Mathematics Graduate Teaching Assistants' Discourse in College Algebra: The Case of Anthony

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Graduate Teaching Assistants (GTAs) play a critical role in undergraduate mathematics, often serving as instructors of record with limited preparation for teaching. This paper examines the case of Anthony, a College Algebra GTA participating in a semester-long professional development seminar focused on anticipating student strategies, planning purposeful questions, and reflecting on teaching. Across three observations, Anthony's lessons shifted from lecture-heavy explanations to student-centered problem solving, supported by targeted guiding questions. Monitoring charts from Anthony's planning and post-lesson debriefs revealed how his instructional decisions were shaped by prior beliefs about student-centered learning and contextual pressures such as pacing demands and coordinator feedback. This case study provides insight into the tensions GTAs face in balancing explanation and inquiry and illustrates how professional development can support incremental shifts toward more student-centered mathematics instruction.

Keywords: Graduate Teaching Assistants, College Algebra, Professional Development, Student-Centered Instruction

Graduate Teaching Assistants (GTAs) play a critical role in undergraduate mathematics, often serving as instructors of record for introductory mathematics courses (Weidert et al., 2012). Despite their instructional responsibilities, GTAs often receive little to no preparation to teach (Judson & Leingang, 2016) and limited ongoing support for effective mathematics teaching (Rogers & Yee, 2018). Because GTAs teach thousands of students nationally, their instructional practices directly influence student learning experiences and retention in STEM pathways (Blair et al., 2018).

One key challenge GTAs face is facilitating student-centered instruction. GTAs often struggle to structure lessons in balancing task setup, group work, and whole-class discussion (Speer & Wagner, 2009). Yet, student-centered teaching practices, such as posing purposeful questions, monitoring student reasoning, and facilitating whole-class discussions, can support conceptual understanding and improved student outcomes (Conner et al., 2014; Smith & Stein, 2018).

Professional development (PD) programs for GTAs seek to bridge this gap by providing structured opportunities to plan tasks, anticipate student strategies, and reflect on instructional practice (Deshler et al., 2015; Maki et al., 2019). However, less is known about how GTAs' beliefs, contextual constraints, and PD experiences interact to shape their classroom practices over time.

This paper reports on one case study from a larger project on GTA professional development in College Algebra. Specifically, it focuses on Anthony, a second-semester master's student GTA enrolled in a semester-long seminar designed to support student-centered teaching. The research question guiding this study was: *How did Anthony structure his College Algebra lessons at the beginning, middle, and end of the semester, and how were these structures influenced by contextual pressures?* By examining Anthony's case, this paper illustrates how a GTA's instructional approach shifted during the semester and highlights the role of external pressures placed on GTAs in shaping the enactment of student-centered teaching.

Literature Review and Theoretical Framing

GTAs struggle to enact student-centered instruction (Speer & Wagner, 2009), challenges that stem not only from limited preparation (Rogers & Yee, 2018), but also from a tendency to reproduce the lecture-based instruction they experienced as students (Kung & Speer, 2009; Sofronas & DeFranco, 2009). Understanding how GTAs navigate these challenges requires attention to both the knowledge they bring to teaching and the contextual factors that shape their practices. This study is framed by Hauk et al.'s (2014) knowledge of discourse, which builds on Ball et al.'s (2008) Mathematical Knowledge for Teaching (MKT) framework.

The MKT framework differentiates between instructors' subject matter knowledge (SMK) and pedagogical content knowledge (PCK). Within PCK, knowledge of content and students (KCS) involves anticipating how learners may approach or misconceive a concept, while knowledge of content and teaching (KCT) focuses on sequencing ideas and instructional moves. Knowledge of curriculum concerns how mathematical topics fit within broader curricular progressions and instructional goals (Ball et al., 2008). This framing emphasizes the multifaceted nature of effective instruction and provides a lens for examining GTAs' instructional decisions.

