Undergraduate Students' Bijective Reasoning about Fibonacci Numbers

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Bijection is an important topic in undergraduate mathematics. Although researchers have studied whether students use bijections to compare infinite sets (Hamza & O'Shea, 2011; Tirosh & Tsamir, 1996; Tsamir & Tirosh, 1992; Tsamir, 1999) and how students understand group isomorphisms (Larsen, 2009; 2013; Leron et al., 1995; Melhuish et al., 2020; Rupnow, 2017; Weber & Alcock, 2004), there is little direct research on how students understand and use bijections. One content area where researchers have looked explicitly at bijections is combinatorics (Lockwood & Ellis, 2022; Mamona-Downs & Downs, 2004; Muter & Uptegrove, 2011). These studies have shown mixed results, suggesting that although students are capable of constructing bijections, it is a challenging topic. More research is needed to understand how students think about and reason with bijections. The research question guiding the current study is: *How do undergraduate students' ways of understanding and thinking about bijections support their combinatorial reasoning and vice versa?*

To investigate this question, I conducted a small-group teaching experiment (Steffe & Thompson, 2000) with six undergraduate engineering majors. I collected 16 hours of data across 12 teaching episodes (each episode was 60-90 minutes). Each episode was video- and audio-recorded, and each student had an iPad which captured screen recordings of their written work. The teaching experiment was broken into three phases: 1) Developing students' combinatorial reasoning, 2) Developing students' bijective reasoning, and 3) Refining students' bijective reasoning. For this poster, I will discuss data from four teaching episodes conducted during phase 2 where students were given a list of written descriptions of sets counted by the Fibonacci numbers and asked to construct bijections between the different sets. Prior to engaging in this series of tasks, students constructed a definition of bijection as a whole class and at the end of these teaching episodes, I asked the students to each write a personal description/definition of what a bijection is. I am currently analyzing this data set to understand how students think about bijections and how they drew on their combinatorial thinking to reason about bijections. To frame my analysis, I draw on Lockwood's (2013) model of student's combinatorial thinking and Harel's (2008) notions of *ways of thinking* and *ways of understanding* to capture students' thinking about bijection

I am currently in the early stages of analyzing this data set to identify students' ways of understanding and thinking about bijections and how these connect to their combinatorial thinking. By the time of the conference, I expect to share emerging findings that illustrate how students reason bijectively when working with Fibonacci-related combinatorial tasks. These findings will contribute to understanding how undergraduate students construct and reason with bijections and can inform instructional approaches for supporting students' bijective reasoning.

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