Student Experience With Collaborative Teamwork in Post-Secondary Discrete Mathematics

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As part of a project developing and researching the use of team-worthy lessons in discrete mathematics in two- and four-year colleges, we conducted a study of three instructors and their 105 students. Classroom observations indicated that the use of team-worthy lessons changed the structure of classroom activity from primarily didactic lecture to at least 75% student-centered activity (e.g., group work, student presentation). However, a change in observed structure is only evidence of an affordance, not of students' experience of it or how it might have been productive for students. After each team-worthy lesson, students completed surveys about what went well (and not). Analysis of student responses used a framework based on social learning theory. Here we present results about students' self-reported experiences in terms of agency, authority, creativity, and self-efficacy.

Keywords: discrete mathematics, professional development, team-worthy lessons

Background

For many students, discrete mathematics is a first exposure to formal logical reasoning, proofs, and proving. Undergraduates entering discrete mathematics have many experiences with procedural problem solving and often attribute the authority for sense-making to textbook, instructor, or software (Herbel-Eisenmann, 2007). Like calculus, discrete mathematics shares a history of enormous disparities in outcomes correlated to demographic variables (Campisi et al., 2024). Research in undergraduate mathematics education, notably in calculus, has indicated that disparities can be addressed by linked changes in student materials and instructional methods (Bressoud & Rasmussen, 2015). This literature has also indicated that group-worthy learning opportunities for students combined with use of socio-culturally informed instructional approaches can build positive relationships among students, between students and instructor (and institution), and foster engagement in and outside of class (Archie et al., 2022; Laursen et al., 2014; Leyva et al., 2022; Yoshinobu et al., 2023).

The research reported here comes out of a multi-year project leveraging lessons learned from calculus reform to develop and research use of team-worthy activities in discrete mathematics (Discrete Mathematics Project, 2026). The project's research aims are to (a) explore how use of team-worthy lessons to replace lecture perturbs/enhances student opportunities for engaging in the intellectual work of discrete mathematics and (b) refine professional learning materials and processes to help faculty incorporate student-collaborative methods into their teaching. This report focuses on a subset of the data collected in the larger project to explore the research question: *What evidence is there from classroom observation and student survey that the use of team-worthy lessons creates opportunities for students to develop self-efficacy, authority, agency, and creativity, as components of their learning experience?*

Theoretical Perspective

Our approach was anchored in social cognitive theory and its attention to self-efficacy, agency, and authority (Bandura, 1997). The conceptual framing also drew on educational equity (Gutiérrez, 2009) and mathematical creativity (Regier & Savic, 2018). The framework we used included five categories (Carmody, 2024):

- **self-efficacy** (self-perception of one's own abilities for a given task),
- **authority** (perception about a source of information that has power for validation, verification, or legitimation of a student's or a group's work in mathematics; can be ascribed to a live human in the room or to others (e.g., in a textbook))
- **agency** (self–perception that is both personal and an aspect of interplay between teacher and student regarding who the agent of action is in doing mathematics),
- **mathematical creativity** (components in the process of intellectual work in mathematics),
- **equity** (in the narrow context of the meeting of student needs for access to ideas and identity development in the classroom, more on this below).

Agency involves both a willingness to act and a readiness to act (Bandura, 1997). Creativity, the use of imagination or inventiveness, influences what a learner may act on (and how they will take action). Agency is a co-requisite for experimentation related to how and what to act on (Regier & Savic, 2018). Beyond creativity and the ideas in social cognitive theory (self-efficacy, authority, agency), are the self-conceptions that make up mathematical identity and contribute to who students see themselves to be as learners and doers of mathematics. The framework offered by Gutiérrez (2009) involves interactions among mathematical identity, power, access, and achievement as aspects of educational equity. Gutiérrez noted, "equity is ultimately about the distribution of power, power in the classroom, power in future schooling, power in one's everyday life, and power in a global society" (p. 5). Note that we do not attempt to define equity here. Rather, we give background for what shaped the narrow use of equity concepts in the study in our attempt to document student experience.

