Approximation, Riemann Sums, and Coordination Class Theory

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Researchers have recognized Adding Up Pieces (e.g., Jones, 2013) to be a critical interpretation when applying definite integrals to problems in physics and other domains. At the same time, researchers have not investigated as closely students' moment-to-moment reasoning when constructing pieces of an approximation and how that reasoning might vary across situations. We interviewed 11 students enrolled in calculus-based physics courses using tasks that asked for approximations, not definite integrals. Individual students approached approximations in more than one way—depending on the problem situation and how it was presented—and they did not always make connections among their different approaches. Based on this observation, we conjecture that approximation in service of Riemann sums is a good candidate for a coordination class (e.g., diSessa, 2002; diSessa & Wagner, 2005). We summarize the theory, present the case of one student, and draw out implications for research and instruction.

Keywords: Approximation, Riemann Sums, Coordination Class Theory

A growing body of research has reported on students' interpretation and construction of integrals, especially when solving problems in physics and other contexts outside of pure mathematics (e.g., Cui et al., 2006; Jones & Ely, 2023; Nguyen & Rebello, 2011; Oehrtman & Simmons, 2023; Thompson, 1994a, Von Korff & Rebello, 2012). One key area of interest has been whether and, if so, how students connect definite integrals with adding up small amounts of a target quantity (Ely, 2017; Jones, 2013, 2015). At the same time, research has not investigated in detail students' moment-to-moment reasoning when constructing pieces that contribute to an approximated target quantity. We report on results emerging from an on-going study that examines how undergraduate students enrolled in calculus-based physics courses respond to tasks that ask for approximations, not definite integrals. Individual students approached approximation in different ways from one task to the next, leading us to conjecture that Riemann sum approximation is a good candidate for a coordination class (e.g., diSessa, 2002; diSessa & Wagner, 2005).

In the sections that follow, we review key strands of research on students' reasoning when applying definite integrals to solve problems, primarily from physics. We follow with a brief description of Coordination Class theory. Then we present our research questions and summarize our methods. Finally, we present the case of one student, Daner, as an exemplar of empirical phenomena we observed across participants.

Literature Review

Applying definite integrals to physical situations requires reasoning in terms of quantities such as lengths, time, work, electric charge, etc. Past research has provided a variety of perspectives on such reasoning, much of it grounded in Thompson's account of quantitative reasoning (e.g., Thompson, 1994b, 2011). In one line of such work, Jones (2013, 2015) has identified several ways in which college students can interpret the $\int_a^b f(x)\,dx$ notation. One of these, the Adding up Pieces interpretation, consists of (a) partitioning a physical situation into small pieces, (b) approximating the target quantity within each partition piece, and (c) summing the small amounts of target quantity across all partition pieces to approximate the total amount.

The Adding Up Pieces interpretation is particularly useful when applying definite integrals to physics and other domains (e.g., Stevens & Jones, 2023; Oehrtman & Simmons, 2023).

In further work, researchers have identified reasoning about local relationships as a key challenge for many students (e.g., Ely, 2017; Jones et al., 2017; Oehrtman & Simmons, 2023; Sealey, 2014; Thompson & Harel, 2021). As one example, Oehrtman and Simmons (2023) proposed *Emergent Quantitative Models* as a tool for tracing how students' attention shifts across three scales when constructing definite integrals. They named these scales the Basic, Local, and Global models. A Basic model relates quantities with constant values, a local model restricts a basic model to small regions where varying quantities are approximately constant, and a Global model is an accumulation based on local models. Oehrtman and Simmons highlighted the interaction and co-evolution of the three scales.

Past research has established *that* students can have trouble reasoning with local approximations but has stopped short of examining *how* students reason in contexts that afford constructing approximations in ways compatible with Riemann sums. We take this next step.

Theoretical Framework

Coordination Class theory (e.g., diSessa, 2002; diSessa & Sherin, 1998; diSessa & Wagner, 2005) provides an account of certain concepts within the knowledge-in-pieces epistemological perspective (e.g., diSessa, 1993, 2006). diSessa (2002) motivated the notion of coordination classes with the following description of expert accomplishment: "The fundamental assumption behind the idea of coordination classes is that information is not transparently available in the world. Instead, we have to learn how to access different kinds. Indeed, in different circumstances, we may need to use very different means to determine the same kind of information" (p. 43). This description implies that (a) a collection of knowledge resources supports "seeing" the same concept-specific information across situations, a phenomenon referred to as *span*, and (b) different subsets of concept-related resources might be needed in different situations, a phenomenon referred to as *concept projections* (diSessa & Wagner, 2005; Wagner, 2010). Researchers have begun applying coordination class theory to gain insight into students' reasoning about several topics in mathematics and science (e.g., Izsák et al., 2021; Levin, 2018; Parnafes, 2007; Wagner, 2006), but not to approximations or Riemann sums.

