An Argument for Teaching Logic With Euler Diagrams Instead of Truth Tables

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This theoretical report integrates insights from various lines of research to posit an argument that early logic instruction for proof-based mathematics should use set-based conditions for the truth of conditional statements rather than the widely used truth table definition. We claim the truth table definition poses unnecessary barriers for students, in part because it does not adequately express mathematical practice. In contrast, the set-based meaning seems highly productive and may better support students in reasoning about conditionals as objects, which our experiments suggest is a highly propitious development for learners.

Keywords: logic, conditionals, truth tables, Euler diagrams

It is standard practice in early proof courses to teach mathematical logic so that students can learn precise conditions for declaring mathematical statements true or false. This occurs especially often in transition to proof courses and discrete mathematics courses (David & Zazkis, 2020; Schumacher, Epp, & Solow, 2015). The most common approach to teaching the logic of statements uses truth tables (Alcock & Sa, 2025), which define the truth of compound statements – such as disjunctions, conjunctions, and conditionals – as a function of the truth of their component propositions. For instance, the conditional "if p, then q" is only false under the condition that p is true and q is false. Based on our joint experiences on two multi-year projects researching the teaching and learning of logic for mathematical proving, we have become convinced that this approach should be replaced with set-based definitions and Euler diagrams. In this theoretical paper, we set forth our arguments for this recommendation drawing on relevant literature from psychology, mathematics education, and our own research experiences. While many of these arguments have appeared in our prior work, we have not put them together in a unified argument meant to prompt a shift in teaching, as we intend for this argument to do.

To build this argument, we pursue a few lines of inquiry that challenge the preference for truth tables to help students understand mathematical language, specifically mathematical conditionals. The first principle we challenge is whether the truth table expresses a common-sense meaning of "if… then" statements from everyday language or how people reason about such statements. We share evidence from psychological research that the truth table definition of conditional truth (often called the "material conditional") fails as a descriptive account of everyday usage. Although this does not undermine the utility of this definition for mathematical usage, it points to the fact that it is a novelty being imposed upon student language use. Second, we challenge whether the material conditional adequately expresses what conditional claims mean in mathematics. We share some mathematics education research that challenges it as a descriptive model for how mathematicians reason. Further, we identify some counter-intuitive "facts" that arise when the truth table definition of "if p, then q" is taken to correspond to the mathematical notion that "p implies q."

The third component of our argument focuses on the adequacy of set-based truth conditions and Euler Diagrams to support student reasoning about conditionals in mathematics.

Specifically, a universally quantified conditional "For all $x \in S$, if $P(x)$, then $Q(x)$" is true whenever the truth set of P is a subset of the truth set of Q, which we shall refer to as the subset meaning (see also Dawkins, 2017; Dawkins, Roh, & Eckman, 2023). We offer three main points in favor of teaching with the subset meaning and Euler diagrams. First, we review some psychological research that suggests that Euler diagrams better facilitate novice learning, suggesting that there may be greater ease of use than has been found for truth tables (Hawthorne & Rasmussen, 2015). Second, our research suggests that some parts of the proof-based curriculum require that students learn to reason about conditionals as objects, meaning they conceptualize implication as a fixed relationship between two predicates. This kind of understanding is better facilitated by Euler diagrams than by truth tables. Third, we argue that set-based truth is a better descriptive account of how conditionals operate in mathematical language.

Point 1: The Truth Table Definition Fails as Descriptive Model for Everyday Conditionals

As Toulmin (1958) argued, scholars use the term *logic* to refer to at least four different kinds of endeavors. Two of them are descriptive, and two of them are prescriptive. In its descriptive mode, *logic* refers to either how individual people reason (Toulmin's *psychology*) or to how groups of people reason in some idealized or normative sense (Toulmin's *sociology*; note this second meaning blends in a prescriptive component). In its prescriptive mode, *logic* refers either to a means for building desirable arguments (Toulmin's *technology*) or to a formal system akin to mathematizing language (Toulmin's *science*). The descriptive sense of logic is at play when someone asks, "What is the logic in that?" while mathematical logic tends to focus on prescriptions that render language rule-based and consistent. Although the association of logic with processes of reasoning has been deeply criticized by some logicians (e.g., Frege and Husserl; see Schroyens, 2010, for helpful discussion), the potential association has led to two extensive lines of inquiry. Philosophers and logicians have pursued the question "can we build formal systems of logic that incorporate elements of everyday argumentation and language use?" This has led to novel logics such as fuzzy logic (in which claims can have truth values other than *true* or *false*) or modal logics that incorporate different senses in which a claim is true (e.g., *possibly true* in addition to *necessarily true*). Psychologists have pursued the converse question, "can we identify which formal system best describes how people interpret and reason about conditional claims?" It is this line of research that most directly informs our current argument.

