Epistemological Frameworks of Advanced Math Majors: A Case Study


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*Although students' epistemological frameworks have been extensively studied in introductory undergraduate courses and lecture-based advanced mathematics courses, there has been less work on students' epistemological frameworks in advanced student-centered courses. The purpose of this case study is to examine the beliefs held about mathematics learning of nine students in a flipped real analysis course using Schommer et al.'s (1992) mathematical epistemological framework as a theoretical perspective. The findings suggest that although there were minimal differences in beliefs when grouping students by major or final course grade; overall, the typical participant's epistemological framework was grounded in effort, a desire for understanding, and belief that there must be at least some reliance on their instructor and peers to provide instruction and support their learning in the course.*



*Keywords:* epistemological framework, flipped classroom, real analysis


An epistemological framework is the set of assumptions a learner holds that taken as a whole define how they believe knowledge is understood. Students' epistemological frameworks influence how they perceive the quality of a class, their motivation, and even what content they focus on learning in a course (Ji et al., 2021). Although students' epistemological frameworks have been extensively studied in introductory classes (e.g. Krupnik et al., 2018), there has been less work on students' epistemological frameworks in advanced student-centered courses. If we better understood what advanced mathematics students believed about mathematics and mathematics learning, we could help them adopt more productive beliefs and more effectively tailor our pedagogies in advanced student-centered mathematics courses. Prior literature on lecture and student beliefs suggests that although mathematics students vary widely in their beliefs of what understanding is and how to be a successful mathematics student, students agree lecture is key to learning, which makes student-centered courses with little lecture challenging to conduct (Zolt et al., 2023). Although lecture is a common instructional method, formal understanding of what is written vs informal understanding of what is said vary widely from class to class (Umgelter & Geisler, 2023). Students tend to struggle to record or recall the informal content of lectures, as it is rarely written down (Fukawa-Connelly et al., 2017).

Student beliefs about mathematics and mathematics learning affect their motivations, behaviors, and academic success (Limeri et al., 2023). However, student beliefs about learning have been primarily studied in early undergraduate mathematics students (Ji et al., 2021), even though students' interpretation of instruction is filtered through their epistemological framework and beliefs about learning, even in advanced coursework (Krupnik et al., 2017). Further, there have been calls for more research on beliefs and affect in advanced mathematics students (Melhuish et al., 2022). The purpose of this study was to investigate the epistemological frameworks of real analysis students in a flipped real analysis course, and was guided by the research question: what is the typical epistemological framework of an undergraduate enrolled in an advanced student-centered mathematics course?

## Theoretical Perspective

For this qualitative study, we chose Schommer et al. (1992) as the theoretical perspective for students' epistemological frameworks. There are ten implicit assumptions in Schommer et al. (1992); although these assumptions were developed for students reading mathematics textbooks, we chose this theoretical perspective because it encapsulated all of the beliefs that have emerged as themes in prior epistemological framework literature (e.g. Limeri et al., 2023). These assumptions are: (1) learning is supposed to be quick (2) you can't learn how to learn (3) concentrated effort is a waste of time (4) success is unrelated to effort (5) ambiguity is something to be avoided (6) single answers are preferred (7) external sources authority are the preferred way to check answers (8) the ability to learn is innate (9) integration of concepts makes learning harder (10) learning typically occurs in the first attempt. We based the interview questions on these assumptions, reverse coding half of the questions.

## Methods

Participants were recruited from a flipped real analysis class with extensive daily activities and class discussions at a rural research university in the southern United States. Nine students consented to a 45-minute semi-structured interview about their epistemological frameworks and perceptions of the course. All participants were mathematics majors or second majors; a second math major is larger than a math minor but contains fewer required courses and credits than a full major. Eight participants were male or male-presenting non-binary. Seven participants self-identified as white and two self-identified as Hispanic. At the time of the interview, four participants were earning A's, two were earning B's, and three were earning a C or below. Real Analysis requires a prior course in proofs as a prerequisite, so it was at least the second proof-based mathematics class for all participants, though it was the first student-centered proof-based class for most participants.

After the interviews were transcribed, the authors coded each question for evidence of one of the ten beliefs in the theoretical perspective for each individual student. The initial coding was then discussed until all differences were reconciled. Although we tried comparing participants based on achievement level in the course and whether their primary major was math or physics, there were no clear differences in belief patterns when grouping students using these variables and we did not have enough female participants to compare by gender. Instead, student beliefs were classified as universal (all nine participants held the same belief), common (six to eight participants held the same belief) or uncommon (five or fewer participants held the same belief).