When analyzing what made successful schools successful, Gutiérrez (2012, p. 26) found "a key feature of this successful context lay in teachers placing student needs, not just mathematics, at the center of their work." Whose needs are met and who decides they have been met depend on context. Some needs are established by and for an institution by majority society expectations (as defined in the curriculum and requirements for graduation or certification) and other needs are established by students themselves (e.g., needs behind why they chose to gain an education, seek certification, take a particular course). Needs being met for students is, therefore, part of a personal experience, established by the student's own background and supported by the institution in its practices, environment, and standards — including what happens at the intermediate level of the classroom. Thus, the analysis for this report centers on the key feature of the fulfillment of a student's asserted educational needs. Unlike Gutiérrez, who considered schools in the context of society, the focus here is students in the context of the classroom. Equity is far more than meeting a student's needs; it includes how this fulfillment is achieved. This report is not an attempt to address the broader issues of educational equity but to exemplify a means of documenting evidence for the particular aspect of equity related to students' perceptions of their needs being met and the openings offered to students to build personal agency, authority, self-efficacy, and mathematical creativity in the classroom setting.

Methods

Setting and Participants

The data for this study were collected from multiple visits to discrete mathematics classes at two- and four-year colleges. Conducted with appropriate institutional review board approval, data gathering and analysis included three instructors (pseudonymously: Antilles, Peter, and Miles) and the 105 students enrolled in their classes. During the study semester, the three instructors themselves engaged in a short-course for professional learning about the student-centered methods for which the lessons were designed. The short-course focused particularly on noticing student thinking and strategies for orchestrating classroom discussion. Instructors had access to both a user guide giving an overview of how to teach with the activity-based lessons and detailed instructor roadmaps for each lesson. Instructional resources included:

- basic principles of team-worthy tasks and role-taking by students in team groups (Facilitator, Reporter, Documenter, Questioner)
- guidance on how to form, structure, and manage student teams
- lesson materials (e.g., student handouts) and recommendations for lesson preparation
- suggested questions to pose during instruction
- commentary on common student difficulties and strategies for addressing them
- written-out sample solutions for each task, including optional assessment items.

All three campuses were designated as Hispanic serving. Attendance in the on-campus classes varied, with an average of 80% of enrolled students present for observed classes. We visited at least two different team-worthy lesson uses for each of the instructors, each lasting 60 to 75 minutes (4 for Miles, 3 for Antilles, and 2 for Peter) in addition to at least 150 minutes of typical classroom teaching for each instructor (i.e., not using team-worthy lessons). Instructors interspersed team-based lessons with typical lecture across the semester, at least 1 lesson per month. Students completed a short survey after each team-worthy lesson (average $\leq$ 10 minutes). The descriptive statistics presented here were the result of quantitative analysis of observation protocol details across all observed classes, summary statistics from selected survey responses, and relative frequencies from qualitative coding of open-ended responses.

Observation

The observation method used for documenting classroom activity was based on the *Toolkit for Assessing Mathematics Instruction - Observation Protocol* (TAMI-OP; Weston et al., 2023). The TAMI-OP tool divides a classroom experience into two-minute intervals where checkmarks for a variety of actions by instructors and by students are recorded in a spreadsheet. Once documented, the actions can be grouped into four profiles based on the predominant actions that were checked: interactive lecture, group work, student presentation, or didactic lecture. An "interactive lecture" environment is teacher-centered yet includes questions and answers being exchanged between the teacher and one or more students in a whole class setting whereas the "didactic lecture" environment is teacher-centered and has little or no expectation of feedback from students to teacher or vice versa. The other two profiles are made up, respectively, of actions involved in "group work" (e.g., students work together in small groups, the instructor moves through the room guiding group activity) and "student presentations" (e.g., a student presents on their own or as a reporter for a group).

