Exploring Aspects of Student Facility With Taylor Series in the Context of the Electric Dipole

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Using a grounded theory approach, we investigate students' facility using Taylor series expansions in the context of the electric dipole. Eight students participated in one-on-one teaching-learning interviews, during which audio and video data were recorded while they completed a written activity. We show that while physics students at this level are very familiar with the procedural aspects of Taylor series expansions, they may not recognize several common cues to use them to simplify functions. We find that sketching out the exact function and an approximation of it can help students learn to associate these common cues with Taylor series. In addition, we probe students' conceptions of the meaning of the expansion point and find that one difficulty in choosing an expansion point in a physics context is associating that 'point' with a spatial location. We conclude with recommendations for physics and calculus instructors.

Keywords: Taylor Series, physics, applications

Introduction

In their studies, physics students must often approximate complicated functions that describe physical systems. While crude approximations—such as setting small parameters to zero—may be sufficient in early coursework, they lose pedagogical value as students advance. A more versatile and effective method involves using a small ratio of parameters to construct a Taylor series expansion, which serves as an approximation with adjustable accuracy. This use case of Taylor series, which Jones et al. (2019) refer to as an "extension of a linear approximation," is an indispensable mathematical technique in a physicist's toolkit and plays a pivotal role in connecting mathematical rigor with physical intuition.

Usually students first encounter Taylor series towards the end of their introductory calculus sequence, although they use results of Taylor series in physics courses earlier. One early use is the 'small angle approximation', that $sin\,\theta \approx \theta$ for $\theta \approx 0$. However, applications like these are often treated as facts to be memorized, justified only with plausibility arguments. In calculus courses, they are expected to remember the general formula for the Taylor series expansion of a function *f(x)* expanded about some expansion point *a* $f(x) = f(a) + \frac{\partial f}{\partial x}\Big|_a (x-a) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}\Big|_a (x-a)^2 + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!}\frac{\partial^n f}{\partial x^n}\Big|_a (x-a)^n$ and be able to apply it to complicated functions that are already in terms of the expansion variable. However, in physics explicit prompting for Taylor series is rare: students are usually expected to pick up on contextual cues, such as one quantity being much smaller than another, or that an approximate expression is required (Wilcox et al., 2013). Physics courses expect students to understand when a Taylor series expansion is needed, to create their own small and dimensionless expansion variable, and only then to perform the expansion (Loverude & Li, 2013).

A common context for Taylor series expansions is to approximate the electric field at a distance *L* from an electric dipole, which consists of a pair of charged particles of equal magnitude and opposite sign, separated in space by a distance *s* (see Fig. 2). An expert would treat the variable *L* as a parameter that takes on values either much smaller or much larger than *s*, and would use the *parameter ratio* $\frac{s}{L}$ as the expansion variable for the Taylor series in order to understand the field's behavior at relative extremes. The expansion point is the approximate value of the parameter ratio, and the zeroth-order term *f(a)* is often physically meaningful. Students may struggle with this even if they have mastered the computational techniques in a calculus course, as these contexts require sense-making with physical quantities.

In this paper we investigate problem-solving strategies of sophomore-level students when applying Taylor series to the electric dipole. We aim to identify specific student strengths and conceptual barriers—specifically those related to cue recognition and interpretation of the meaning of the expansion point—to better inform instructional scaffolding. Following a review of the literature on Taylor series in physics education, we outline our study design, present and discuss our findings, and conclude with instructional implications for both physics and calculus courses.

Our research questions are as follows:

1. Research Question 1 – Cuing with the Parameter Ratio: Do students in this population recognize that the request to find an approximate expression when the charge separation is much smaller than the observation distance—indicating a very small parameter ratio—serves as a cue to apply a Taylor series expansion, without explicit prompting? If not, what prompts allow students to come to the conclusion themselves that these phrases are cues to do so?
2. Research Question 2 – Conceptions of the Expansion Point: What are students' conceptions of the meaning of *a* (the expansion point) in a physics context?