One of the dominant means by which psychologists have studied how people interpret conditional claims has been the Wason (1966) card task. We will describe the task in its generalized form. Participants are asked to test a rule of the form "if p, then q" regarding four cards displayed. Each card has one side relevant to p and another side relevant to q. The four displayed cards show cases corresponding to p, not p, q, and not q. Participants must indicate which cards need to be turned to check whether the rule is followed/violated. It is far beyond the scope of this paper to review this vast literature (using variants of the Wason task and related study designs), but we shall recount a few key elements from the findings.

First, though the Wason card task seems a simple operationalization of the material conditional definition for conditional claims, the standard rate at which untrained undergraduates provide the normative answer is about 10% (Evans & Over, 2004). According to the material conditional definition that a conditional is false whenever p is true and q is not, the normative solution to the task is to turn the p and not q cards, since these are the two that might yield a counterexample to the rule. The most common answer provided by most undergraduates to many forms of the task are to turn the p and q cards, meaning they select an irrelevant card and omit a relevant one. Second, performance on the task varies greatly depending upon the context for the rule. If the rule corresponds to a known social rule such as "if someone is drinking, then they must be at least 21 years old," then performance on the task greatly improves. This runs contrary to the underlying notion that logical reasoning pertains to the form of an argument, not to its content. These findings drive a wedge between the prescriptions of formal logic and empirical findings about human reasoning about statements in conditional form. For these and similar reasons, Schroyens (2010) claims that "Classical propositional calculus is vastly inadequate as a theory about how people reason. Whatever merits one (e.g., a logician) wants to attribute to the propositional calculus, one merit it cannot possess is being a descriptively adequate theory of human reasoning" (p. 71).

What role should a descriptive account of everyday reasoning hold for the teaching and learning of logic in mathematics? We see two main implications. First, instruction should not assume as intuitive any logical notions that do not have empirical support as being widely held apart from mathematics instruction. Second, instruction should privilege ways of teaching the logic of mathematical statements that do not needlessly introduce contrasts with everyday language use, all other things being equal. The descriptive account that has better empirical support relies heavily on probabilistic notions. According to the suppositional account, one's affirmation of "if p, then q" in the everyday is most closely linked with the conditional probability of q given p is true (see Evans & Over, 2004; Oaksford & Chater, 2020). This account helps explain why some human reasoning in experiments aligns to the *defective truth-table* for conditionals, which only includes the cases where p is true (Evans et al., 2007; Wason, 1966), as the meaning of "if p, then q" entails assuming p true. Cases in which p is false are taken as merely irrelevant to the conditional. Indeed, most anyone who has tried to teach the logic of conditionals using the standard truth table have found that convincing students that the conditional is true when p is false constitutes a steep uphill battle. The suppositional account justifies students in their resistance to this aspect of the material conditional.

We find these findings from non-mathematical settings insightful, and they help contextualize the rest of the paper. However, we should note that everyday meanings for conditionals need not restrain mathematical usage. There are fundamental distinctions. As Oaksford and Chater (2010) note, proponents of the suppositional account "argue that logical methods need to be supplemented with probabilistic notions not because classic logic is incorrect, but because they contend that outside mathematics, it rarely, if ever, applies to real human inferences and arguments" (p. 21). By contrast, mathematical inferences and arguments certainly align to logic in some manner (though it is worth considering the nature of this link carefully). Furthermore, we are concerned with student learning of technical meanings, rather than untrained usage of language (Alcock, 2025a). Next, we shall examine how the truth-table definition fails to express how mathematicians reason about conditionals, even in mathematics.

Point 2: The Truth Table Does Not Capture the Mathematical Meaning for Conditionals

As we noted in the introduction, the standard way of presenting the truth conditions for mathematical conditionals is the full truth-table. However, there are multiple reasons to question whether the material conditional truly aligns to mathematical usage. We will provide a few different strands of support for this claim.

Do Mathematicians Reason About Conditionals According to the Material Conditional?