We sketch how Riemann sum approximations could be interpreted in terms of Coordination Class theory. First, we acknowledge that researchers have taken different positions on whether one can always interpret the integrand, $f(x)$, as a rate in a definite integral, $\int_a^b f(x)\,dx$. If one does interpret $f(x)$ as a rate, then proficient reasoning about Riemann sums involves approximating $f(x)$ using locally constant rates over small changes in x. Furthermore, if one interprets areas of rectangles as products of constant rates and small changes in x, then such areas can serve as generic representations of small pieces that can be accumulated to approximate target quantities. Students might use such reasoning to approximate a target quantity in one context but then use quite different reasoning when approximating target quantities in other contexts. Employing different reasoning to construct approximations across contexts is consistent with using different concept projections. Furthermore, seeing that quantities across situations can be interpreted as rates need *not* be transparently available. As an example, students might have experience thinking of locally constant speeds when estimating distance travelled in a straight line over an interval of time, but not see density as a locally constant property that can be used in a similar way to estimate mass. Constructing parallel reasoning across situations is an example of increasing span.

Research Questions

1. What knowledge resources do students engage when constructing approximations of target quantities across problem situations?
2. To what extent are students' approaches to constructing approximations compatible with Riemann sum?

Methods

We recruited 11 students from second semester, calculus-based physics (Five in Fall 2024, six in Spring, 2025) at a selective university in the northeast United States. Each student participated in a series of three, semi-structured interviews (e.g., Bernard, 1994; Ginsburg, 1997) and was paid $50 per interview. The interviews lasted 60 to 70 minutes each and were spaced a few weeks apart. Students worked on tasks that afforded opportunities to construct approximations through addition of pieces, consistent with constructing Riemann sums. We began with approximation in contexts familiar to students (e.g., length • width = area, speed • time = distance) and then moved to approximation in contexts we conjectured would be less familiar (e.g., density • volume = mass, pressure • area = force). These choices were deliberate: We wanted to see whether and, if so, how students would draw on prior experiences constructing approximations in what, for them, would be novel contexts. We omitted formulae that related quantities, because they could short cut students' reasoning about multiplication when forming pieces of target quantities. In addition to text, several of the tasks included diagrams or graphs. Students worked on two or three tasks per interview, pacing that allowed for multiple follow-up questions to explore their reasoning in detail.

We recorded the interviews using two cameras, one to capture the student and interviewer and one to capture close ups of students' written work. After each interview, we collected all written work, combined the two video recordings into a single file, and transcribed verbatim. We analyzed each interview individually and met weekly to discuss our observations. We used students' utterances, inscriptions, and hand gestures to infer the knowledge resources they relied on moment-to-moment for each task. After group discussion we often rewatched videos to refine our analyses. We constructed accounts of reasoning for each task and looked for similarities and differences in the knowledge resources and approaches to approximation students used across tasks.