Background

Knowing when and how to apply Taylor series in physics is often difficult, even for students who have attained procedural proficiency in their use from calculus courses (Smith et al., 2013; Wilcox et al., 2013; Loverude and Li, 2013). Some of this may be due to the increased number of steps required to use a Taylor series expansion in physics as compared to in calculus courses. One such step is determining the expansion point: Wilcox et al. (2013) and Smith et al. (2013) showed that students have difficulty when the expansion point is non-zero. Another step is to recognize the cues to use a Taylor series that experts recognize. Loverude (2025) observed that while students readily employ basic approximation methods when prompted, they may not yet intuitively select more sophisticated techniques that experts use to extract physical insights.

Common cues for students to use a Taylor expansion are outlined in the Wilcox et al. study. These cues are: explicitly asking for a Taylor series expansion, asking for an approximate expression for a complicated function, and the use of language or symbols implying that one parameter is much smaller than another. Their students were successful in picking up the cues, but their population was also from a course in which these cues would be taught. We sought to evaluate whether the second and third cues are sufficient for students—who had not had explicit instruction identifying these as indicators—to infer the appropriateness of a Taylor series expansion when given an intellectual need for such an expansion (Harel, 2008). We predicted, given the documented difficulties students have in transferring knowledge between courses (Loverude, 2017), that students would struggle to come to the conclusion that a Taylor series expansion is necessary. We wanted to explore their reasoning at this stage.

Methods

The student population sampled in this study consists of physics majors and minors at a large-enrollment R1 institution in the Pacific Northwest that graduates about 100 students annually. Approximately 15% of students self-identify as members of underrepresented minority groups and about 27% identify as female. Math Methods I is a sophomore-level course, usually taught with minimal physics context and from the standard textbook (Boas, 2005). The course is typically taken after one year of introductory calculus and one year of introductory calculus-based physics. Similar to patterns across the U.S., the introductory physics sequence followed by most participants did not require students to solve problems using calculus (Loverude, 2025).

Eight students, all enrolled in Math Methods I, volunteered for this study in exchange for $200. All participants identified as female and used the self-selected pseudonyms Abby, Chad, Grace, Jake, Jenna, Priscilla, Summer, and Susan. While this sample was not representative of the course demographics, time constraints prevented further recruitment. Two participants, Jake and Summer, were non-traditional students who had transferred from community colleges; Jake had completed the same course in the prior quarter.

Participants engaged in one-on-one teaching-learning interviews (Nguyen & Rebello, 2011), completing a Taylor series-based activity with intermittent input from an interviewer. Previous studies on the use of Taylor series expansions in physics have asked students to complete the entire problem at once, with written responses analyzed afterwards (Loverude, 2025; Loverude & Li, 2013; Wilcox et al., 2013). Other studies have conducted teaching interviews in which students were explicitly prompted to use Taylor series (Smith et al., 2013). Teaching-learning interviews allow for deeper probing of student reasoning than analysis of written responses alone. Interviews were conducted in quiet rooms, lasted 1–2 hours, and were screen-recorded on an iPad. Audio was transcribed using AI software and manually corrected. Grace's audio was not recorded, so her data were only used when video and notes were sufficient.

Data processing followed the procedures outlined by Otero and Harlow (2009). After reviewing interview notes and recordings, the lead researcher identified sections of interest, usually where students struggled most. Content logs were written for about one-third of the activity questions; these logs helped us focus on three questions from the activity, reproduced at the end of this section. Student written data were integrated into the corrected transcripts.

Figure 1. Schematic of the data analysis process. We used grounded theory to generate codes which were grouped by theme (such as 'Expansion Point'), and report on patterns that emerged from within those themes (Attride-Stirling, 2001; Glaser & Strauss, 1967; O'Callaghan et al., 2024).