First, if we assume mathematicians as paragons of logical reasoning, we may wonder how mathematicians respond to standard psychological tasks such as the Wason Card task. Inglis and

Simpson (2004) administered this task (with abstract content) to mathematicians and found that their answers differed from what is commonly observed among mathematics or history undergraduates, but still did not give the normative answer (conforming to the material conditional) more than half of the time. Mathematicians were more likely to avoid irrelevant cards (each irrelevant card was selected 19% of the time), but still only considered the not q card only 67% of the time.

In a more qualitative investigation of the process of reasoning, Inglis (2006) found that the best way to explain how mathematicians reasoned about mathematical conjectures (in conditional form) was the suppositional account. Specifically, he claims they assumed p true and then considered the probability that q was also. The distinction is that in the everyday, people will affirm a conditional even in the presence of counterexamples (instances of p and not q) while in mathematics, the observed reasoners sought to affirm that q was necessarily true without exception, ultimately by producing a proof. Excepting the affordance of various degrees of certainty in the truth of q, Inglis' account of the mathematicians' reasoning is compatible with the defective truth table account, which Hoyles and Kuchemann (2002) endorsed as adequate for usage in the mathematics classroom.

Attridge and Inglis (2013) provide a final piece of evidence supporting the link between mathematical conditionals and assuming p is true (consistent with the suppositional account or the defective truth table). In this study, the authors presented abstract reasoning tasks to students in mathematical and non-mathematical courses of study near the beginning and near the end of one year of secondary school studies. The goal was to compare the influence of mathematical study on abstract inference tasks with non-mathematics students as a comparison group. A year of studying mathematics (including no lessons on logic) decreased the rate of affirming the (invalid) converse inference, but also slightly depressed the rate of affirming the (valid) modus tollens inference from not q to not p. The authors explain this pattern as aligning to the defective truth table pattern of conditional inference.

Conditional Truth Versus Valid Inferences

Weber and Alcock (2005) asked mathematicians whether they affirmed the statement "if 7 is prime, then 1007 is prime," which everyone in the experiment took to be true according to the material conditional definition (1007 is non-obviously composite). Those authors found that mathematicians did not affirm this claim because 1007's primality does not follow from 7's primality (e.g., the inference "if x is prime, then $x + 1000$ is prime" is not valid). The authors concluded that the criterion by which these mathematicians evaluated the conditional was stronger than the truth table definition. Mathematicians required that q be a warranted inference from p. Along similar lines, Vargas (2020) notes, "This is the case for conditional statements which are not seen as formulas with truth values in themselves, but more as "licenses for inference" in logic programs" (p. 58). Janke and Wambach (2013, quoted in Vallejo-Vargas, 2023) add,

> a mathematical proof does not establish *facts*, but *implications*. A proof proves 'if-then-statements'. That means a mathematical theorem is not about a fact B, but about an implication 'If A then B'. Thus, the *absolute certainty* of mathematics does not reside in the fact, but in the *inferences*. (p. 471)

So, for teaching logic we should desire an account of conditionals that facilitates the important link between conditional statements and implications (Alcock & Sa, 2025).

It is precisely this intended link between conditional statements and implications that renders the material conditional an inadequate account of conditionals in mathematics. To see this disconnect more vividly, consider that according to the material conditional, $(p \rightarrow q)\ or\ (q \rightarrow p)$ is a tautology. However, we would never want to affirm that for any claims p and q, one must imply the other. We raise this challenge because it allows us to endorse what we see as a more productive alternative to the material conditional. Most mathematical conditionals of interest are not conditionals involving propositions with a fixed truth-value, but rather universally quantified claims between predicates: "$\forall x \in S, P(x) \rightarrow Q(x)$." If we introduce this modification, the resulting expression $(\forall x \in S, P(x) \rightarrow Q(x))\ or\ (\forall x \in S, Q(x) \rightarrow P(x))$ is no longer a tautology. We will argue in the next section that an approach to conditional truth that builds upon universal quantification is more productive and instructionally adequate, namely the subset meaning represented using Euler diagrams.

Point 3: A Set-Based Definition for Conditionals is More Adequate for Teaching Logic

If we are to argue against the truth table definition of the truth of mathematical conditionals, we must offer a viable alternative. In our instructional design projects (NSF DUE #1954768, #1954613, and #2141626), we have explored such a viable alternative in the subset definition of conditionals. In this view, the conditional "$\forall x \in S, if\ P(x), then\ Q(x)$" is true whenever the truth set of P is a subset of the truth of Q. We encourage students to explore such relationships between truth sets using Euler diagrams. In this section, we provide three strands of support for the preferability of this approach.

