Mathematical Mnemonics as Tools for Shaping Concept Images in University Mathematics Tutoring

Keith Gallagher
University of Nebraska at Omaha

Deborah Moore-Russo
University of Oklahoma

Nicole Infante
University of Nebraska at Omaha

In mathematics, the teaching and use of "tricks," known formally as mathematical mnemonics, is commonplace. However, little research has focused on this topic, particularly in university mathematics tutoring. In this study, we investigated when and why undergraduate mathematics tutors implemented mathematical mnemonics in their tutoring practice. Video recordings of online tutoring sessions were collected from eight tutors working at an R1 university in the United States. Of the eight, five were observed using mathematical mnemonics in their tutoring practice. Our results show that these tutors used mathematical mnemonics for three reasons: in response to a student committing an error or demonstrating a misconception, as a response that represents how the tutor reasons about the topic, and as part of a predeveloped curriculum script. We argue that each of these reasons is an implicit attempt by the tutor to reshape a student's concept image of the topic under consideration.

Keywords: Tutoring, mathematical mnemonic, concept image

The teaching and use of "tricks" in mathematics education is commonplace. These tricks are formally known as *mathematical mnemonics*. Miller and Obara (2017) defined a *mathematical mnemonic* (referred to herein simply as a *mnemonic*) as "a visual cue or verbal strategy used to aid initial memorisation and recall of a mathematical concept or procedure" (p. 13). While some mnemonics help with recalling definitions and concepts, others help with the application and interpretation of procedures.

The National Council of Teachers of Mathematics documented that, in many mathematics classrooms, "... too much focus is on learning procedures without any connection to meaning, understanding, or the applications that require these procedures" (National Council of Teachers of Mathematics, 2014, p. 3). Indeed, although many mnemonics can help students correctly answer assessment questions, they can obscure the conceptual underpinnings that make them work, reinforcing the common perspective that mathematics is simply a set of rules to be memorized rather than a set of concepts to be understood (Kalder, 2012; Miller & Obara, 2017). While the use of mnemonics can improve student understanding (Greene, 1999; Mastropieri, Scruggs, & Levin, 1987), over-emphasis on such tools can lead to decreased conceptual fluency (Karp, Bush, & Dougherty, 2014; Kilpatrick, Swafford, & Findell, 2001).

Despite this potential pitfall, mnemonics have been shown to produce positive outcomes for conceptual, as well as procedural, learning when they are accompanied or followed by conceptual instruction (NCTM, 2014). Although mnemonics often allow students to succeed in math classes despite lacking the conceptual understanding that makes them work, they can also act as scaffolds for students until they are developmentally prepared to adopt the conceptual understanding (Karp, Buch, & Dougherty, 2014; Manalo, Bunnell, & Stillman, 2000; Miller & Obara, 2017).

In this exploratory study, we observed online university mathematics tutors sharing a variety of mnemonics with the students they were tutoring. The research team's curiosity about the pedagogical value of these mnemonics led to posing the following research question: What are university mathematics tutors' implicit goals when sharing mnemonics with students during tutoring sessions?

The Importance of Tutoring and the Influence of Tutors

Tutoring brings with it myriad benefits, including improved course grades (Bloom, 1984; Rickard & Mills, 2018), increased mathematical confidence (Lepper & Chabay, 1988), and improved sense of mathematical identity (Bjorkman & Nickerson, 2019), among others. Indeed, Merrill et al. (1995) observed that, while doing problems is the best way to learn, it is essential that this effort is guided such that learners receive positive feedback when they are moving in a productive direction and corrective feedback when making mistakes.

Citing Vygotsky (1978), Mascolo (2009) argued for the importance of the learner's interaction with a more knowledgeable other and that such interactions "pull the less expert partner's functioning to levels that are higher than that which he is capable of sustaining alone" (Mascolo, 2009, p. 5). Although experienced tutors take very deliberate actions during tutoring sessions to provide individualized instruction for their students (Gallagher et al., 2025; Lepper & Woolverton, 2002), tutors also sometimes proceed through a largely preordained set of interactions known as a *curriculum script* (Putnam, 1987). "A curriculum script is an ordered set of goals and actions for teaching a particular topic – the skills and concepts students are expected to learn and activities and strategies for teaching this material. ... [I]t is the teacher's curriculum script, rather than a model of the student based on detailed diagnosis, that determines the sequence of instruction" (Putnam, 1987, p. 39). Tutors, as well as teachers, can use curriculum scripts where they speak and act in a particular sequence that can fail to take the learner's needs into account.