Building on this model, Hauk et al. (2014) introduced knowledge of discourse to highlight the communicative dimension of teaching. Knowledge of discourse examines how instructors foster interactions among students, exchanges between students and instructors, and connections between classroom activity and broader mathematical communities. They linked this knowledge to three forms of instructor thinking: **anticipatory thinking**, which connects to KCS as teachers predict and prepare for a range of student responses; **implementation thinking**, which connects to KCT as teachers adapt instructional moves in real time; and **curricular thinking**, which connects to knowledge of the curriculum as teachers situate lessons within broader mathematical goals and trajectories. Later work reinforced that discourse is integral to both the enactment of mathematical tasks and the construction of collective understanding (Jackson et al., 2020).

Student-centered instruction in undergraduate mathematics has been consistently tied to opportunities for classroom discourse (CBMS, 2016; Yackel, 2002). Mathematics is not only a collection of procedures but also a way of reasoning, justifying, and engaging in argumentation (Conner et al., 2014; Sfard, 2001). Yet facilitating productive discussions is challenging, as instructors must guide students toward conceptual goals while maintaining space for exploration (Sherin, 2002). The five practices for productive mathematical discussion (Smith & Stein, 2018; Stein et al., 2008) provide a framework for supporting these instructional decisions: anticipating student responses, monitoring strategies during class, selecting contributions to highlight, sequencing them to build toward conceptual understanding, and connecting responses to key mathematical ideas. Questioning is central to each of these practices, both in assessing students' current thinking and in advancing their reasoning (Smith et al., 2017). However, research also shows that without targeted professional development, many GTAs continue to rely on procedural questions and extended explanation rather than orchestrating open-ended discourse, underscoring the need for sustained support (Paoletti et al., 2018; Roach et al., 2010). Thus, PD is essential in supporting GTAs from moving beyond procedural questioning toward facilitating mathematical discourse.

Although GTAs often possess strong mathematical backgrounds, content expertise alone does not ensure effective pedagogical practices (Begle, 1979; Musgrave & Carlson, 2017). PD programs vary widely and range from brief orientations to semester-long seminars (Deshler et al., 2015; Rogers & Yee, 2018). Sustained PD models emphasize iterative cycles of planning, instructing, and reflecting (Kung & Speer, 2009) as well as peer observation and mentoring

(Deshler et al., 2015). Recent recommendations highlight three priorities for GTA PD: fostering student engagement, developing knowledge of student thinking, and providing ongoing, sustained pedagogical training for GTAs (Maki et al., 2019). The seminar at the center of this study was designed with these principles in mind, engaging GTAs in planning cognitively demanding tasks, anticipating student strategies, and developing purposeful questions.

Methods

This paper reports a sub-study from a larger convergent mixed-methods investigation of GTAs teaching College Algebra. The larger project integrated qualitative (seminar recordings, reflective journals, interviews, planning monitoring charts, classroom transcripts) and quantitative (observation protocols, student surveys) data to examine GTA instruction and student perceptions. This paper focuses specifically on lesson structure across the semester using the Classroom Observation Protocol for Undergraduate STEM (COPUS; Smith et al., 2013) and a focal case analysis of one GTA, Anthony, a second-semester master's student. This was Anthony's second time teaching College Algebra. Other data sources are referenced to provide context for interpretation.

Context and Participants

This study was conducted at a large public university during one semester of a weekly professional development seminar, the Planning for Approaches to Teaching High-demand tasks (PATH) seminar, for GTAs serving as instructors of record for College Algebra. In that semester, twelve GTAs were assigned to teach College Algebra, and six volunteered to participate in this study. All participants were master's students in mathematics and were in their second semester of teaching. Each had previously completed a first-semester seminar on teaching mathematics, where the researcher was their instructor of record. Participation in the PATH seminar was voluntary, and GTAs who participated earned two credit hours and a $200 stipend provided by the mathematics department.