Survey

Two open-ended items on the survey asked students to:

O1. Share at least one aspect of your experience today with math teamwork that NEEDS IMPROVEMENT

O2. Share at least one aspect of your experience today with math teamwork that was POSITIVE.

Four Likert scale items on the survey asked students to rate aspects of their class session experience (see Table 1). The first two prompts are proxy measures for student sense of belonging and efficacy for engaging in the face of the challenge of working with others. The pacing questions are a proxy for identifying access to intellectual work experienced by students.

Table 1. Likert-like Rating Items on the Survey

Question	Scale
L1. Overall, how would you rate your experience of class today?	1=Satisfying to 5=Not Satisfying
L2. Overall, how would you rate your experiences today in working with others?	1=Fantastic to 5=Terrible
L3. How would you rate the pace of today's activities?	1=Too Slow to 5=Too Fast
L4. Thinking about all class meetings, including today, how would you rate the pace of the class so far?	1=Too Slow to 5=Too Fast

Qualitative Coding

Inductive analysis of students' responses began with the framework. Through three rounds of coding (open, thematic, categorical), the definitions of each subarea were refined. Distinguishing evidence of authority and of agency in student reporting was challenging. A student's reported actions might arise from being an agent in mathematics or might be more connected to their own sense of authority (e.g., validating or verifying their own legitimacy). A further complication, in a group setting, is that students participating in discussion may show agency or authority together (e.g., as a group collectively) and individually. In fact, the authority of the group may yield to any one person. Thus, for the purposes of this report, authority was considered a personal exercise. Consequently, in coding student's responses we distinguished between agency (acting on one's own work) and authority (verifying the legitimacy of one's or others' work).

Results

The first two rating questions were about overall experience and experience working with others. These were proxy measures for student sense of belonging and for engaging in working with others. For both items, the vast majority of responses were in the top two positive categories, with 98% in the top three (i.e., fewer than 2% of student responses in negative categories, see Figures 1 and 2).

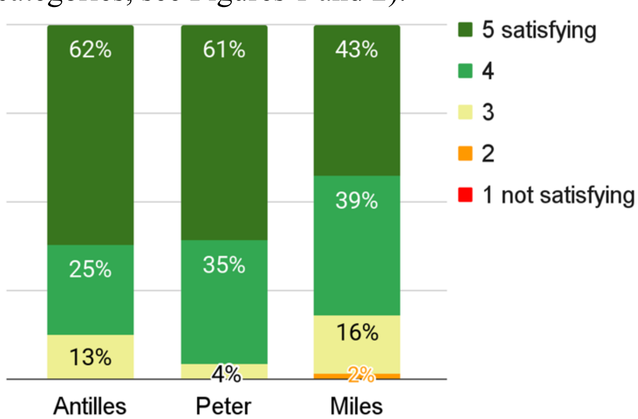

Figure 1. Student rating of overall experience today.

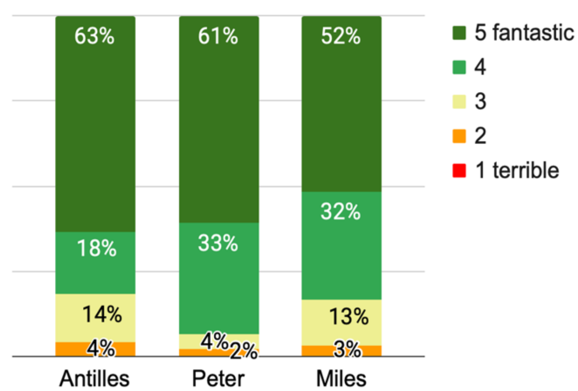

Figure 2. Student rating of experience working with others on mathematics today.

The questions about pace of the day's team-worthy lesson activities and for the class so far, were a proxy for identifying the access to intellectual work experienced by students. If pacing is about right (in the middle), that suggests students felt the work was accessible. For both items, most responses were that pacing was reasonable (not too fast, not too slow, see Figures 3 and 4).