Concept Image

Tall and Vinner (1981) define an individual's *concept image* for a concept as "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes" (p. 152). Although one's concept image for a concept can be rather expansive, only certain parts of an individual's concept image may be evoked when solving problems. Problematically, however, an individual's concept image need not be accurate or even internally consistent, and the portion of one's concept image that is evoked while solving a particular problem may not be helpful for obtaining a solution. These challenges may lead to struggle or even failure during problem solving.

Individualized tutoring often reveals problematic inaccuracies in students' concept images. Fortunately, concept images are malleable and can be updated with further learning (Tall & Vinner, 1981; Vinner, 1983). Our data suggest that the university tutors in our sample used mnemonics to reshape their students' concept images to bring them more in line with their own, which were seemingly more productive.

Theoretical Framing

For this work, we adopted Merrill et al.'s (1995) conceptualization of *tutoring as guided learning by doing*. A significant body of research demonstrates the benefits of active learning (Abell et al., 2018; Bressoud et al., 2015; Deslauriers et al., 2019; Freeman et al., 2014; Prince, 2004; Theobald et al., 2020), in which students engage in idea generation, problem solving, and practice. While this work appropriately places the onus of knowledge creation on students, much of it neglects the importance of having a more knowledgeable other (Vygotsky, 1978) present during this process. Merrill et al. (1995) noted that, although students' active participation in problem solving is the key to learning, leaving students to practice without the appropriate supervision and support can lead to floundering, confusion, and frustration, which hinders learning. Mascolo (2009) echoed this point, emphasizing the importance of the teacher's role in an active learning environment: "... the idea that students must actively construct their skills and understandings *for themselves* is not the same as suggesting that [they] must actively construct their skills and understandings *by themselves*" (p. 7). While Mascolo's work focused on classroom instruction, Merrill and colleagues' work is focused on tutoring. Still, both agree that the more knowledgeable other guiding the lesson is a critical component of effective instruction.

Tutoring as guided learning by doing (Merrill et al., 1995) emphasizes the importance of the individualized instruction involved in tutoring. Similar to Mascolo's (2009) views on the importance of the teacher, it has been argued that effective tutoring involves the tutor taking an active role in their student's learning. Merrill and colleagues (1995, p. 316) argued that "rather than letting students solve problems on their own, with occasional advice, tutors carefully monitor the problem solving to ensure that it stays on track and to help direct students back toward a productive solution path when needed." Merrill and colleagues referred to this as "guided learning" and pointed out that this practice allows "students to attain the benefits of learning by doing while avoiding some of the costs" (p. 316).

Methods

The current analysis is part of a larger ongoing project studying different aspects of online mathematics tutoring at the university level. The data set consisted of online tutoring sessions focused on topics ranging from College Algebra through Multivariable Calculus that were recorded at an R1 university tutoring center from January 2022 through May 2024. There were eight online tutors who worked 5-10 hours per week. As the research team members were cataloging the mathematical content of these sessions, they made notes of other interactions they found interesting. The data analysis reported here occurred over five months between the research team of co-authors on this paper.

While initially going through the videotaped tutoring interactions, the research team observed several instances where five of the eight tutors shared "tips and tricks" for being successful on a variety of problem types during their tutoring sessions. These tips and tricks met Miller and Obara's (2017) definition of *mnemonics* provided earlier. The research team decided to focus on these instances of mnemonic use for this study.

One researcher reviewed all videos during the timeframe noted above and identified when mnemonics were being used by the tutor. A second researcher then reviewed the videos that the first researcher categorized; there was 100% agreement on the identification of the mnemonics. The whole research team discussed each mnemonic, decided on categorizations, and came to agreement on the attributes of each category. The third researcher then classified each video into the mutually agreed upon three classifications discussed below in results; note that the categories as not mutually exclusive. Thus, a video could have been identified as falling into more than one classification. Finally, the research team reviewed each classification assigned and discussed until consensus was reached.

