given the agency in making assumptions, creating a model, and interpreting the results in meaningful ways. Olivia's understanding of task characteristics also influenced her expectations for the task. For example, Olivia explicitly stated and collected students' poster reports that included the problem statement, assumptions, model formulation, validity, and improvements, addressing the model documentation principle. Finally, the task exhibited the reality principle through drawing on real-world issues of plastic waste that support students' socio-ecological awareness—a critical component of modeling tasks.

Table 2. Olivia's Eliminating Bottles task prompt

The Natural Resources Defense Council claims that 20 billion plastic bottles are thrown away each year. To reduce plastic waste, many public spaces are installing bottle filling stations to encourage the use of reusable water bottles. Develop a procedure to calculate the amount of plastic the [institution name] filling station has eliminated based on the reported number in the photo. Relate this measurement to something you are familiar with by describing how much space the plastic would cover (e.g., number of football fields, gym courts, etc.,).

Knowledge about Intervention

Olivia's knowledge about intervention was evident in her modeling task implementation report, where she documented her strategies to support students' understanding of the modeling process. For example, Olivia cited using several intervention strategies, including probing students' thinking when she felt that the students were stuck, incorporating contingent scaffolding or breaking tasks into smaller chunks for diverse learners, monitoring student work, facilitating small group interactions through the use of pocket questions (ready-to-use questions), creating groups based on her (Olivia's) knowledge of students, providing manipulatives for collecting data, and offering extension activities for those who finished early. These instructional approaches demonstrate Olivia's ability to design targeted interventions for implementing modeling tasks in her classroom.

Knowledge about Assessing Students' Modeling Competencies

Olivia assessed students' modeling competencies using an assessment rubric that evaluated specific stages of modeling cycles. The rubric reflected the modeling stages that Olivia prioritized and that she expected students to understand. These include making sense of the problem, assumptions and model creation, data collection, validating, and communicating the process clearly. She also added a quantifying description (Figure 2) to each modeling stage to communicate her expectations and provided fair feedback on students' work to support students' model development. Prior to students engaging in the task, Olivia shared the rubric and reviewed both the rubric and project expectations with the entire class, ensuring that the assessment process was open and transparent (Assessment Standards for School Mathematics, 1995). In addition, Olivia also diagnosed students' blockages informally as she walked around the class and listened to students' conversations, which supported Olivia's contingent interventions.

Modeling Task Rubric

	Excelling	Progressing	Emerging
Make Sense of Problem	Problem statement and conclusion are both made in context. Objective of the solution is identified. **30**	At least one, either the problem statement or conclusion, is made in context. The objective of the solution is not clearly identified. **20**	Problem statement and conclusion are identified but lack context. **10**
Assumptions and Model Creation	All assumptions listed are justified or based on background knowledge. All assumptions made are utilized in the model. **50**	One or more assumptions made lack justification. OR Not all assumptions made were necessary for the model. **40**	At least one assumption is listed but lacks justification. OR Model lacks evidence of use of assumptions. **30**

Figure 2. Excerpt of Modeling Task Rubric Developed by Olivia

Constructive Orientations Towards Teaching Modeling

Olivia demonstrated multiple constructive orientations towards teaching modeling in her task reflections and implementation reflections. For example, in her implementation reflection, she expressed that she wanted to challenge herself in implementing more modeling tasks in her classroom due to the overwhelming engagement she received from her students. She also expressed that implementing modeling tasks can be interdisciplinary. This idea is evident in her task reflection, where she reflected that if she were to implement modeling tasks with historical contexts, "[she] likes the idea that [she] could work with the history department and create a lesson that ties in with [the history department] and can integrate history into [her] classroom and mathematics into theirs." In other constructive orientations, Olivia provided guardrails for students who may struggle with open-ended questions and designed modeling tasks that draw on students' personal interests, mathematical experiences, and local community-based knowledge to support mathematically rich discourse.

Positive Perceived Competence in Modeling

Through her engagement with the modeling tasks in the first part of the PD program, Olivia developed confidence in her modeling skills. For example, in her modeling task reflections, Oliva made statements such as "I think in the future I will be less afraid to create multiple models knowing that I will eventually adjust them to make them fit the task better," "I am gaining more confidence in my ability to attack these modeling tasks. I'm becoming more eager to attempt them, and I am realizing that I can be very creative in how I approach problems," and "I am impressed with my ability to clearly explain my processes and procedures in context and effectively discuss the applications my model has to the real world." These statements demonstrate her positive perception of her ability to model, which in turn has an impact on how she may teach modeling.

Discussion

The framework presented in this paper advances the holistic understanding of competencies required for teaching modeling by positioning cognition and affect as inseparable domains (Chamberlin, 2019). On the cognitive side, teachers require both SMK-M and PCK-M for teaching. On the affective side, teachers require positive emotions and attitudes, together with confidence in tackling and promoting modeling with an optimistic vision of its broader impact on learners and society. We argue that competencies along both domains must co-exist for teaching modeling successfully. This is because teachers with strong pedagogical content knowledge may avoid implementing modeling tasks if they lack self-efficacy in their own modeling competencies or in addressing students' difficulties during modeling. Conversely, teachers with positive affective orientation but with limited cognitive resources for modeling knowledge (Hammer, 2000) may struggle to design and implement modeling tasks effectively.

The integration of cognition and affect in our framework has several implications for teacher education in modeling. First, the design of PD programs can foster teachers' positive attitudes towards modeling and teaching modeling by incorporating tasks with various educational, cultural, and societal contexts that are equally challenging but also feasible at the same time. This PD environment will provide opportunities for teachers to (i) experience struggling in tasks in a productive manner and to (ii) appreciate the utility of modeling in describing and predicting real-world issues. PD programs can also consider incorporating book studies and journal reflections, where teachers are continuously encouraged to reflect on their learning both as a modeler and as an educator of modeling. Second, teachers' growth in modeling competencies for teaching should be assessed through both knowledge and affect domains. Evaluators can leverage tools such as longitudinal cross-sectional analysis of focus group interviews (Tuthill et al., 2020), and pre-post surveys of modeling specific attitudes (e.g., Asempapa & Brooks, 2022), to track changes in teachers' affective orientation towards teaching and learning of modeling. Finally, researchers may use the framework both to inform the design of modeling-specific PD programs and data collection, and to undertake empirical validation through its application to data. This work contributes to the fields of teacher education and modeling by challenging the prevailing emphasis on PCK for modeling. We argue that attention must also be given to *what* teachers need to know as *learners of modeling* and *how* teachers should *feel* towards the teaching and learning of modeling for its successful implementation in K-16 classrooms.

Acknowledgement

This work was supported by the National Science Foundation, Award Number 2345194.

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