## Findings

There were three universal beliefs, five common beliefs, and two uncommon beliefs held by participants. Overall, the beliefs held by participants were consistent across participants, and generally positive. However, our sample was not particularly representative of a typical real analysis course, as two-thirds of our participants were taking this class as an advanced mathematics elective for a second major rather than as a requirement for a primary major.

### Universal Beliefs

Although there were no clear epistemological framework patterns when considering students by achievement, there were beliefs held by all nine participants: it is possible to learn how to learn, effort aids success, and connecting concepts is important in mathematics. Maya, a math major and prospective high school teacher, believed it was possible to learn how to learn:

> Yeah. I think I would have to say yes [you can learn how to learn], just because the whole metacognition part of the taxonomy and the exercises you find to help you get to that part of that pyramid. I'm like, yeah, I could see how you would need to learn how to learn. We see this because not everybody learns in the exact same way.

Leon, a physics and math double major, gave a typical response when asked if mathematics can be learned:

> I think it's possible to learn it [mathematics]. There are people who are going to enjoy it and it'll become easier, but generally, I think it can be learned. I sure hope it can be learned. I'm tutoring people to do it.

The second belief about mathematics held by all participants was that effort aids in their success in both mathematics in general and real analysis in particular. Luca, a physics major with a second major in math, explained that without effort, he would likely not be passing, "Probably, I would say, if I didn't put forth effort I would be failing right now." Carter agreed, adding, "Yes, very much so. Highly correlated. Yeah, sometimes it's slower than other times. Sometimes it's very grinding. You've already experienced some of that with my proofs."

The third universal belief about mathematics was that making connections between concepts was very important in learning mathematics. Albert, a physics major with a second major in mathematics, gave a typical answer when he was asked how important was it to connect concepts in mathematics in general and real analysis specifically:

> Very, because then I'll know how to use them in other problems that are similar to this. Like you said earlier about how a problem could have multiple answers. I say yes, because I remember that we could solve a certain problem using different methods, and with how those methods tell us, like, the result of those methods could also tell us the same answer, but in different forms. That's what I say.

Taylor, also a math and physics major, agreed, emphasizing the importance of the building connections between his two chosen majors:

> I think it's very important. As a person who's taking and has taken a lot of physics classes, taking the math classes makes physics a lot easier and sometimes there'll be something I learned in math class, and then we talk about it in physics on a deeper level, and it's using calculus or analysis that totally makes the physical concept come quicker.

**Common Beliefs**

Five of the ten components of the epistemological framework were classified as common beliefs, where six to eight of the participants held the same view of mathematics in general and real analysis in particular. The first was that repetition is important to learning, a belief eight participants held. Maya, a mathematics major with an emphasis in secondary education, explained that repetition was learning:

> That's how I learn, period. A lot of the stuff is me looking at an example of how to do it, doing it, and kind of figuring out how you get from step one to B. And a lot of it is just doing it over and over again until I understand the thought process behind why I need to do it. And I'm like, okay, yeah, I can replicate it. And even if the question is not exactly like the one I just did, I'll still be able to figure it out.

Wade, a physics major with a second major in math, was the only one who disagreed with the consensus:

> [Repetition is] Not so important. I think, I don't necessarily find myself going back on all notes and rereading them. I find it more beneficial to try to use those ideas and things in different contexts. I find that to be the best strategy for learning for me.

Two common beliefs had seven participants holding a consensus belief: It is not normal to learn real analysis quickly and instructor quality is important. Taylor, a math major who switched to a physics major during real analysis, explained that he found the proof-based class challenging, and did not agree that it was normal to learn real analysis quickly: "Well, I don't know [if it's normal to learn real analysis quickly]. I've only taken one Real Analysis class. It feels like for me it's faster than what I was ready for. I'll say that." The two students who believed real analysis was relatively quick to learn were both high achieving students; as Jeremy, a mathematics major, explained: "Um, if it's just what's covered in this class, I don't think it'd be too hard to learn it at the same rate as we go through it in class on our own." The other belief held by seven students was that instructor quality was important to mathematics learning. Wade provided a typical explanation of why an instructor matters so much in advanced mathematics:

> It's very important. I think it's important in keeping passionate about a subject. Recently, I've done a lot of self-study, so I suppose in that case, quality of the book that you choose is important to keep you motivated. I think having a good teacher can inspire you to want to do more on the subject, which then is probably the best thing that they could do for you. And that's, I think, a good way.