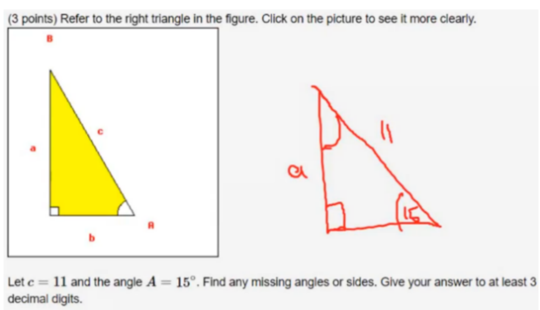
Results

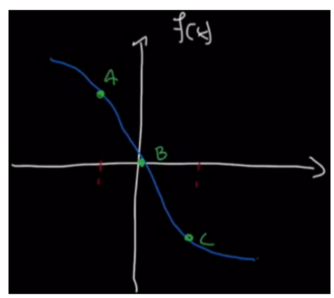
We identified reasons why tutors use mnemonics when tutoring. First, almost all mnemonics were used in response to a student error or misconception. Second, a tutor introduced a mnemonic because it seemed to be how they thought about the topic under consideration. Third, tutors used mnemonics “automatically,” even after students had self-corrected and figured out a correct path forward. We now present examples of each of these situations.

Mnemonics as Response to Student Errors and Misconceptions

Mnemonics were almost always used after the tutor saw an error or a misconception. Working with a trigonometry student to find the length of side *a* of a triangle in which the opposite angle and the hypotenuse were known (Figure 1a), Aaron used the mnemonic *SOH-CAH-TOA*. Aaron drew and labeled a picture with the known information and asked the student which trigonometric function should be used to find *a*. When the student suggested cosine, Aaron responded, “I would probably use sine, because remember *SOH-CAH-TOA*. So, sine of 15 degrees would be equal to the opposite side, *a*, divided by the hypotenuse, which is 11.”

Another tutor, Bernardo, used a mnemonic for thinking about increasing and decreasing to help a student sketch the graph of a derivative from the graph of a function that was decreasing over an interval from point *A* to point *C* but its concavity changed from concave down to concave up at an inflection point *B* (Figure 1b). The student explained that, starting from the left, the graph of the derivative should be negative and would decrease until point *B* was reached, conflating the notions of *increasing function* and *increasing derivative*, explaining that since the derivative is increasing from point *B* through point *C* and beyond, the graph of the derivative should be drawn in the first quadrant over this interval. The tutor used an analogy of walking on a hill, explaining that in this case, even after passing through point *B*, he would still be “walking downhill” even though “the hill was becoming less steep.”





Figures 1a and 1b. Situations prompting mnemonics

Mnemonics as Tutors’ Genuine Reasoning

Tutors brought up mnemonics because that is how they, themselves, think about certain problems. More than once, Selena described a mnemonic for determining concavity by drawing eyes on the graph to make happy and sad faces (Figure 2). Selena worked with two students to distinguish between intervals on which a function is increasing and intervals on which the function is concave up. Selena explained, “I’m not gonna lie, I do this really dorky thing whenever I do this – I kinda draw sad and happy faces. And if they’re sad, they’re concave down. If they’re happy, they’re concave up.” Selena’s students found this technique comical, but they immediately began correctly answering questions about concavity by implementing it.

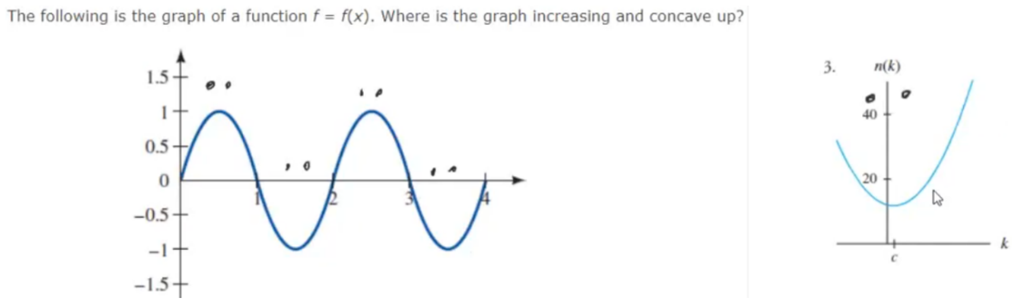


Figure 2. Selena's drawings of eyes to determine concavity of graphs.

Marian seemed to use the mnemonic *rise-over-run* as a command to apply the slope formula. On an algebra review assignment for a calculus class, the student Marian was tutoring had been asked to write equations for two lines (one passing through the origin, and the other with a nonzero *y*-intercept). The student was confused because he had applied the method of dividing a single point's *y*-coordinate by its *x*-coordinate, resulting in the correct slope for the line passing through the origin but the incorrect slope for the other. Marian explained that "You can just find the rise-over-run of that... you just take those two points, and you find their rise-over-run." As she said this, Marian wrote down the algebraic slope formula, that is $(y_2\text{-}y_1)/(x_2\text{-}x_1)$, and then applied it to the line for which the student had found the correct slope, then used this as a model to repeat the process for the other line. She continued to describe this process as "finding the rise-over-run" throughout the entire tutoring session.