PATH Seminar Overview

The PATH seminar emphasized planning cognitively demanding, group-worthy tasks, anticipating student strategies, and crafting both assessing and advancing questions aligned with the five practices (Smith & Stein, 2018). Each week, GTAs engaged in a four-part cycle: (1) working through a focus task (adapted from Kuennen, 2021) as learners, (2) recording anticipated strategies and potential questions on a monitoring chart (Smith et al., 2020), (3) implementing the task the following week in one of their class sessions and (4) reflecting after implementation of the task on unanticipated student strategies. GTAs were expected to incorporate each focus task into their class the following week and share reflections during the subsequent seminar session.

Data Collection

Each GTA was observed three times during the semester, at the beginning (weeks 1–3), middle (weeks 7–8), and end (weeks 13–14), in their 80-minute College Algebra class. Observations were video recorded with the camera focused on the instructor and GTAs wore microphones to capture audio during student work. COPUS (Smith et al., 2013) was used to code observations minute-by-minute for both instructor and student actions.

In addition to the classroom observations, GTAs completed monitoring charts (Smith et al., 2020) for the seminar focus tasks. Monitoring charts asked instructors to anticipate a range of student strategies, potential misconceptions, and plan assessing and advancing questions they might pose during instruction. The charts also included space to document which students used each approach and how to order student strategies in whole-class discussions. These documents provided a window into GTAs' planning and in-the-moment noticing of student thinking. A blank sample is provided in Table 1 to illustrate the structure of this planning tool.

Table 1. Sample Monitoring Chart

Strategy	Assessing Questions	Advancing Questions	Who and What	Order
Anticipated Strategy 1				
Anticipated Strategy 2				
…				

Finally, each GTA participated in three semi-structured interviews following their observed lessons. Interviews followed a common protocol designed to elicit GTA reflections on planning, classroom interactions, questioning, and student engagement.

Data Analysis

All classroom observations were initially coded with the full set of COPUS categories. For interpretability and comparison across lessons, codes were collapsed using the COPUS categories for instructors (Presenting, Guiding, Administration, Other) and for students (Receiving, Talking, Working, Other). Proportions were calculated for each category to generate a profile of lesson structure for each GTA across the three observations. For more details on these categories and individual codes, see Smith et al. (2013).

Monitoring charts were analyzed descriptively with attention to the strategies GTAs anticipated, the assessing and advancing questions they prepared, and the number of students noted for each approach. Particular attention was given to alignment (or misalignment) between GTA planning and classroom enactment.

Although COPUS and monitoring charts were collected for all six participants, this paper highlights Anthony's case. Anthony was selected because his trajectory illustrated both common challenges and noteworthy shifts in practice across the semester, making his case illustrative of the tensions GTAs experience in balancing instructional demands. The analysis traced changes across his three lessons, with attention to how his distribution of presenting, guiding, and student engagement shifted over time. To interpret these shifts, Anthony's interview reflections were triangulated with his monitoring charts and COPUS data, providing multiple perspectives on his planning, enactment, and instructional decisions.

Results

COPUS: Lesson Structure Across Observations

Table 2 summarizes the data for Anthony's three observations on the collapsed COPUS categories. With the COPUS data, multiple codes can occur within the same minute, so instructor and student percentages do not add to 100%.

Table 2. Proportion of Class Time for Anthony's Observations

Observation	Instructor				Student			
	Presenting	Guiding	Admin	Other	Receiving	Talking	Working	Other
1	73%	63%	5%	2%	73%	32%	27%	0%
2	71%	95%	7%	0%	73%	51%	27%	0%
3	17%	88%	5%	0%	2%	0%	98%	0%

Anthony's first two observations were dominated by presenting, with nearly three-quarters of class time coded as instructor presenting and students receiving information. Although guiding was coded relatively high (63% and 95%), this largely reflected procedural questions posed during lecture rather than student-centered activities. In contrast, Anthony's third observation reflected a shift in his instructional practices. He devoted only 17% of class time to presenting, while student work jumped from 27% to 98% of the class period. Guiding remained high (88%) in this lesson, which reflected his monitoring during group work and one-on-one questioning rather than procedural questions during lecture. Students' time receiving information dropped from 73% to 2%, highlighting the move from lecture-driven to student-centered instruction.