Results

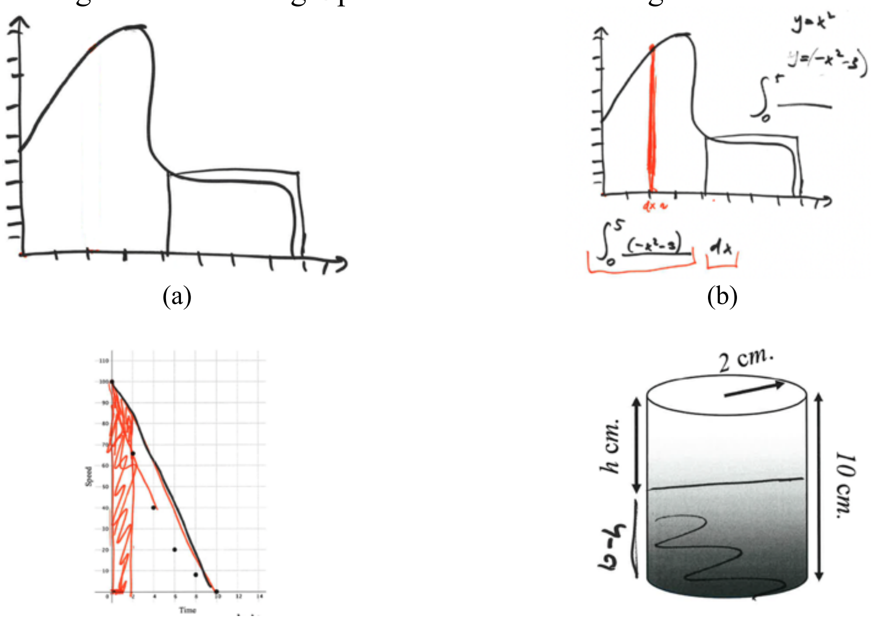
Our top-level result is that all students approached approximation in different ways as they moved from one problem situation to another, and we conjecture that students' reasoning about approximation may be well captured using constructs from Coordination Class theory, as discussed above. Given space constraints, we have room to present key features of one case. Daner (a pseudonym) self-reported receiving a 4 on the AP Calculus BC exam, was enrolled in second-semester calculus-based physics (Spring 2025), and was majoring in mechanical engineering. The second author led all three interviews with Daner.

Interview 1: Multiple Concept Projections When Constructing Local Approximations

During the first interview, Daner demonstrated different approaches to approximating area from lengths and widths, distance from speeds and times, and mass from densities and volumes. The Lawn task asked students to approximate the area of a lawn whose boundaries were presented through a diagram that showed two perpendicular line segments and a third, curved line. Daner immediately recognized the relevance of an integral but was stuck initially without "the equation of the line." He then partitioned the lawn into two regions, a parabola and a rectangle (Figure 1a). Here, his sense of approximation relied on guessing an algebraic expression for the parabola, choosing five as an upper bound for the definite integral, and treating the second region as a rectangle. He then wrote $\int_0^5 (-x^2 - 3)dx$ for the area under the parabola (Figure 1b). When asked to explain the meaning of dx, he described a second method for approximating area more compatible with a Riemann sum:

> Like this is the process that does it, uses the equation of the parabola that is given, takes the height, multiplies by dx and it's like a small rectangle, multiplies by dx, and does it infinitely many times until five.…The integral, what it does, it adds it up together.

When asked to illustrate his explanation, Daner drew a generic "small rectangle" (Figure 1b). Thus, he demonstrated two approaches to approximating the area of the lawn, and only the second was well-aligned with Adding Up Pieces and constructing a Riemann sum.



(a) (b) (c) (d)

Figure 1. Daner approximates (a) Area using a parabola and rectangle, (b) Area using small rectangles only, (c) Distance using trapezoids, (d) Mass using basic models.

The Car task presented the speed of a car every 2 seconds as it slowed to a stop. The data were presented in a table with columns labeled "Time" in seconds and "Speed" in feet per second. Similar to the Lawn task, Daner commented that he did not have an equation with which to work. When he tried to recall an equation, he multiplied speed not by time, but by distance. At this point, the interviewer introduced the same data presented as discrete points on a graph whose axes were labeled "Speed" and "Time." Daner stated that "the distance is the area under the curve" and explained how to estimate using averages for each 2-second interval:

> I would find like, the velocity, the mean, like the mean velocity between the first one and the second one, and do the same like here, here, here, here, and multiply it by 2 seconds, and add them together.

Daner then used the given data to compute two examples of average speed correctly, (100 + 66) ÷ 2 = 83 and (66 + 40) ÷ 2 = 53. He multiplied both results by the duration of 2 seconds but, at first, said he could not "visualize" the corresponding 166 and 106 values on the graph. Then he changed his mind, drew a curve that connected the graphed points, and said that 166 would be the area "under this curve from 0 to 2." Daner then clarified that 166 would not be exact, drew a straight line connecting the points (0, 100) and (10, 0), shaded the resulting trapezoidal region for the interval from 0 to 2 seconds (Figure 1c), and stated that 166 would be exact if the deceleration was linear.

From the data, we could not tell why Daner switched his focus from lengths and widths of rectangles on the Lawn task to average speeds and trapezoids on the Car task. Several interviewed students expressed a preference for midpoint Riemann sums over lefthand or righthand Riemann sums because they are often more accurate. The associated calculations can be associated either with areas of trapezoids or with areas of rectangles. The important point is that Daner maintained a focus on Adding Up Pieces but used different knowledge resources in each task, ones related to areas of rectangles versus ones related to averages and trapezoids.