Mnemonics as Curriculum Scripts

Tutors introduced mnemonics even after students detected their own errors and self-corrected, suggesting the mnemonics are part of curriculum scripts the tutors have developed for certain concepts. When asked about a question in which the student needed to determine the sine of an angle whose terminal side lay in the third quadrant, Marian drew and labeled a picture to represent the angle's terminal side and the right triangle it forms with the *x*-axis (Figure 3).

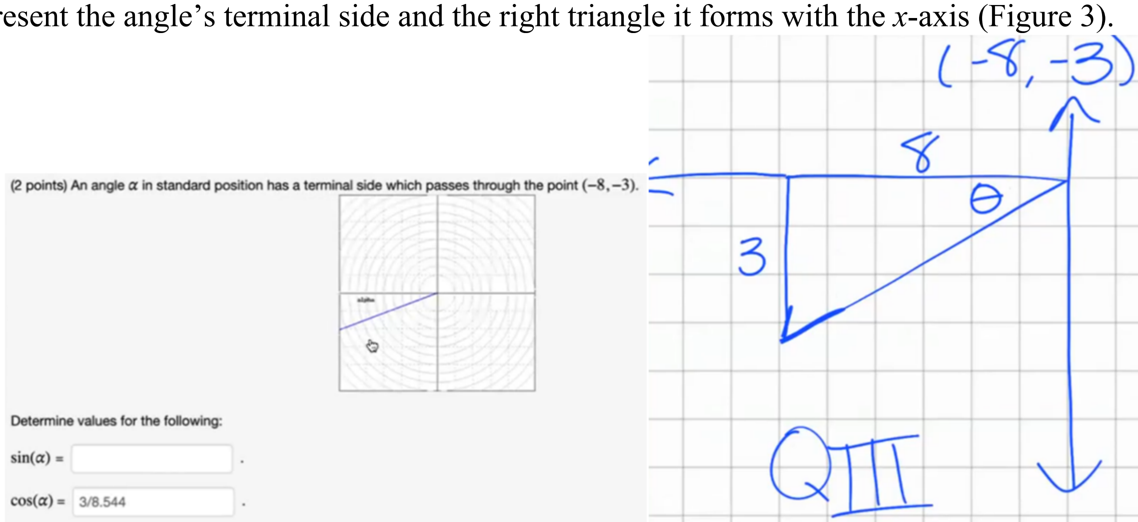


Figure 3. Marian's drawing of an angle with its terminal side in QIII.

As Marian was drawing, her student acknowledged aloud that his mistake was a sign error; he had neglected the fact that the vertical side length should be negative because the terminal side lay in the third quadrant. Marian continued to explain the problem even though the student's question had been resolved and described the mnemonic *All Students Take Calculus* that she used to identify which trigonometric functions are positive in each quadrant (**A**ll are positive in I, **S**ine is positive in II, **T**angent is positive in III, and **C**osine is positive in IV; see Figure 4).

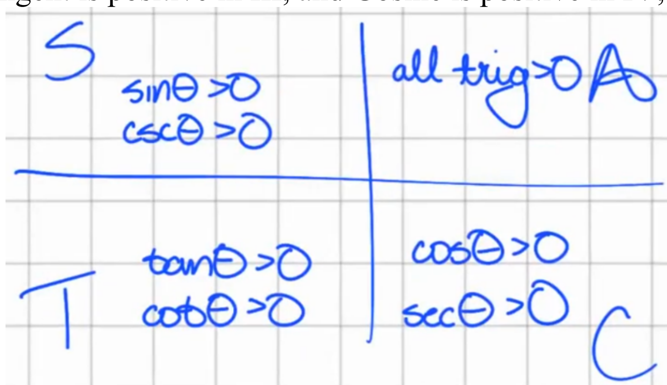


Figure 4. Marian's diagram explaining the All Students Take Calculus mnemonic.

In a session with Carey, a student mistakenly suggested that $a^2 - 4$ should be factored into $(a - 2)(a - 2)$ but quickly corrected himself to $(a - 2)(a + 2)$ after realizing that his initial factoring would yield a positive 4 in the constant term. Although the student had self-corrected, Carey was intent on ensuring the student understood factoring by explaining the "x-method" (Figure 5).