Carter, a mathematics major, disagreed. He said that although instructor quality is nice to have, learning is possible even without an instructor:

> Again, ideally, high quality instruction is great, right? It only improves what you get. You could work with a lot less, that's for sure. You could work with nothing. I mean, these things weren't around. These weren't originally ideas. Eventually, they came about.

There were two beliefs with a six-student consensus: effort helps you understand things more and problems with multiple solutions are in general a positive thing. Interestingly, all nine students classified a proof as a problem with multiple solutions. Wade gave a typical answer, explaining that pushing past the initial urge to quit and focusing on a problem he did not immediately know how to solve was generally successful:

> Yeah. You know, a lot of problems, you look at them and think you want to just give up right away, and I think if you force yourself to sit down, like consider for however long, you're at 20 minutes, whatever, I think that some progress is better than no progress. With some progress, you do understand things better. It usually is.

Three participants partially disagreed with the consensus, stating that effort only sometimes helps, and occasionally there is no amount of effort that will move you forward in a problem. Luca, a physics major with a mathematics second major, when asked if trying harder helped him understand better, replied, "Sometimes, sometimes I just get more confused."

Problems with multiple solutions were generally seen as at least an interesting phenomenon. Carter, a mathematics major, explained, "I mean, those are the exciting ones, right? I think that's enough said." The three students who disagreed with Carter tended to be among the lowest-achieving participants. As Maya explained:

> I don't like them. Because I guess, I say as I'm a mathematics major, it's like math is very logical, and so it should only have one specific answer. And so you really cause problems. I'm like, I know this is technically right. I don't like it.

## Uncommon Beliefs

The two beliefs where there was little consensus among participants were whether or not it was normal to learn mathematics quickly in general (4/9 yes, 4/9 depends on the context, and 1/9 no), and how to verify correctness on solutions (4/9 external sources such as peers or instructor, 3/9 internal sources such as intuition and logic, 2/9 a combination of internal and external sources). Lewis, a physics major with a second major in mathematics, felt that mathematics was generally something he learned quickly, “It usually doesn't take too long for me to figure out concepts or I don't feel like anything goes too fast for me to follow usually.” Jeremy felt that the speed of learning was context dependent, “Like what is the math and what is quick? So it's very context-dependent.” Wade, was the only student who felt that it was not normal to learn math quickly, as he explained, “No, I don't think so. I think it depends what you want to learn, I guess. If you want to learn it well, I think you need to spend a lot of time digesting topics.”

Albert used external sources to verify his work, “[I use] My peers, the book, and the instructor [to verify my work].” Taylor trusted his intuition that his first answer was correct, “Often I don't check that it's right. I think I'm pretty careful in my calculations. And so, usually I'll just go with the first answer that I get, and trust my gut.” Carter, one of the participants that used both internal and external sources of authority to verify his answers, discussed the value of office hours to fine tune answers, but felt that the most valuable way to check his answers was internal consistency:

> Well, I'll make sure it's consistent with other things, so... It's hard to think about this specifically, but pull any other facts, I suppose, and make sure that it doesn't lead to inconsistency. Check for consistency.

## Discussion

Overall, the typical participant’s epistemological framework was grounded in effort, a desire for understanding, and belief that there must be at least some reliance on their instructor and peers to provide instruction and support their learning in the course. These beliefs were remarkably consistent across participants, which suggests in part that there may be a set of adaptive beliefs students must adopt to be successful in advanced mathematics courses; it would be interesting to compare students’ epistemological frameworks in an introductory proofs course to these participants. Our findings align with prior work on undergraduates’ epistemological framework (e.g. Limeri et al., 2023; Krupnik et al., 2017) in that most students hold fairly productive beliefs about learning, but are still very reliant on external authority as a source of conviction for their work.

However, there are several limitations to this inquiry. The participant pool was overwhelmingly white, male, and majored in physics with a second major in math, so they may not have been particularly representative of a typical population of real analysis students since this class was an elective and not required for 6/9 participants. Despite these challenges, the consistency of beliefs the participants in this study hold provide a foil for future research on traditionally underrepresented groups in advanced mathematics classes. Future research on advanced students’ epistemological frameworks should include a variety of advanced math classes as well as more diverse participant pools.

For the RUME conference, we hope to receive feedback from our peers about the theoretical framework of our study. What alternative framings might provide additional insight on the data? How does the context of a student-centered class add to the findings?

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