Transposition Instability Due to Novel Mathematical Organizations for Life Sciences

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This study examines how institutions navigate praxeological uncertainty when implementing Modeling Life, a mathematical modeling curriculum for life sciences students. Using the Anthropological Theory of the Didactic, we analyzed implementation across five institutions where didactic transposition processes remain under active negotiation. We identify four interconnected patterns: positioning uncertainty, epistemological conflicts, human resource demands, and systematic adaptations. These findings illustrate how novel mathematical organizations whose requirements exceed existing institutional conditions both generate and are shaped by institutional adaptation processes during ongoing curricular development.

Keywords: mathematical modeling, ATD, biology majors, institutional perspective

The growing importance of mathematical modeling to the research methodologies and conceptual frameworks of the life sciences has prompted numerous calls for reforming the mathematical experiences of life science majors (Brewer & Smith, 2011; Steen, 2005). However, research has documented persistent barriers to implementing modeling-focused approaches in undergraduate mathematics education (Barquero et al., 2015; Bennoun, 2022). Studies identify institutional constraints including faculty's epistemological commitments to *applicationism* and *monumentalism* (Barquero et al., 2015), rigid corpora of mathematical knowledge at university level (Bosch, 2014), and institutional conditions that favor traditional calculus approaches over modeling-centered curricula (National Research Council, 2013). With highly standardized topics like calculus or linear algebra, "the scholarly mathematical *praxeologies* that are selected as knowledge to be taught and the *praxeologies* that are actually taught in the classroom constitute a very stable corpus of knowledge" (Bosch, 2014, p. 115). This stability is not present with bodies of knowledge that are new to mathematical programs at institutions, creating opportunity to study transposition processes during active development. The UCLA *Modeling Life* curriculum represents a modeling-centered approach that has been adopted by multiple institutions. Previous work documenting challenges to implementing novel curricula indicate that the Anthropological Theory of the Didactic (ATD) would allow for examination of the institutional dynamics that underly curricular transformation processes. Adopting this lens, we analyze *Modeling Life* implementations across five institutions to examine these dynamics.

Theoretical Perspective

ATD explains how mathematical knowledge transforms as it shifts between institutions, such as scholarly communities that use knowledge and scholastic communities that teach it. ATD concerns itself with how institutional conditions affect mathematical teaching and learning by coordinating analysis of didactic transpositions, praxeologies, levels of codetermination, and operating epistemologies (Bosch, 2014; Chevallard 2002). These tools aid in understanding why curriculum innovation often encounters systemic, institutional resistance and how implementation challenges arise from mismatches between innovative praxeological requirements and existing institutional conditions.

Knowledge also undergoes systematic transformation as it moves between institutions; this *didactic transposition* reconstitutes scholarly knowledge as knowledge-to-be-taught. The transformation is not so simple as articulating a list of topics and techniques. It emerges from activity within the noosphere, the institutional sphere where curriculum designers, policymakers, textbook authors, and educational experts select, organize, and adapt mathematical content for institutional contexts. External didactic transposition refers to processes within noospheric activity while internal didactic transposition subsequently transforms this prescribed knowledge into "knowledge actually taught" and ultimately "knowledge learned" through classroom interactions between teachers, students, and mathematical content (Bosch & Winsløw, 2020). Both the process and the mathematical knowledge are shaped by the institutional conditions.

ATD's primary object of analysis is *praxeology*. These are organized blocks of knowledge and practice that describe how mathematical work is accomplished within institutions. A praxeology comprises two interconnected components: (i) a praxis block consisting of types of tasks (T) that individuals encounter and techniques (τ) used to solve them and (ii) a logos block containing the technology (θ) that describes, explains, and justifies the techniques, supported by theory (Θ) that provides broader theoretical foundation (BarbÉ et al., 2005). Mathematical organizations (MOs) are structured collections of praxeologies that make up the mathematical knowledge, and they operate within specific institutional contexts. An important undertaking for ATD is to elaborate the ecological conditions that enable praxeological development, maintenance, and evolution, as well as which institutional factors support or hinder the construction and survival of MOs (Bosch & Gascon, 2006). Chevallard's (2002) levels of didactic codetermination show how each level influences lower levels, explaining why curriculum implementation challenges stem from inter-level conflicts that require coordinated institutional responses rather than isolated pedagogical changes.

ATD has also examined the epistemologies underlying mathematical instruction that influence how MOs develop and function within institutions. For example, the "visiting monuments" paradigm treats mathematics as established works to be consumed and appreciated. This creates praxeologies where tasks are phrased in terms of procedures and techniques focused on algorithmic implementation of those procedures. In contrast, a "questioning the world" paradigm treats mathematics as tools for addressing genuine problems, creating praxeologies where tasks emerge from authentic inquiry and techniques developed in response to problem-solving needs (Bosch & Winsløw, 2020). Similarly, "applicationism" views mathematical modeling as mere application of pre-established mathematical knowledge to external domains (Barquero & Bosch, 2015). This leads to modeling appearing as end-of-chapter exercises following content presentation, creating a weak logos block where the technology and theory present modeling as supplementary to mathematical development rather than as mathematically generative. In contrast, an epistemology that positions modeling as integral to mathematics itself enables praxeologies with strong logos blocks where the technology and theory establish modeling processes as fundamental to mathematical understanding and concept formation.

