From the Polar to the Pole: The Enactive and Embodied Choreomathematics of the Polar Coordinate System

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The polar coordinate system (PCS) is typically introduced in Pre-calculus, where it provides a foundation for undergraduate coursework in mathematics, physics, engineering, and other STEM fields. Regrettably, students' engagement with it is often limited to procedural tasks, such as plotting points, rote memorization of the relationship between the shapes of graphs and the parameters of their polar equations, and converting these equations between Cartesian and polar forms. And too little attention is paid to how radius and angle measure co-vary in the graphs of polar equations (e.g., Moore et al., 2014; Paoletti et al., 2013). This project seeks to remedy the anemia of this conventional approach by discerning the features of an enactive educational ecosystem for emergent learning that enables learners to graph and reason within the PCS.

The enactive theory of cognition (Maturana & Varela, 1987; Thompson, 2007) and the theory of inclusive materialism (de Freitas & Sinclair, 2014) frame our investigation of mathematical thinking as a complex and emergent phenomenon constituted in embodied and material engagement, and in the affect and aesthetics of relational interactions between learners and their environments. Owing to its attention to the essence of in-the-moment, lived experience, these theories take inspiration from the philosophy of phenomenology (Husserl, 1977; Merleau-Ponty, 1962). Prior research has demonstrated the viability of such an approach, which enabled young learners to make the sophisticated distinctions between qualitative and quantitative change (Greenstein et al., 2024a, 2024b).

At the onset of design-based research (Brown, 1992; Collins, 1992; DBRC, 2003) into what might be the preconceptual (Varela, 1992), enacted, and embodied groundings for understanding the PCS, we found ourselves in need of an actual "pole" as well as the use of gestures and other bodily movements to enact the roles of radius and angle measure through movement. This experimental and exploratory experience led to the decision that pole dance could situate our approach to emergent learning, and that we would seek to move in new ways (Abrahamson & Sánchez-García, 2016) that might enable us to predict the shapes of polar curves. As soon as we moved forward, though, an obstacle we encountered involved the reliance of the PCS on references to trigonometric functions in the Cartesian coordinate system (CCS; e.g., substituting $x = rcos\theta$ and $y = rsin\theta$ to move from Cartesian to polar). And as we recursively moved between the two worlds of the PCS and CCS in order to coordinate simultaneous changes in the covarying quantities, we got the sense-ation that these parametrized experiences (Dimmel et al., 2025) might illuminate the roles of radii and angle measures in dynamic traces of polar curves. This proved to be a surprisingly joyful aesthetic experience, the essence of which we hope to illuminate through phenomenological inquiry (e.g., van Manen, 2023). Our next step is to determine what pole-dance choreographies and other practices make this interworldly movement productive. We will share these and other findings, which lend credence to our emergent learning approach to teaching the polar coordinate system.

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