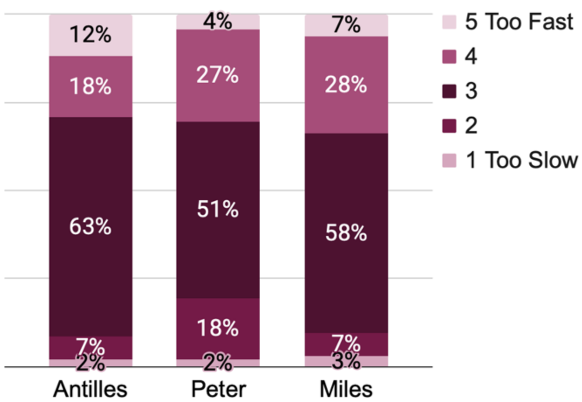

Figure 3. Student rating of pace of activity.

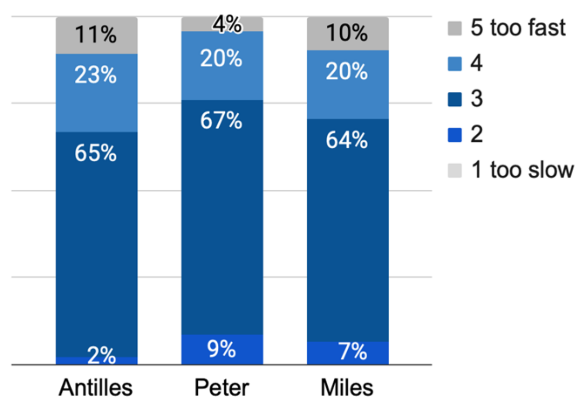

Figure 4. Student rating of pace of class so far.

Analysis of observation data indicated that the use of team-worthy lessons changed the structure of classroom activity from primarily didactic lecture (see Figure 5) to at least 75% student-centered activity (e.g., group work, student presentation; see Figure 6). Figures 5 and 6 suggest that the professional learning opportunities offered as support for instructor use of the materials was sufficient for a change in classroom practice when lessons were substituted for typical practice. However, a change in observed structure is only evidence of an **affordance**, not of students' experience (i.e., if and how the much larger proportion of group work and student presentation time was productive for students' engagement or learning). The examination and coding of written self-reports from the student surveys shed light on what went well (and not).

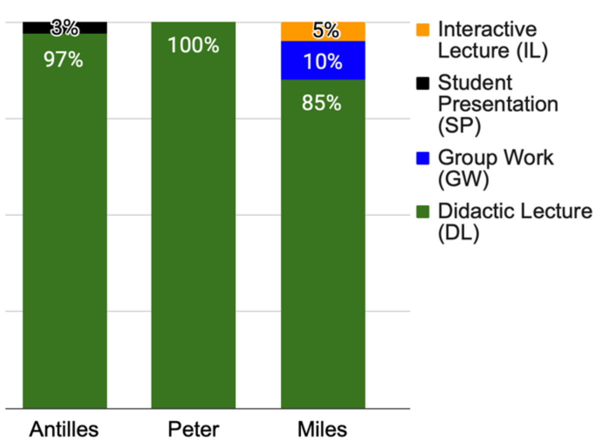

Figure 5. Distribution of instructional formats during typical classes.

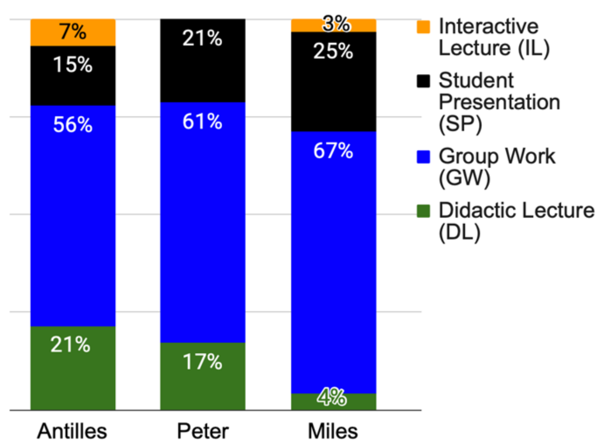

Figure 6. Distribution of instructional formats during team-worthy lessons.