Our data were analyzed using thematic analysis informed by grounded theory (TAG) (Attride-Stirling, 2001; Creswell & Creswell, 2018; Glaser & Strauss, 1967; O'Callaghan et al., 2024). We used a grounded theory approach in generating a codebook from the raw data (Fig. 1, left). A primary focus of the analysis was on students' initial responses to the activity questions; codes for these were initially developed by the lead researcher during a review of the content logs and later validated through application. A second focus was on students' responses to interviewer prompts. Both the prompts and the corresponding responses were coded inductively. Both coding efforts contributed to the development of the final codebook. The lead researcher and two undergraduate research students applied the completed coding scheme on one interview before independently applying it to other interviews and reaching above 85% agreement.

We subsequently used our codebook to identify themes, which formed the basis of a thematic analysis (Fig. 1, center). Individual codes were grouped into broader themes (e.g., Expansion Point), and we describe several reasoning patterns that emerged as clusters of codes within each theme (Attride-Stirling, 2001; O'Callaghan et al., 2024). This analysis yielded the reasoning patterns which inform the discussion throughout this paper.

The activity used was adapted from the Tutorials in Mathematical Methods in Physics by Loverude, Thompson, and Christensen (Loverude et al., n.d.) to provide an intellectual need for a Taylor series expansion (Harel, 2008). The goal of the activity was to model the electric field produced by an electric dipole at an observation point P far away from the center of the dipole; that is, $L \gg s$ in the diagram.

Figure 2. The figure given on the activity. The two circles ('+Q' and '-Q') are the equal magnitude, oppositely signed charges that constitute the dipole. L is distance from the dipole's midpoint to the observation point P and s is the distance between charges in the dipole. Students were to work in the regime where L $\gg$ s.

In their study, Loverude and Li (2013) report that students have difficulty finding an appropriate expansion variable and getting their equations into a productive form for expanding in a Taylor series. Since we were unsure to what extent students would be cued to use a Taylor series—if at all—they were guided to write their exact expression in terms of the parameter ratio $u = \frac{s}{L}$ before expanding it as a Taylor series.

The relevant questions in the activity are listed below. Note that the 'expression in question 6' is the exact equation for the electric field as a function of u: $E = \frac{kQ}{L^2}\left[\frac{1}{\left(1-\frac{u}{2}\right)^2} - \frac{1}{\left(1+\frac{u}{2}\right)^2}\right]$.

10. Let's say instead that we want to know what the electric field is when u is very small, but not exactly zero. What could you do to your expression in question 6 to approximate the electric field for small values of u? (Answer: Expand it as a Taylor series and truncate.)
13. One of the most important Taylor series expansions to learn is the expansion for $(1 + x)^P$. Derive the Taylor series to three terms for this expression.
14. Look again at your expression in question 6. If we were to use the expansion that you just derived, what should p be equal to? What should x be equal to? *Asked verbally: What should a be equal to?* (Answers: p=-2, $x = \pm\frac{u}{2}$, and a=0 since x is small.)

Findings and Discussion

RQ 1: Cuing with the Parameter Ratio

Findings. Our first research question is whether students would recognize that a very small parameter ratio and a request for an approximate expression are cues to use a Taylor series expansion, and if not, how they come to recognize this. Prior to Question 10, students were asked to describe the physical and mathematical results of setting the parameter s to zero and the parameter L to infinity separately, and subsequently about the results of setting the quantity $u = \frac{s}{L}$ to zero.

To answer Question 10, Chad, Jake, Susan, and Summer initially suggested that (in Susan's words) they "do the limit as u approaches zero". Abby, previously informed by the interviewer that setting $u=0$ was not correct, settled on "a limit approaching one". Summer and Jenna suggested 'dropping' u, which appeared to mean setting it to zero rather than excluding the symbol. Priscilla only algebraically rearranged the symbols without altering the expression. After clarification that taking the limit as u approaches zero would yield the same result as setting $u=0$—which they had already done—Jake and Susan proposed plugging in a number close to zero (such as 0.001). No others had alternate ideas before discussing with the interviewer.

