Students' Perturbations and Accommodations in the Context of Graphing

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Covariational reasoning can support students' thinking across mathematical domains including functions, graphing, calculus, and modeling. Researchers have developed several frameworks for classifying students' covariational reasoning. For example, Thompson and Carlson's (2017) levels of variational reasoning apply broadly across mathematical contexts and attend to the nature of the students' images of variation and the construction of a multiplicative object in students' reasoning. While researchers have documented the ways students' reason about and construct graphs of covarying quantities, it remains an open question how covariational reasoning develops in students' thinking and whether the levels articulated by Thompson and Carlson are developmental.

Our work seeks to provide empirical evidence for the extent to which Thompson and Carlson's levels of covariational reasoning might be developmental, and to understand the mechanisms and instructional decisions that might contribute to the development of students' covariational reasoning. To address these goals, we conducted one-on-one task-based clinical interviews (Hunting, 1997) with college Calculus I students to understand the extent to which task sequencing of non-normative graphing tasks might facilitate the development of students' covariational reasoning.

In this poster, we focus on one student, Meg, to explore how her engagement with a sequence of non-normative graphing tasks contributed to her demonstration of more propitious levels of covariational reasoning. Our analysis is grounded in Piaget's conceptualization of schemes where students' schemes to develop through processes of assimilation and accommodation (Piaget, 1985). We examine how Meg assimilated tasks into her existing graphing schemes, we document the perturbations Meg experienced when assimilating these tasks, and classify the nature of the accommodations she made to her graphing scheme to resolve her perturbations. Through this process, Meg's reasoning evolved from gross images of variation to chunky images of covariation in four distinct phases. We take this as preliminary evidence that there might be a developmental progression between the levels of covariational reasoning.

Additionally, our analysis revealed that the interplay between Meg's activity constructing graphs and then reasoning about her constructed graph resulted in Meg experiencing cognitive perturbations that ultimately led her to make accommodations to her graphing scheme. This finding builds on our previous work that students reason differently when constructing graphs and interpreting graphs and that students' graph interpretations more often align to their images of covarying quantities than their graph construction (Frank, 2018). Thus, as Meg interpreted her constructed graphs in ways that did not align with her images of covarying quantities, she experienced perturbations that challenged her graphing scheme.

This work provides empirical support that levels of covariational reasoning might be developmental and if a student has robust images of covarying quantities, then their work constructing and reasoning about their constructed graphs can serve as a productive mechanism for the development of their graphing scheme.

References

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