The distribution of coded categories in responses to the question about what was positive differed across instructors (Figure 7, next page). For this report, only one primary code was assigned to each student's response, thus the columns sum to 100%. The code of *creativity* was the primary code in fewer than 6% of responses for any instructor. Similarly, the equity-related category of *no unmet need* was explicit in only a few responses in each class. Students commented on a positive experience of *agency* by remarking on their pleasure with exploring mathematical ideas and engaging with teammates. A primary code of *authority* arose when a student reported satisfaction with their own or their group's competence or legitimacy in mathematical reasoning. In about 10% of responses in each class, students reported a positive experience of calibration of *self-efficacy*, in understanding the mathematics in the lesson. Finally, the most notable positive experience for many students was that they *worked together* in coming to a deeper understanding of the mathematics at hand (more on this new category, not in the original framework, below).

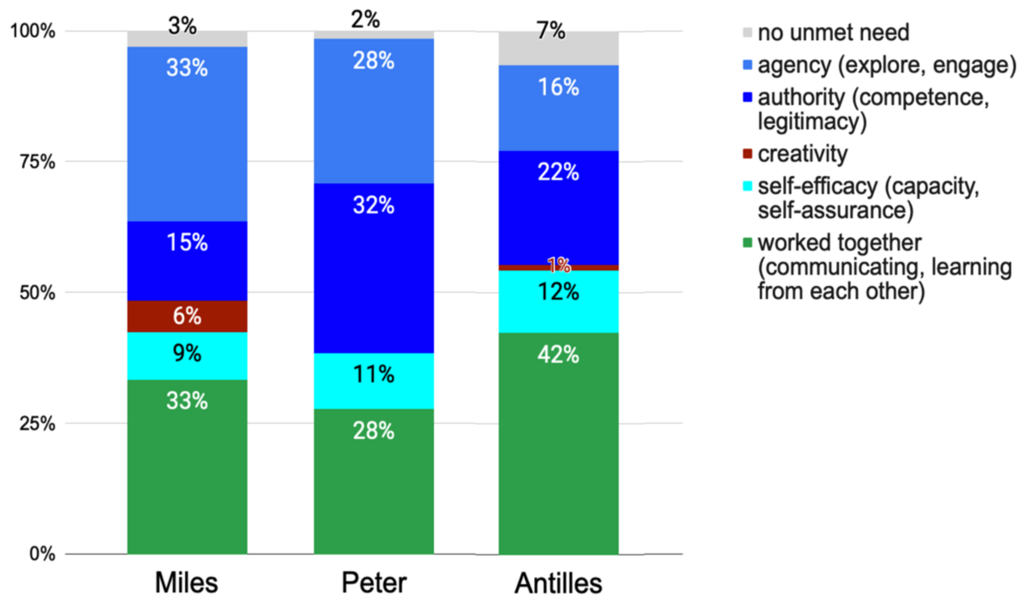

Figure 7. Distribution of percentage of coded student response across all activities, by instructor for "Share at least one aspect about your experience today with math teamwork that was POSITIVE."

Comments about what needed to improve to meet students' felt needs also varied by instructor (Figure 8, next page). As with the positive responses, each student comment got one primary code. Students sought better *access to tools* (e.g., whiteboard pens) or *directions* (e.g., details from the instructor about what was expected for success). Students also reported a feeling of not being clear on if/how they might have *choice* or make *decisions* related to questioning others within their group or on other teams. For each instructor, nearly 20% of students had concerns about how to behave to engage in *collaboration* so that all in the team were involved in interpretation, problem-solving, and/or communication of ideas. The phrase *no suggestions* indicates students commented that all was well or left the answer blank.