The interviewer then asked each student a variant of: "In introductory physics, you used the small angle approximation $sin\, x \approx x$. Why is that valid?" Only Jake immediately related this to the Taylor series of sine. Four (Abby, Grace, Summer, and Susan) were asked to sketch graphs of $y = sin\, x$ and $y=x$ overlaid and three of them eventually concluded that the approximation was due to a Taylor series. Among those who did not graph, none came to this conclusion. These results are shown in Table 1.

Table 1. Student reasoning in Question 10. Three of the four students who concluded that a Taylor series expansion was necessary drew the functions $y = sin\, x$ and $y=x$ on the same axis. Three of the four who did not conclude such also did not draw any graphs.

	Concluded Taylor Series	
	Yes	No
Drawing	Abby, Grace, Susan	Summer
No Drawing	Jake	Chad, Jenna, Priscilla

Discussion. That five out of seven of our students initially suggested using a limit is not surprising, as prior questions likely primed them to think about limits. However, all of their initial suggestions would result in exact values for the electric field at various values of u, rather than in approximations valid over a range of values of u. These data agree with Loverude and Li (2013), who noted that students often interpreted $L \gg s$ as a reason to set $s=0$. Combined with the fact that their alternatives also give exact values rather than approximate expressions, this suggests that students at this level may struggle to conceptualize approximations of functions.

The graphing task proved useful in helping students connect mathematical tools to contextual use, though the interviewer's prompts also mattered. Before coming to the correct conclusion Abby and Susan were asked how to increase the domain of validity of the approximation, while Summer was asked how to improve its accuracy and did not conclude that the approximation was the result of a Taylor series. This suggests that prompting students to broaden the approximation's domain may be more effective, possibly because it avoids assumptions about the series' exactness; according to Martin et al. (2011), students commonly view truncated Taylor series expansions as being exact throughout the domain of validity.

These results show that the cues from Wilcox et al. (2013) may be valid if they are taught, but students who have not learned Taylor series in a physics context may not recognize them. While visualizations help, additional prompting about the nature of the graph may be necessary. This illustrates how effortful the transition from mathematical abstraction to physical application is for learners while also supporting the idea that students have emerging graphical-mathematical blended reasoning, which is present in experts (Zimmerman et al., 2023).

RQ 2: Conceptions of the Expansion Point

Findings. Before investigating students' conceptions of the expansion point, we first assessed their procedural facility with Taylor series. In Question 13 we asked students to find the Taylor series expansion of $(1+x)^p$ without giving them the general formula. Six of the seven immediately started taking derivatives to get the coefficients, while Jake attempted a summation expression. All of them ended up with the correct formula with minor errors. Most errors involved the expansion point a: three students dropped all powers of x-a, three omitted $-a$ from the powers of x (two—Jenna and Susan—did so intentionally, stating that $a=0$), and one had to be prompted to evaluate the derivatives at a.

In Question 14, students matched the result of their expansion to the exact expression of the electric field from earlier. Most incorrectly chose $p=2$ rather than the correct $p=-2$, which is consistent with Loverude and Li (2013). While most correctly or nearly correctly identified x, only Jake immediately concluded that $a=0$ without interviewer guidance.

When asked for the meaning of a, Jake paraphrased the interviewer's earlier explanation of a and concluded that $a=0$ since the expansion variable was close to zero. Priscilla offered that a is "something really close to the value of whatever x is" but needed prompting to identify its value. Chad also produced an acceptable definition of a without prior explanation by the interviewer but initially believed that 'point' referred to the spatial location point P, as did Abby. When told by the interviewer that $a=0$, Abby explained that zero makes sense because s and L are measured from the origin, another spatial location. Summer reported that a is "where we are basing the expansion", but then was unsure what it would represent, guessing the variable u, Coulomb's constant k, "the electric field", and $\frac{1}{2}$.

