Exploring Covariational Reasoning Pathways in Undergraduate Mathematical Modeling

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Modeling plays a critical role in helping students make sense of and analyze complex biological systems, which is important for those pursuing life science or health professions (AAMC, 2009). Participants in this study previously completed a life science modeling course designed to improve their ability to use modeling as a tool for understanding the dynamics of biological systems. This work investigates students' covariational reasoning (CR) when engaging in tasks requiring movement across multiple representations, such as developing a trajectory from a time series and, conversely, generating a time series from a trajectory. A time series showcases how a single variable changes as a function of time, while a trajectory illustrates how two variables change in a space formed by the two variables that describe the system (Bennoun, 2024). Covariational reasoning refers to an understanding of how two quantities change with respect to one another (Carlson et al., 2002) and is foundational to understanding the concept of function, reasoning with multiple quantities, and modeling dynamic functional events (Thompson & Carlson, 2017), all critical skills for success in mathematics.

This work builds on Bennoun's (2024) work on examining the extent to which undergraduate students who completed a modeling course have learned how to sketch the trajectory associated with a time series and vice versa. Extending this line of work, we focus on how students reason through these representational shifts and how their covariational reasoning manifests across tasks. Our research question is: *How do students engage in covariational reasoning when working with time series and trajectory representations of mathematical models in biology?*

To investigate students' CR, we conducted 10 task-based think-aloud interviews (Zazkis & Hazzan, 1999) with students who had taken a mathematical modeling course at a research-intensive university in the southwestern United States. For the interviews, students were asked to sketch a time series associated with a given trajectory and compare their sketch to alternative solutions. To analyze the interview data, we deductively coded the data using Thompson and Carlson's (2017) six levels of covariational reasoning as a priori codes (Miles & Huberman, 1994), followed by reflexive thematic analysis (Braun & Clarke, 2006) to identify overarching themes that emerged from the data.

Our preliminary findings highlight CR's dynamic and flexible nature, as students shifted between reasoning levels across tasks, leveraging different levels depending on the demands of each task. We present three empirical cases that illustrate different CR pathways and discuss how classroom norms and teacher expectations may shape students' conceptions of what counts as "correct." For example, all students' responses for sketching task were categorized into the smooth, chunky, or coordination levels. When presented with solution comparison problems, they either remained at the same level across tasks or shifted to another level, sometimes moving from smooth to chunky, or from coordination to smooth, depending on the task. We then compare the three paths, noting that students often referenced classroom expectations, emphasizing the interplay between individual understandings and classroom norms.

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