Noteworthy in Figure 8 is the smaller proportions of Miles' students who indicated a need for improvement in *choice/decision* or *access to tools/directions*. Miles regularly referred to group roles and reinforced students enacting those roles when he visited groups. The other two instructors used the roles mostly for the launch (facilitator come get supplies) and wrap-up (reporter share group's conclusions). Miles encouraged students to attend to their roles and be purposeful in participating in the team roles (e.g., during a visit to a group Miles sometimes suggested a Questioner visit other groups to ask about ideas).

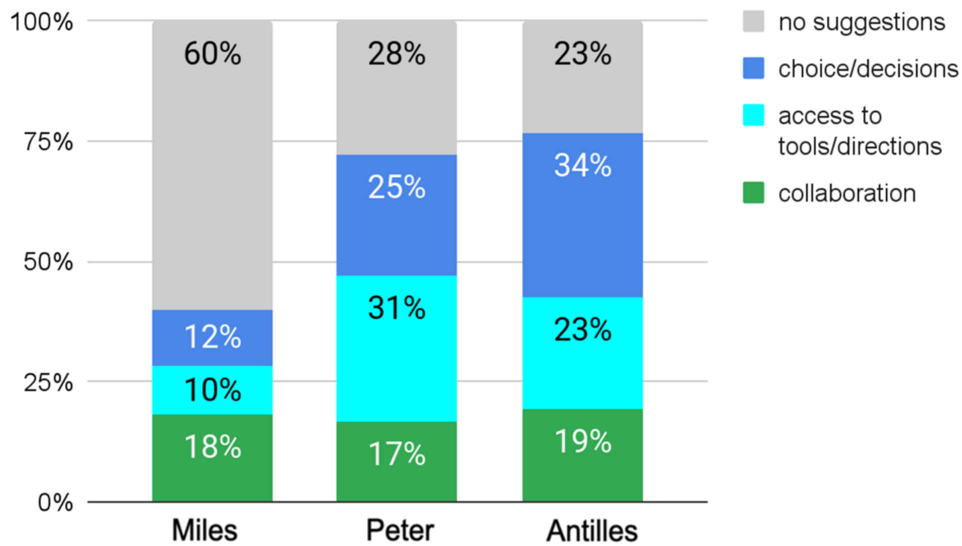

Figure 8. Distribution of codes for student post-lesson survey comments about what needed to improve.

Students generally enjoyed the teamwork and felt it helped them grow as users and doers of mathematics. At the same time, student survey responses suggested that more scaffolds for instructors to implement and orchestrate structures within and across teams for working together well would make student collaboration more effective.

Discussion

In addressing the research question, the findings do indicate there is evidence from classroom observation and student survey that the use of team-worthy lessons creates opportunities for students to develop and demonstrate self-efficacy, authority, agency, and creativity, as positive aspects of their learning experiences. For the "needs improvement" question, most students in Miles' class indicated no suggestions while Antilles' and Peter's students highlighted needs for clearer instructions, improved participation, and better collaboration. For the "positive" question, collaboration emerged most frequently of all positive characteristics noted by students, with at least 28% for each instructor. In future work we plan to add this category to the analysis framework. The distributions in Figures 7 and 8 reflect students' reports of their needs, both met and unmet, for creating and sustaining team-based learning.

Though the extent to which those experiences were equitable was not the focus of the student survey, there seems to be evidence that student needs for engaging in the intellectual work of discrete mathematics were being met in many ways. The majority of students indicated collaboration as an aspect they deemed positive. Yet, collaboration was also the area they felt needed the most improvement! A clear next step is revising the instructor short-course to share with instructors the results of this study and to build up supports for instructor learning in the areas students indicated need improvement.

A next research area is: What (re)balancing of instructor skills is needed to realize the potential of the (re)balancing of activity structures afforded by the lessons? Future work can examine how instructional skills, combined with the affordances of lesson content, contribute to the development of agency, self-efficacy, authority and additional aspects of equity (e.g., belonging) in mathematical sense-making among discrete mathematics students. Research on impact in terms of student mathematical knowledge growth is part of future work.

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