Table 2. Students' initial responses to Question 14. A question mark indicates that the student did not give a definite response without aid from the interviewer. Jenna and Susan had already chosen a=0 in Question 13, though with incorrect/no justification.

	p	x	a
Abby	2	±u/2	Point P
Chad	2	±u/2	Point P
Jake	2	±u/2	0
Priscilla	2	±u/2	?
Jenna	2	u	0*
Summer	2	Q or k/L^2	?
Susan	?	u	0*

Discussion. Students generally showed comfort deriving the Taylor series expansion for $(1+x)^p$. Errors in Question 13 mostly involved omitting or misplacing the expansion point a, suggesting that students are accustomed to dealing with expansions where $a=0$ (Martin, 2009; Habre, 2009) and have limited experience with making sense of a in physics contexts.

When applying the Taylor expansion to the physics context, identifying p and x became a straightforward extension of students' mathematics knowledge. Arriving at the conclusion that $a=0$ was more difficult. Though many students were able to offer workable definitions of a, this did not always translate into finding its value. Priscilla and Summer may also have resisted concluding that $a=0$ from a mental distinction between Taylor and Maclaurin series.

We did not expect to observe conflation of the expansion point with a spatial location. This issue likely arises from the diagrams used in physics problems: they are often spatial diagrams of the problem set-up and not graphs of the expression to be approximated, as are found in calculus.

While students demonstrated a good grasp of the procedural aspects of Taylor expansions, their difficulties identifying and interpreting *a* reveal that applying Taylor series in physics requires scaffolding extending beyond the procedural proficiency developed in calculus courses.

Conclusion

This study adds to evidence that while sophomore physics students are often procedurally fluent with Taylor series, they may not recognize when and how to apply them in physics contexts. Our findings focus on how students interpret common cues for approximation (Wilcox et al., 2013) and how they adapt the formula using physical parameters.

Consistent with Loverude (2025) and Loverude and Li (2013), no students identified the need for a Taylor expansion without prompting, often preferring to 'do the limit' instead. We found that graphing both exact and approximate expressions in the case of the small angle approximation helped students recognize that "find an approximate expression" is a cue to Taylor expand. We further conjecture that, for students who are not immediately helped by graphing, prompting them to consider how to extend the domain of validity of the approximation yields better results than prompting them to focus on increasing its accuracy within a fixed domain. We hypothesize that this is because the former avoids the conceptual challenge of reconciling the inexactness of a truncated Taylor series.

Choosing the expansion point was also challenging for students. Both Smith et al. (2013) and Wilcox et al. (2013) report on student difficulties with non-zero expansion points, but in neither study were students' conceptions of the meaning of the expansion point investigated. We show that even students who could self-generate reasonable definitions for *a* struggled to determine that *a=0* in this context. Some interpreted the expansion point as a spatial location, a confusion which is likely more prevalent in physics and engineering contexts than in mathematics because diagrams in these disciplines often depict spatial configurations rather than graphs of the function, so students cannot visually locate the expansion point.

Our evidence indicates that students need support in identifying when a Taylor series is an appropriate tool in physics contexts. To help them develop this sense, we recommend that math instructors use physical examples that are within students' zone of proximal development (Vygotsky, 1978) to illustrate the practical uses of Taylor series. For instance, the potential energy between two objects is $U = -\frac{GMm}{r}$, where *M* and *m* are the masses and *r* is the distance between the objects. The potential energy between the Earth (radius *R*) and an object at a height *h* above Earth's surface is $U = -GMm(R + h)^{-1}$. Using $x = \frac{h}{R}$ as the (very small) parameter ratio and expanding about *x=0*, this yields $U = -\frac{GMm}{R} + mgh$; the first-order term is the familiar expression for potential energy above the surface of the Earth. Examples like these give physical meaning to the parameter ratio and the expansion point while illustrating common cues to Taylor expand, like a small parameter ratio.

Lastly, we recommend that physics instructors be aware that even algorithmically proficient students may not understand or remember the mathematical properties of the tool they are using if those properties don't help them to perform the algorithm. In addition, math courses make definitional distinctions between similar concepts (like Maclaurin series, Taylor series, and Taylor polynomials) which students may retain in contexts where physics instructors are not.

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