Monitoring Charts

Anthony's monitoring charts offered insights into how his planning aligned (or failed to align) with enactment. In his first observation, which focused on solving quadratics using multiple approaches, Anthony anticipated that "no help was needed," assuming that students would work through the task independently. However, his after-lesson notes indicated that students struggled to recall certain solution methods, and he had to provide additional support.

By the second observation, on graphing quadratics and generalizing transformations, Anthony anticipated that students would copy the x-values from $y = x^2$ (e.g., $x = -2, -1, 0, 1, 2$), and not focus on the vertex of other functions such as $y = (x - 4)^2$. This did occur with some students, though in his reflections, he noted that limited time prevented many from reaching generalizations about parameters.

In his third observation, comparing exponential and linear growth models, Anthony's expectations narrowed, with his monitoring charts predicting that students would "get stuck". His reflection notes, however, documented a wider range of strategies than he had anticipated: students mistaking the exponential function for linear, attempting to compute initial population values, and experimenting with equivalent exponential functions. He also critiqued one of his planned questions, "How can we get the function?" as "terrible," noting that he revised in the moment, instead asking about patterns in the data, drawing on questioning strategies discussed in the PATH seminar the previous week. This shift illustrates both his persistent challenge in anticipating strategies and his growth in adapting questions in-the-moment.

Interview Reflections

Anthony's first interview sheds light on his rationale for lecture-dominated instruction at the beginning of the semester. Drawing on his experience, he explained:

> Last semester, the students actually were okay just reading the [ALEKS] explanation and trying to solve whatever they could. But I noticed not all of them would do that. Some of them just played on their phones or did other assignments, and that's not ideal.

As a result, Anthony saw explanations from him during lecture as "safe" structure: "Even if no one pays attention, at least they are hearing the solution… it feels like the safe side." While he worried about being “too direct” at times, he valued the sense of security and coverage provided by lecturing.

Anthony’s second interview revealed both his rationale for maintaining lecture at the start of class and reflected concerns about pacing and control: “I don’t like them to start and not finish what they started… that’s one of the reasons I decided to start explaining before giving them the paper.” Yet, Anthony admitted that only “four or five” of the twelve students followed during lecture, while others “just ignore the lesson and try to work on something else.” To address this, he focused his questions on computational struggles, like squaring negatives. He thought his questioning was a mix of single answers and explanations. Anthony tied his instructional decisions to coordinator and student feedback because he believed that this approach improved students’ completion rates of the homework but constrained time for in-class exploration.

In his final interview, Anthony described a noticeable shift in his instructional approach. Having already lectured on prerequisite content in a prior class, he devoted nearly the entire session to the focus task on modeling exponential and linear population growth: “I wanted them to have the biggest chance to work on [the focus task] or figure it out.” Although he was still uncomfortable with this and felt students might view his circulating as “lazy,” but argued it was more demanding: “Maybe they think I’m lazy… but actually it’s a lot more difficult, because you need to follow the students.” He noted moments of unanticipated reasoning, such as when students connected exponential growth to the growth of world populations, or constructed different but equivalent exponential functions. He intentionally allowed them to pursue their own approaches before discussing alternatives: “Even if their ideas are really complicated, I like to let them approach the problem with the same idea they have in mind… after they are done, they will be happy because they were able to get it.”

Taken together, the COPUS data, monitoring charts, and interviews illustrate Anthony’s shift from lecture-dominated to student-centered instruction. Initially underestimating student needs and relying on his explanations, he gradually became more responsive, devoting more time to the focus tasks and adapting his questioning in-the-moment. At the same time, his reflections highlight ongoing tensions between content coverage and study inquiry, shaped not only by his own beliefs but also by coordinator and student feedback.