Euler Diagrams Are Accessible for Novice Learners

One affordance of universally quantifying conditionals is that it brings them closer to the kinds of statements classically used in Aristotle's system of syllogisms. That system featured statements of the forms "All A are B," "Some B are C," and "No C are A." The development of this system preceded the development of modern logic by about 2000 years, which may speak to its relative intuitiveness, though this evidence is clearly only suggestive. Nevertheless, this allows us to draw upon research investigating how students reason about syllogisms in addition to the Wason Card tasks and inference tasks featured in the studies reviewed above.

Sato, Mineshima, and colleagues have conducted a series of experiments exploring how students solve syllogism tasks with relatively brief training (Mineshima et al., 2012, 2014; Sato & Mineshima, 2015; Sato et al., 2010, 2011). In many cases, they compare student success on syllogism tasks after one of three instructional treatments: brief training in Euler diagrams; brief training in Venn diagrams; and training only in sentential, non-diagrammatic approaches. Venn diagrams differ from Euler diagrams in that they always show the closed curves overlapping and use shading and x's to indicate which regions are empty or occupied. Euler diagrams by contrast will show closed curves non-overlapping either to show one set is disjoint from another or is a subset of another (when one curve surrounds another). A brief summary of these studies' findings is that students perform better on syllogism tasks when using diagrams, and Euler diagrams support more accurate responses than Venn diagrams.

These authors claim that these findings demonstrate that such diagrams are highly accessible to novice learners. "The manipulation of diagrams in the inferential process is triggered without effort, if the spatial relations holding on external diagrams are governed by *natural* constraints" (p. 29, Mineshima et al., 2012). Although we do not echo their emphasis on the lack of effort involved for learners, we value their evidence that Euler diagrams are particularly helpful. "Our hypothesis is that Euler diagrams are *self-guiding* in the sense that the construction of diagrammatic proofs could be automatically triggered even for subjects without explicit prior knowledge of inferential strategies or rules, whereas Venn diagrams are not" (p. 7, Sato et al., 2010). Their argument for this contrast, by which they explain their empirical findings, relates to the contrast between the underlying accounts of logic. Whereas the standard textbook treatment (based on truth tables) analyzes statements using quantification over individuals,

> in the field of natural language semantics, by contrast, quantifiers in natural language, such as *all*, *some* and *no*, are analyzed as denoting *relations* between sets… Thus, a sentence of the form *All A [are] B* is analyzed as $\boldsymbol{A} \subset \boldsymbol{B}$, rather than as the first-order representation $\forall x(Ax \rightarrow Bx)$. Similarly, *No A [are] B* can be analyzed as expressing $\boldsymbol{A} \cap \boldsymbol{B} = \emptyset$. (p. 2183, Sato et al., 2011)

They argue that Venn diagrams foster analysis by region of the diagram, while Euler diagrams foreground relations between the sets as units.

We argued in a previous section that findings about student thinking outside of mathematics inform logic instruction by helping us not place unnecessary barriers in students' ways. These studies of student performance on syllogism tasks suggest that Euler diagrams are particularly facilitating, likely because they foreground conditionals as relations between sets. The fundamental object relevant to truth-table analysis is the specific example object or a non-quantified statement about specific objects. Set-based truth conditions, especially as represented by Euler diagrams, support analyzing the truth of mathematical claims with the truth sets as objects. This means the truth of a conditional hinges on the relationship between two sets rather than a potentially infinite range of individual objects. Both views are ultimately useful in their way, but we contend that the view relating sets may be more facilitating for early logic learning. Insights from our studies in proof-based mathematics courses further support this claim.

Euler Diagrams May Support Students in Reasoning About Implications as Objects

In our investigations of student learning in introduction to proof courses, we have found that it is important for students to learn to reason about implications as objects, rather than actions or pseudo-objects (Norton et al., 2025). Building upon Sato et al.'s (2011) discussion, we can identify the distinction between noting a subset relationship between the sets $\boldsymbol{A}$ and $\boldsymbol{B}$ versus reasoning about the subset relationship as an object. As noted in Norton et al. (2025), this becomes particularly relevant when trying to act on a particular implication via operations such as composing implications ($p \rightarrow q$ and $q \rightarrow r$ yield $p \rightarrow r$), negating implications (which is distinct from negating either or both components), or transforming implications such as by inversion or conversion and reasoning about the properties of the transformed implication. We claim that set-based meanings for conditionals, especially when explored using Euler diagrams, may help foster understanding of implications as objects. While we acknowledge that this claim needs nuance in terms of how and how frequently this claim bears out in student learning, we also see evidence that truth tables are particularly unfit for fostering this kind of understanding.

