Instances of First-year Calculus in a Calculus-based Physics Textbook

Maxwell Ichinose David Robles Michael E. Loverude
California State University, Fullerton

We examine the presence of calculus concepts and skills in the portions of a standard textbook that are generally covered in the second and third courses of a typical introductory calculus-based physics course. Using the Calculus Concept Framework (Sofronas 2011) three researchers independently coded calculus concepts and skills in eighteen chapters of a calculus-based physics textbook covering electricity and magnetism, geometrical and physical optics, and modern physics. Along with prior work on the first twelve chapters covering mechanics, this analysis suggests that the calculus content is uneven and not well aligned with prerequisites.

Keywords: physics, calculus in the disciplines, instructional practices, textbook analysis

In previous work [redacted], we described analysis of the classical mechanics portion of a common introductory physics textbook using the Calculus Concept Framework (Sofronas 2011) to examine the prevalence of calculus skills and concepts in the first semester of introductory physics. In this paper, we continue this analysis to cover the portions of the textbook that cover electricity and magnetism, geometrical and physical optics, and the introduction to modern physics.

Our goals in this project have been to document and characterize instructional practice in introductory physics with respect to the use of mathematics in general and calculus in particular. We have sought to understand the extent to which calculus is used to scaffold student understanding, following the framing of Biza et al who contrasted calculus as a scaffold vs a filter (Biza 2022). As a scaffold, calculus is used to support learning and understanding; as a filter, it serves to prevent students from progressing without successful completion of calculus.

Introduction and Previous Research

As a means of characterizing calculus practice in introductory physics, we have followed the work of several other scholars in adopting the Calculus Concept Framework [CCF] (Sofronas 2011). The CCF was developed through analysis of interviews with a set of disciplinary experts who were asked to characterize the goals of first-year calculus. One element of the CCF identifies a list of concepts and skills that are essential to calculus understanding and that are generally taught in a calculus course sequence.

Several researchers have used the CCF to guide their analysis of the calculus content in later courses, including later courses in mathematics as well as those in so-called client disciplines. Czocher et al. used the CCF to describe the concepts and skills from calculus were used in a course in differential equations (Czocher et al., 2013). Subsequent work adapted this analysis in disciplines that require calculus, including introductory engineering (Faulkner et al., 2020), and general chemistry (McAfee and Rodriguez, 2024). Until our prior work, there was no similar analysis in introductory physics.

Research questions and methods

The primary research question driving this study has been: What concepts and skills from calculus are encountered by students in introductory calculus-based physics, and when are they encountered? A secondary question that has emerged is, what additional mathematical concepts and skills are encountered by students? In previous submissions we have reported initial results from the mechanics portion of the text. For this paper we focus on the portion of the text that is generally covered in the second and third semesters of a typical introductory sequence. In addition, we reflect on these results as well as those from prior work to characterize the role of calculus for the whole introductory sequence.

The methodology involved three researchers coding instances of the concepts and skills from the CCF in a commonly used introductory physics textbook. The three researchers independently coded chapters 2 through 12 and 21 through 34 of the text (Resnick et al., 2014) for the presence of absence of each of the skills described in the CCF. We neglected chapter 1 in our analysis, which introduces quantities and units, and chapters 13-20 which cover topics that are skipped in the introductory sequence at our university (fluids, thermal physics).

After each section was coded, the researchers met to discuss codes and coding principles and re-negotiated how codes were to be interpreted. As we proceeded, we recorded inter-coder agreement after independent coding. By the end of the first-semester analysis, the agreement between coders was very high, e.g., 97% agreement for Chapter 11.

Three examples of portions of the text that were coded can be found below in the Results. Each example states what concepts or skills were coded from that passage. Note that the coding was based on the entire section, so the absence of a concept or skill in the included passage does not necessarily mean that the concept or skill is not present in the section that includes it.

There were several repeated instances of mathematical practices that were not present in the CCF. Some were instances of calculus concepts beyond the scope of the CCF, due to being covered in multivariable or vector calculus: dot and cross products, partial derivatives, and ordinary differential equations. We also added concepts and skills to our codebook for units, vectors and unit vector notation, and reasoning with graphs and diagrams. While one might argue that none of these are distinctly calculus, each of these were more common than many of the skills and concepts identified in the CCF and part of mathematical practice for the course.

While all three researchers identify themselves as physics faculty or students, one of the three is a supplemental instruction leader in calculus courses and so is regularly in contact with instructional practices in mathematics. This researcher initially coded some concepts based on practice in a calculus classroom. This made it clear that our coding is from the point of view of physics practice, and that coding might be different were mathematicians doing the coding. As the project has progressed, we have also sought to characterize the ways in which mathematical practice in introductory physics differs from what students encounter in their calculus courses, as well as a broader characterization of the mathematical skills beyond the CCF. These aspects are ongoing and will include interviews with instructors and students.