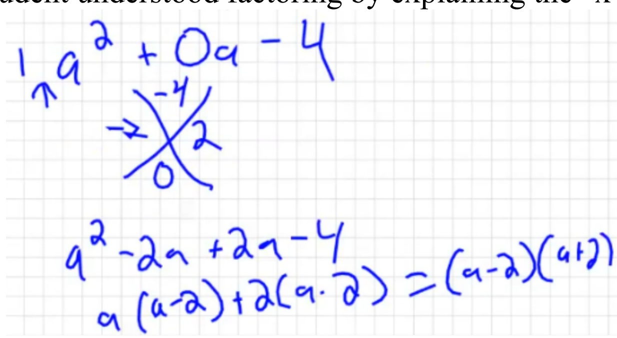


Figure 5. Carey's explanation of the "x-method" used to factor a difference of squares.

Discussion

We found that tutors in our sample used mnemonics for three reasons: in response to the detection of a student error or misconception, because it represented how they think about a concept or problem, and as part of predeveloped curriculum scripts they use for tutoring certain topics. Underlying all of these reasons is an attempt to reshape their student's concept image of the topic under consideration to bring it more in line with their own.

Merrill and colleagues (1995) argued that one of the major benefits of working with a tutor is the tutor's ability to curtail students' unproductive activities. When the students in our data demonstrated a misunderstanding, tutors' uses of mnemonics "direct[ed] students back toward a productive solution path" by providing students with a useful way of conceptualizing the topic, gently suggesting that the student should reconsider their current approach and update their way of thinking to be more in line with how the tutor thinks. Furthermore, as more knowledgeable others (Vygotsky, 1978), tutors modeled modes of thinking that they knew to be reliable and encouraged their students to adopt them, even when students identified and corrected their own mistakes.

Considering the use of a mnemonic as part of a curriculum script, we echo Mascolo's (2009) suggestion that, although students must build their own knowledge, they need not do so on their own. Curriculum scripts provide a structured framework in which students work on specific kinds of activities designed to help them construct specific knowledge. Although it may not have been necessary for our tutors to describe mnemonics in these situations, as their students had already detected their own errors and self-corrected, they chose to share them to reduce the likelihood of such student mistakes recurring in the future by updating or reinforcing concept images already held by their students.

Implications and Directions for Future Research

This research expands the current body of research related to the use of mathematical mnemonics, specifically by addressing tutors' motivations for teaching mnemonics to the students they assist. Although the students in the sessions we observed seemed to appreciate and derive benefit from the mnemonics tutors showed them, it is essential that the research community ascertains exactly what benefits and drawbacks students might experience from learning and applying such mnemonics. It has been noted that the use of mathematical mnemonics (and, indeed, much of modern mathematics teaching) places a heavy focus on getting the right answers to problems, often at the expense of conceptual understanding (Kalder, 2012; Miller & Obara, 2017; NCTM, 2014). However, it has been suggested that mathematical mnemonics can lead to improved conceptual understanding, either by using them as a scaffold building toward conceptual understanding or by teaching mnemonics and concepts side-by-side (Karp, Buch, & Dougherty, 2014; Manalo, Bunnell, & Stillman, 2000; Miller & Obara, 2017; NCTM, 2014). In our review of the data presented here, we hypothesized that the use of certain mnemonics, such as "rise-over-run" or the use of happy and sad faces to determine concavity, might foster conceptual understanding, while other mnemonics, such as "ILATE" (an abbreviation of *inverse trigonometric*, *logarithmic*, *algebraic*, *trigonometric*, and *exponential* used by Marian but not discussed in this paper) for integration by parts, might obscure or at least not develop concepts. Future studies of mathematical mnemonics need to explore the effects of individual mnemonics on conceptual and procedural fluency among mathematics students at various levels of their mathematical education.

Acknowledgments

This work was supported by NSF IUSE award DUE-2201747. All findings and opinions are those of the research team and do not necessarily reflect the NSF's position.

References

Abell, M., Braddy, L., Ensley, D., Ludwig, L., & Soto-Johnson, H. (2018). MAA instructional practices guide https://www.maa.org/programs-and-communities/curriculum%20resources/instructional-practices-guide

Bjorkman, K., & Nickerson, S. (2019). This is us: An analysis of the second groups within a mathematics learning center. In A. Weinberg, D. Moore-Russo, H. Soto, & M. Wawro (Eds.) *Proceedings of the 22nd annual Conference on Research in Undergraduate Mathematics Education*.