The Sand task presented a cylinder containing sand suspended in water. The cylinder had a height of 10 cm and a radius of 2 cm. The density of sand in water was given by $\rho(h) = \sqrt{h}$, where h measured distance from the top surface. A diagram showed the cylinder with graduated shading to reinforce that the density of the sand varied continuously from the top to the bottom of the cylinder (Figure 1d). Daner recalled the formula "density is mass over volume" and the formula for the volume of a cylinder. At the same time, he interpreted the $\rho(h) = \sqrt{h}$ function to give a single value of density that applied to the top h cm of the cylinder. He then considered whether the sand would be in the top region with a height of h or in the bottom region with a height of $10 - h$. He wrote two expressions for mass based on basic models, one for each case: $m = \rho \bullet 4\pi h$ and $m = \sqrt{10 - h} \bullet 4\pi(10 - h)$. Daner explained the meaning of density using qualitative comparisons, "if there's a lot of density, that means that there's a lot of mass in a more compact volume." He then relied on this meaning to explain:

> In this case, if we're looking at sand, the lower part is more dense than the upper part, because there's less sand in it for the same amount of volume compared to the other one where there's more sand for a specific volume.

Finally, when asked if he interpreted $\rho(h)$ as the density for the entire top h cm or just for a thin slice of the cylinder, Daner reported, "I would think of it, yes, from the top to the bottom of h, not in terms of layers."

Daner's approach to the Sand task both resembled and differed from his approach to the Lawn and Car tasks. Parallels included both a correctly stated relationship among mass, density, and volume that paralleled a correct relationship among distance, speed, and time and a description of density in terms relevant for conceiving a unit rate—an amount of sand "for a specific volume." At the same time, he gave no evidence of constructing small pieces of a target quantity. It is plausible that his interpretation of $\rho(h)$ as a fixed value for a cylinder of height h obstructed his attention to small pieces of mass. We conclude that Daner drew on different sets of resources across the Lawn, Car, and Sand tasks tasks and that features of task presentation (e.g., diagrams, graphs, explicit variables) shaped which resources Daner employed. Such data are consistent with concept projections of resources relevant to approximating small pieces of a target quantity in some situations but not others.

Interview 2: Coordinating Knowledge Resources Toward Increasing Span

The second interview began with the Rod task, a task similar to the Sand task that involved approximating mass given a varying density. The rod was a horizontal cylinder of length L, radius R, and density $\lambda(X) = X^2 + 4$, where X was the distance from the left-hand end. The first version of the task used uniform gray shading on the cylinder, the second version used gradated shading to help communicate that density increased continuously from left to right (Figure 2a). Daner reused knowledge resources evident in his reasoning on the Car and Sand tasks: He interpreted density, $\lambda\,(X)$, first to be a single value for a cylinder of length X and then reasoned about midpoints and averages.

(a) (b)

Figure 2. Daner approximates mass (a) Partitioning the rod into fourths, (b) Using a graph.

Daner began by recalling density equals "mass over volume" and the formula for volume of a cylinder. Once again, he treated density as a single, fixed value and used a basic model to multiply density and volume for the entire rod: $M\ = (L^2 + 4) \bullet \pi R^2 \bullet L$. At this point, the interviewer sought to help Daner perceive density as varying. He began by presenting a second image of the cylinder in which gradated shading reinforced density increasing from left to right. Daner stated that the new diagram did not change his interpretation of density as a single value: "It just doesn't make any sense why the density would increase if it goes to the right."

When the interviewer asked to view density as varying anyway, Daner partitioned the rod into fourths (Figure 2a) and explained that he would "subtract the distances from each other and plug it into L again." He subtracted $\frac{3L}{4} - \frac{2L}{4}$ and generated: $M\ = (\frac{L}{4} \bullet \pi R^2) \bullet ((\frac{L}{4})^2 + 4)$. Thus, he interpreted $\frac{L}{4}$ as a new length and reasoned in ways that paralleled his work on the Sand task and his initial work on the current task. When the interviewer asked about the mass of the segment between $\frac{L}{4}$ and $\frac{2L}{4}$, Daner saw that his work implied that the mass of two regions would be the same while the actual masses should differ. At this point, he was stuck: "The answers are going to be the same for mass. So I don't really, I don't really know how to find the different masses."