Hawthorne and Rasmussen (2015) conducted one of the primary studies of how students reason about truth tables in learning logic in mathematics. One contrast they found was that the instructor conceived of the truth table as a unified definition of the truth of a statement while students often displayed a fragmentary understanding of the truth table. They understood each line as assigning truth values from the parts to the whole but did not reason about the truth table in a unified manner. We find it unsurprising that patterns of truth values do not support students in conceiving of a conditional statement as a fixed relationship between p and q, much less that it would support them to operate upon that object. Actions like composition, negation, and transformation can be carried out with truth tables by producing ever larger lists of T's and F's, but they do not provide a unified image of the meaning of the conditional to be acted upon.

By contrast, if students associate the conditional "if p, then q" with a subset relationship on their truth sets $\boldsymbol{A} \subset \boldsymbol{B}$, then we see easily how students could compose ($\boldsymbol{A} \subset \boldsymbol{B}$ and $\boldsymbol{B} \subset \boldsymbol{C}$ yields $\boldsymbol{A} \subset \boldsymbol{C}$), negate ($\boldsymbol{A} \not\subset \boldsymbol{B}$), and reason about transformations of this relationship (given $\boldsymbol{A} \subset \boldsymbol{B}$, what is known about $\boldsymbol{B} \subset \boldsymbol{A}$ or $\boldsymbol{A^c} \subset \boldsymbol{B^c}$?). We have explored some of these affordances in our teaching experiments (Dawkins et al., 2023; Hub & Dawkins, 2018). These experiments suggest these are viable learning pathways for at least some students, though further work is in order.

Another endorsement of the set-based meaning over the truth table definition comes from the literature we reviewed on the suppositional account and the defective truth table. As we noted before, students are most often troubled by the standard truth table because it claims conditionals are true when p is false. This whole question becomes a non-issue when analyzing conditionals using the set-based definition. Students are aware that to decide whether $\boldsymbol{A} \subset \boldsymbol{B}$ only objects in $\boldsymbol{A}$ are relevant and that objects outside of $\boldsymbol{A}$ certainly do not falsify the relation.

Subset Relations Capture Some Aspects of Mathematician Reasoning

In line with the potential compatibility between the subset meaning of conditionals and the suppositional account/ defective truth table, we think this could help explain some of the psychological findings about mathematician reasoning on the Wason Card task and abstract inference tasks. In particular, mathematical training particularly supports reasoners in avoiding fallacious inferences such as $q \to p$ and $\neg p \to \neg q$, but is less successful in promoting attention to the *modus tollens* inference $\neg q \to \neg p$. This seems reasonable given a strong association of a conditional with a subset relation, as subset relations are asymmetric (suppressing $q \to p$) and instances outside the subset are irrelevant (suppressing $\neg p \to \neg q$). It stands to reason that this same de-emphasis on cases where p is false would similarly suppress the *modus tollens* inference, as for a true conditional the truth sets of $\neg q$ and p are disjoint.

We will reference one final bit of evidence for the compatibility of mathematical practice and the subset condition. Alcock (2025b) investigated what diagrams mathematicians preferred for portraying conditional relationships. She found almost uniform preference for Euler diagrams over Venn diagrams. This means they chose representations that emphasize the truth conditions of a conditional, as Venn diagrams portray the region in which p is true and q is false, which may then be empty (the conditional being true) or non-empty (the conditional being false). While further work is needed to understand the nature of this preference, this supports our claim that the subset relation captures some aspects of how mathematicians reason about conditional statements in mathematics.

Conclusion

In this theoretical paper, we have drawn on a range of scholarship from different fields to build a case against the use of truth tables to teach logic for proof-based mathematics, which at present holds dominance in textbooks. We claim this approach puts unnecessary barriers between student reasoning from the everyday and mathematical practice, in part because the truth table definition does not adequately capture mathematical usage of conditionals. We offer the subset meaning as a viable alternative that avoids these challenges. Some studies we reviewed suggests that reasoning about conditionals in terms of set relationships is quite accessible, especially when explored using Euler diagrams. We do not mean to imply that there are not major challenges along that learning pathway, but we are convinced that this alternative approach has ample evidence to support a move away from the current status quo.

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