Results

We summarize the results in tabular form in Figure 1. The number in each cell refers to the number of codes for this concept or skill in each chapter. While we coded all the concepts and skills in the CCF (and a few others), the table displays only a few of the categories. The reason for this is twofold. First, many of the categories were coded very infrequently (or even not at all). CCF categories that were seldom / never coded included: (concepts) Riemann sums, Parametric and Polar Equations, Continuity, Optimization, Transformation; and (skills) Parametric Equations, Polar Coordinates, Epsilon Delta Proofs. Many of these were lower prevalence in the original CCF study. There were two additional skills that we omitted because their coding was so ubiquitous to be meaningless: Listening and Reading Comprehension and Facility with Definitions and Notation.

We added to our analysis several concepts and skills that were not included on the CCF, either because they are not considered calculus or are part of courses after the first year: vectors, units, graphs, diagrams, dot and cross products, and partial derivatives. In the diagram we show only the last three of these. Vectors, units, and diagrams were ubiquitous in the data set.

Some of the integrals that students encounter are no longer integrals of a single variable as students would encounter in first-year calculus. In this textbook, students encounter partial derivatives and functions of multiple variables, as well as path integrals, surface integrals, and volume integrals. These concepts are concentrated in the early sections of second semester physics, which at many institutions has Calculus 2 as co-requisite.

Concepts	Chp22	Chp23	Chp24	Chp25	Chp26	Chp27	Chp28	Chp29	Chp30	Chp31	Count	% of Total Codes
Derivative	0	0	5	0	3	6	0	0	20	9	43	8.62%
Integral	9	8	10	9	6	0	0	12	9	3	66	13.23%
Limit	0	0	0	0	0	2	2	2	0	0	6	1.20%
Approximation	4	0	3	0	0	0	0	2	0	0	9	1.80%
Skills												
Derivative Computations	0	0	0	0	0	3	0	0	2	0	5	1.00%
Techniques of Integration	6	0	0	0	0	0	0	0	0	0	6	1.20%
Area / Volume	0	15	0	10	6	0	0	0	6	0	37	7.41%
Trigonometric Manipulations	9	3	15	0	1	0	15	12	0	15	70	14.03%
Manipulating Algebraic Expressions	15	14	22	15	13	9	19	15	20	18	160	32.06%
Facility with Logarithms and Exponentials	0	0	2	2	0	3	0	0	3	3	13	2.61%
Advanced skills												
Dot Product	2	5	15	6	3	0	0	4	6	0	41	8.22%
Cross Product	2	6	0	0	0	0	20	9	3	0	40	8.02%
Partial Derivatives	0	0	3	0	0	0	0	0	0	0	3	0.60%
											499	100.00%

Figure 1. The prevalence of calculus concepts and skills in the chapters of introductory physics corresponding to electricity and magnetism. The corresponding data for optics and modern physics are not shown, as explained.

Algebra and trigonometry appeared more frequently than any other codes. Manipulating Algebraic Expressions was the single most common code, appearing in all chapters, and represented over 30% of all codes. Trigonometric Manipulations was second more frequent, representing approximately 14% of all codes, though it was more present in some chapters than others. Notably, several of the physical quantities and relationships in electricity and magnetism use vectors and the dot and cross product are both prevalent in this portion of the course.

Whereas derivatives were more prevalent than integrals in the mechanics portion of the class, this trend was reversed in electricity and magnetism, with integrals being coded approximately twice as frequently as derivatives. In addition, for integrals and derivatives, the concept was coded an order of magnitude more frequently than the corresponding skills of computing derivatives or using techniques of integration.

The three semesters of introductory physics have considerably different calculus usage. As described in prior work, the first semester of introductory physics (classical mechanics) features intermittent use of calculus. Derivatives and integrals appear most heavily in chapters on kinematics, work and energy, and rotational motion (2, 4, 7, and 10) but were completely absent from three of the ten chapters in that course. In electricity and magnetism, in contrast, each of the first five chapters, and seven of ten overall, include heavy emphasis on integration, frequently of vector quantities. Students encounter integrals of cross products (Biot-Savart law) and results from vector calculus, including the divergence theorem and Stokes' theorem. Derivatives are less common but are prominent in Chapters 24, 27, 30, and 31.

When the course turns to optics and modern physics in the third semester, calculus essentially vanishes entirely. While we coded through Chapter 37 in this portion of the project, the table in Figure 1 stops Chapter 31. This is because the chapters covering optics and modern physics had essentially zero calculus concepts coded. These sections follow a common pattern; the only codes were for algebra and trigonometry (as well as diagrams and units, not included in the figure) but essentially zero codes for calculus-specific concepts and skills like derivatives, integrals, or limits. There were single instances of integrals in two sections and a derivative in one, but otherwise calculus is entirely absent for this portion of the course.

Discussion and Implications

Our findings have significant implications for instruction in physics as well as calculus. We make some observations and discuss these implications.

Several of the concepts and skills in the CCF have different interpretations in physics.