Realizing that he needed a new approach, Daner turned his attention to the graph of $\lambda\,(X) = X^2 + 4$. He partitioned the horizontal axes from zero to L into six pieces, considered the interval between $\frac{2L}{6}$ and $\frac{4L}{6}$, and asked how "to find a like density between two lines" (Figure 2b). After considering and rejecting using differences in densities at two points, he returned to notions of midpoints and averages. First, he used the example $\lambda\,(X) = 2X$ to explain that using a midpoint would work for a density that increased linearly: "Like if it was a linear, because it goes linearly the average density would also, like it would be just the half of, the half value." This reasoning resembled his reasoning about average speed in the Car task. Then he computed midpoints *for distances* and generated expressions for mass in the third region from the left, $M\ = (\pi R^2 \bullet \frac{L}{4}) \bullet ((\frac{5L}{8})^2 + 4)$, and in the second region from the left, $M\ = (\pi R^2 \bullet \frac{L}{4}) \bullet ((\frac{3L}{8})^2 + 4)$. Notice that these expressions could contribute to a normatively correct midpoint Riemann sum approximation. Daner did *not* draw anything that looked like rectangles or trapezoids that approximated area under a curve.

At this point, the interviewer asked Daner to identify similarities or differences across the Lawn, Car, Sand, and Rod tasks. Not surprisingly, Daner saw similarities between the Sand and

Rods tasks and found the Lawn task to be less similar to the other three tasks. Daner said that the Car and Rod tasks were similar but, an exchange later, said they were not similar. When the interviewer asked about these two points of view, Daner described in more detail parallels that he saw. First, he compared interpretations of area under the graph in the Car task (Figure 1c) and in the Rod task (Figure 2b). He knew the former represented distance and after a 75-second pause discussed whether the latter could represent mass:

> It has to be density multiplied by length. Because [in the Car task] this region is speed multiplied by time, which gives us distance. And so this region is distance. Whereas [in the Rod task] it also has to be density multiplied by length. But I was thinking like why would you multiply length with density, like what would that ever give? But then I just understood that that's basically, that's basically the volume. Like multiplying by length can be the same as multiplying by volume for a shape. So that will give you mass.

On the one hand, Daner drew sensible parallels where units on axes (speed and time, density and length) determined units represented by areas under curves. On the other hand, he knew that in the Rod task "it has to give you basically mass." He then included πR^2 on the horizontal axes:

> So you can just give pi r squared to L, multiply L by pi r squared, which will give you volume. And when you multiply volume by density, it gives you mass.

Although Daner articulated normatively correct coordination of density, volume, and mass represented as area, he had trouble maintaining that coordination when he wrote: $\lambda\,(X) = (\pi R^2 L)^2 + 4$. In response to further questions, Daner explained that the Lawn, Car, Sand, and Rod tasks were all similar based on the solution process: "what you do basically at the end of each task is you find the area underneath of the, of any like equation that you get." At the same time, when he stated that "density is not similar to speed here at all" we could see that the parallels he perceived between constructing approximate distances in the Car task and approximate mass in the Rod task were incomplete. Thus, he made partial progress towards increased span.

Discussion

In the Background section, we organized past research on definite integrals into two strands, one that has identified different interpretations of integrals and one that has delineated layers of attention (e.g., Basic, Local, and Global models). Our tasks combined with semi-structured interview methods allowed us to investigate in more detail how students construct approximations moment-to-moment within and across problem situations. We presented Daner's case as an exemplar of our overall finding that the students we interviewed often approached approximation in different ways as they moved from one problem situation to next. This suggests that constructing parallel reasoning across situations that include speed, density, and other quantities is not straightforward and involves developing disciplinary perspectives on what counts as a rate—for instance, the independent variable does not always have to be time.

Compared to past research, our results suggest that more precise theory is necessary if we are to account for students' reasoning about definite integrals in ways that can ultimately inform instruction. Tools from Coordination Class theory have helped us describe with some precision the complexity of constructing approximations across situations in ways compatible with Riemann sums. In this regard, we build on Wagner (2016), who took a first step in this direction when arguing that students' various interpretations of definite integrals could be understood as different concept projections. As noted by diSessa (2002), relevant information is not always transparently available, and students may need different means to construct approximations when assembling target quantities and solving problems in areas outside of pure mathematics.

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