Discussion

This case study illustrates how Anthony’s teaching was shaped not only by his developing knowledge of teaching but also by contextual pressures that influenced what felt possible in practice. Anthony’s case demonstrates both growth and limits in his knowledge of discourse (Hauk et al., 2014). A central theme was the tension between Anthony’s prior beliefs and his enacted practice. Early in the semester, he defaulted to lecture as a “safe” structure, believing that at least students would hear the solution even if they were not fully engaged. Additionally, Anthony struggled with anticipatory thinking, as seen in his first monitoring chart, where he assumed that “no help was needed.” His enactment of discourse was also constrained by contextual factors. Coordinator comments about students disliking independent work and low attendance reinforced Anthony’s perception that lecture was safer than leaving time for group work. These findings illustrate how contextual pressures can narrow the enactment of pedagogical content knowledge, even when instructors hold student-centered beliefs.

In the middle of the semester, Anthony attempted to integrate questioning but continued to focus on computational details rather than open-ended reasoning. His reflections about asking “mid-level” questions suggest some development in implementation thinking as he considered the role of the questions he posed. While he recognized questioning as a tool to support engagement, orchestrating whole-class discussions remained difficult to sustain within pacing constraints. This highlights the ongoing challenge that ambitious pedagogy requires both time and tolerance for ambiguity, which Anthony felt constrained by (Stein et al., 2008).

By the final observation, Anthony's practice shifted more substantially toward student-centered instruction. Students spent most of the class working on the focus task while he circulated, adapting his questions in real time. This represented growth in implementation thinking, as he revised a planned question he later critiqued as "terrible" and instead drew students' attention to patterns in the data. His reflection that circulating was "more difficult" than lecturing reveals a developing awareness of the demands of facilitation. At the same time, his anticipatory thinking remained limited. Even in his third monitoring chart, he predicted only that students would "get stuck," failing to anticipate the diversity of strategies that later emerged. Thus, while Anthony demonstrated growth in adapting instruction in the moment, he still had room to grow in predicting and planning for a range of student strategies.

Taken together, Anthony's case demonstrates the interplay between knowledge of discourse and contextual pressures. While he developed greater responsiveness to student thinking across the semester, his ability to enact ambitious pedagogy was mediated by coordinator expectations, pacing demands, and concerns about student perceptions. This aligns with prior research showing that GTAs often struggle to align beliefs with practice when institutional forces pull in other directions (Deshler et al., 2015; Kung & Speer, 2009).

For mathematics departments, this case underscores the importance of supporting not only GTA pedagogical knowledge but also the contexts in which GTAs teach. Professional development should continue to emphasize frameworks such as the five practices (Smith & Stein, 2018) and knowledge of discourse (Hauk et al., 2014), while also explicitly addressing the institutional and cultural pressures that shape enactment. Coordinators should recognize how their feedback is interpreted and ensure that GTAs understand the broader pedagogical goals of the course. Without this alignment, GTAs may feel pressured to prioritize visible coverage over fostering mathematical discourse.

As a single case study, these findings are not generalizable to all GTAs or contexts. Anthony's reflections are shaped by his experiences, the particular coordinator he worked with, and the structure of his course. Nevertheless, the triangulation of COPUS data, monitoring charts, and interviews provides a rich account of the complex interplay between knowledge and context in GTA teaching. Future research could build on this case by examining how GTAs' practices evolve across multiple semesters of teaching, particularly on how they balance lecture with student-centered instruction. Additional studies might also investigate the pressures GTAs navigate, including departmental expectations, student feedback, pacing demands, and course structure, and how these interact with their development of anticipatory and implementation thinking. Such research could inform the design of PD and expectations that not only build knowledge of teaching but also alleviate the contextual constraints that can limit GTA growth.

Anthony's case underscores the complex reality of GTA teaching. While he entered the semester valuing student-centered learning, his practice was heavily mediated by contextual pressures. Only when schedule and content afforded greater flexibility did he more fully enact student-centered approaches. This finding highlights a broader lesson for GTA development: pedagogical knowledge cannot be understood in isolation from the institutional structures and feedback systems that shape practice. To support GTAs in enacting ambitious, student-centered teaching, PD must address both pedagogical tools and the contextual constraints that frame classroom decisions.

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