Bloom, B. S. (1984). The 2 sigma problem: The search for methods of group instruction as effective as one-to-one tutoring. *Educational Researcher*, 13, 4-16.

Bressoud, D. M., Mesa, V., & Rasmussen, C. (Eds.). (2015). *Insights and Recommendations From the MAA National Study of College Calculus*. MAA.

Deslauriers, L., McCarty, L. S., Miller, K., Callaghan, K., & Kestin, G. (2019). Measuring actual learning versus feeling of learning in response to being actively engaged in the classroom.

Proceedings of the National Academy of Sciences, 116(39), 19251-19257. https://doi.org/10.1073/pnas.1821936116
Gallagher, K., Infante, N. E., & Moore-Russo, D. (2025). Three cases representing a variety of online, post-secondary mathematics tutoring interactions. In Howard, J. & Beyers, J. (Eds.), *Teaching and Learning Mathematics Online* (pp. 282-302). Chapman and Hall/CRC.
Greene, G. (1999). Mnemonic multiplication fact instruction for students with learning disabilities. *Learning Disability Research & Practice*, 14(3), 141-148.
Kalder, R. S. (2012). Are we contributing to our students' mistakes? *Mathematics Teacher*, 106(2), 90-91.
Karp, K. S., Bush, S. B., & Dougherty, B. J. (2014). 13 rules that expire. *Teaching Children Mathematics*, 21(1), 18.
Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding It Up: Helping Children Learn Mathematics*. National Academies Press.
Lepper, M. R., & Chabay, R. W. (1988). Socializing the intelligent tutor: Bringing empathy to computer tutors. In H. Mandl & A. Lesgold (Eds.), *Learning Issues for Intelligent Tutoring Systems* (pp. 242-257). Springer-Verlag.
Lepper, M. R., & Woolverton, M. (2002). The wisdom of practice: Lessons learned from the study of highly effective tutors. In *Improving Academic Achievement* (pp. 135-158). Academic Press.
Manalo, E., Bunnell, J. K., & Stillman, J. A. (2000). The use of process mnemonics in teaching students with mathematics learning disabilities. *Learning Disability Quarterly*, 23(2), 137-156.
Mascolo, M. F. (2009). Beyond student-centered and teacher-centered pedagogy: Teaching and learning as guided participation. Pedagogy and the Human Sciences, 1(1), 3-27.
Mastropieri, M. A., Scruggs, T. E., & Levin, J. R. (1987). *Effective Instruction for Special Education*. College-Hill Press/Little, Brown & Co.
Merrill, D. C., Brian, J. R., Merrill, S. K., & Landes, S. (1995). Tutoring: Guided learning by doing. *Cognition and Instruction*, 13(3), 315-372.
Miller, G., & Obara, S. (2017). Finding meaning in mathematical mnemonics. *The Australian Mathematics Teacher*, 73(3), 13-18.
National Council of Teachers of Mathematics (2014). *Principles to Actions: Ensuring Mathematical Success For All*. National Council of Teachers of Mathematics.
Prince, M. (2004). Does Active Learning Work? A Review of the Research. *Journal of Engineering Education*, 93(3), 223-231.
Putnam, R. T. (1987). Structuring and adjusting content for students: A study of live and simulated tutoring of addition. *American Educational Research Journal*, 24(1), 13-48.
Rickard, B., & Mills, M. (2018). The effect of attending tutoring on course grade in Calculus I. *International Journal of Mathematical Education in Science and Technology*, 49(3), 341-354.
Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12(2), 151-169.
Theobald, E. J., Hill, M. J., Tran, E., Agrawal, S., Arroyo, E. N., Behling, S., Chambwe, N., Cintrón, D. L., Cooper, J. D., Dunster, G., Grummer, J. A., Hennessey, K., Hsiao, J., Iranon, N., Jones, n. L., Jordt, H., Keller, M., Lacey, M. E., Littlefield, C. E., ..., Freeman, S. (2020). Active learning narrows achievement gaps for underrepresented students in undergraduate

science, technology, engineering, and math. *Proceedings of the National Academy of Sciences - PNAS*, 117(12), 6476-6483. https://doi.org/10.1073/pnas.1916903117

Vinner, S. (1983). Concept definition, concept image and the notion of function. *International Journal of Mathematical Education in Science and Technology*, 14(3), 293-305.

Vygotsky, L. (1978). *Mind in Society: The Development of Higher Psychological Processes*. Harvard University Press.