Two of the concepts and skills identified in the CCF are limits and approximations. These have specific and formal meaning in mathematics. In physics, students encounter limits and approximations as well, but the instantiation of these ideas looks quite different. The passage shown in Figure 2 came from the chapter on electric fields (22) and this section was coded as including both limit and approximation. This illustrates mathematical moves that are common in physics: a complicated expression is simplified by considering what physicists would call a limiting case. This allows the derivation of an approximate expression (equation 22-8 in the figure) that is conditionally valid. A mathematician might or might not code this differently.

We are usually interested in the electrical effect of a dipole only at distances that are large compared with the dimensions of the dipole—that is, at distances such that $z \gg d$. At such large distances, we have $d/2z \ll 1$ in Eq. 22-7. Thus, in our approximation, we can neglect the $d/2z$ term in the denominator, which leaves us with

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}. \qquad (22\text{-}8)$$

Figure 2. An example of limiting arguments and approximations that highlights differences in instructional practices between calculus and physics.

While the contexts are complex, the actual derivatives and integrals are often not.

While the contexts in second-semester physics involve integrals of vector quantities in multiple dimensions, as a practical matter the integrals that are evaluated are often quite simple. As an example, in Chapter 22 the text derives expressions for the electric field of various configurations of static charges, including continuous charge distributions. To derive the electric field of a line of charge, the text begins with a charged ring. To an observer (probably including many students), this might seem more complex, but the symmetry of the charged ring simplifies

Integrating. Because we must sum a huge number of these components, each small, we set up an integral that moves along the ring, from element to element, from a starting point (call it $s = 0$) through the full circumference ($s = 2\pi R$). Only the quantity s varies as we go through the elements; the other symbols in Eq. 22-14 remain the same, so we move them outside the integral. We find

$$E = \int dE \cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int_0^{2\pi R} ds$$
$$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}. \qquad (22\text{-}15)$$

Figure 3: The conclusion of the textbook derivation of the electric field due to a charged ring. While the integral in principle is a vector integral in three dimensions, in practice the integral is of a single variable.

the task. Figure 3 shows the concluding steps. Despite the complexity of the expression for the electric field, the integral to be evaluated is a definite integral of a single variable: $\int_0^{2\pi} ds$. All the complexity of the situation is in setting up and simplifying the integral; evaluation is, in principle, trivial. (Though we note that there is evidence that this may not be so trivial to students.) White Brahmia [2023] has noted that the functions that students encounter with in introductory physics are simpler than their calculus courses might suggest.

In the following chapter, students encounter Gauss' law, which is a physics application of the divergence theorem. The presentation in the text sets up a surface integral over the surface of a sphere but then simplifies the calculation with physical reasoning and symmetry. Notably, the integral is now with respect to an apparent single variable A, but that simplified expression might conceal the complexity of a two-dimensional surface area of a three-dimensional object.

> The drill here is the same as previously. Pick a patch element on the surface and draw its area vector $d\vec{A}$ perpendicular to the patch and directed outward. From the symmetry of the situation, we know that the electric field $\vec{E}$ at the patch is also radially outward and thus at angle $\theta = 0$ with $d\vec{A}$. So, we rewrite Gauss' law as
>
> $$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{\text{enc}}. \tag{23-8}$$
>
> Here $q_{\text{enc}} = q$. Because the field magnitude E is the same at every patch element, E can be pulled outside the integral:
>
> $$\varepsilon_0 E \oint dA = q. \tag{23-9}$$

Figure 4. A surface integral that is simplified through symmetry and physical reasoning to a more straightforward (to physicists, at least) example of integration.

In summary, we note several implications and future thoughts. First, the sequence of calculus usage in introductory physics is a mismatch with co- and pre-requisite structures. In many institutions, including our own, first semester physics is a co-requisite for Calculus I. For students in first semester physics, the first integral might appear in week two; at this time in Calculus I, students might not have even encountered derivatives. In electricity and magnetism, students encounter integrals of vector quantities in three-dimensional situations by the second week of the semester, when students in the co-requisite Calculus II are learning single-variable integration. The third semester then has minimal calculus.

Second, and perhaps a result of the above, when calculus does appear, there are often steps taken to reduce the complexity of calculations. Situations with two or more dimensions are regularly reduced to integrals of a single variable. While these integrals are conceptually complex, as examples above indicate, structurally they appear as integrals of a single variable.

Third, the CCF is an imperfect match to the mathematics that students encounter in introductory physics. As we previously reported, many of the mathematical skills required in the course were not included in the CCF, and many of the skills and concepts identified in the CCF appeared rarely (or never appeared). For the second semester of the course, this reflects the need to attend to vector quantities and situations in multiple dimensions, each normally the province of multivariable calculus and outside the scope of the CCF.

While the textbook strongly guides instructional practice, we note that analysis of a textbook provides only an incomplete description of what instructors and students do in class. We have performed several interviews with instructors related to first semester content, and while preliminary analysis indicates that they perceive their practice to be closely aligned with our findings, that work is still ongoing.

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