Whereas in previous studies, external and internal transposition occur simultaneously (Bergsten et al., 2010; Bosch et al., 2021; Bosch & Winsløw, 2020; Strømskag & Chevallard2024; Winsløw et al., 2014), we use didactic transposition and praxeology to describe situations where external transposition (noospheric selection and prescription) has not settled into a reference MO by the time instructors begin internal transposition. We refer to this situation as *transposition instability*. Under this condition, institutions attempt to position, implement, and legitimate courses in the absence of reference MOs, even though instructional practice is already underway. As a result, stakeholders face *praxeological uncertainty*, understood here as indeterminacy regarding which praxeologies should be taught, how they relate to existing curricular pathways, and on what grounds they can be justified within the institution. In this study, we examine how praxeological uncertainty emerges when institutions attempt to adopt novel MOs under conditions of transposition instability.

Mathematical Organizations of Modeling Life

The MOs of *Modeling Life* are constructed around dynamical systems and biological modeling. The praxis block centers on task types that require students to formulate, simulate, interpret, and revise mathematical models for phenomena like disease transmission, population dynamics, and gene regulation, in order to predict system behavior. The tasks demand techniques that combine analytic methods (linear stability analysis and equilibrium classification), numerical approaches (Euler's method, simulations), and computational tools (Python, SageMath). Freshman life science students construct meaning for differential equations by identifying feedback relationships in biological systems, entailing interdisciplinary reasoning about biological processes and their mathematical representations simultaneously. The logos block comprises technology that emphasizes concepts directly relevant to life sciences: state space representations, vector fields, bifurcation theory, and covariational reasoning. This technological discourse positions derivatives as linear approximations of vector fields rather than rules or slopes, and integration as accumulated change rather than antiderivatives. The underlying theory proposes an epistemological shift because the tasks emerge from biological questions (e.g., what is a genetic switch?) and the justifications for the mathematical techniques follow. Two characteristics distinguish these MOs from calculus praxeologies. First, the primacy of modeling over mastery of procedures entails the creation of a logos block that explicitly bridges disciplinary boundaries. The technological discourse brings to the fore both mathematical concepts and biological processes, and the theoretical framework positions mathematical validity as dependent upon biological plausibility. Second, the computational praxis block assumes simulation, visualization, and manipulation of parameters as standard mathematical practices, which enable tasks about sensitivity analysis, for example. The presence of these praxeologies create specific institutional demands: faculty must possess or develop interdisciplinary expertise, computational infrastructure becomes important, and assessment practices must evaluate conceptual synthesis rather than procedural execution. The MOs themselves thus create the conditions for transposition instability, as their requirements often exceed existing institutional capacities while their epistemological foundations challenge established mathematical teaching cultures.

Methods

We visited all five sites implementing *Modeling Life* materials, which represent variation in departmental context, institution size, and implementation maturity. We focused exclusively on adopting institutions since non-adopting cases provide limited insight into implementation processes (Ragin, 2006). At each site, we conducted semi-structured interviews with stakeholders including *Modeling Life* instructors, faculty teaching related courses, administrators, teaching assistants, and support staff (site characteristics in Table 1). Interviews elicited stakeholders' salient experiences pertaining to developing, implementing, and sustaining the course (Leech, 2002). We treat absence of a reference MO as an observable condition when stakeholders report contested or changing learning goals or prerequisite expectations. We interpreted stakeholders' reports of adaptations and adjustments to curriculum, contested positioning of the course, or shifting academic oversight as evidence of transposition instability.

Table 1. Site characteristics

Institution	Years Operating	Department	Approx. Enrollment	# Interviews
UCLA	10+	Life Sciences Core	2400	12
UC Santa Cruz	1	Mathematics	160	7
U. Arizona	6	Mathematics	120	7
Utah State	1	Mathematics	60	10
Harvard	1	Mathematics	30	10

Results

We present short vignettes of each site, organized to show evidence of the state of external transposition (e.g., noospheric activity) and institutional responses that instantiate praxeological uncertainty (e.g., positioning moves, epistemological tensions, resources adaptation). We obscure interviewees' identities and attribute conditions they mention to their institution.

At Utah State, praxeological uncertainty emerged through transposition instability operating under conditions of perceived external disruption. At the time of our site visit, a state legislative mandate for general education, including Quantitative Literacy (QL), was interpreted by stakeholders as consistent with the university's land-grant mission to prepare citizen scholars for workforce needs. Participants anticipated a potential reconfiguration of curricular priorities towards critical thinking, data science, and meaningful quantitative work. Although the concrete implications of the mandate for the mathematics department were not yet fully specified, the transfer of curricular authority from home disciplines to a general education center was understood by some as an opportunity for reform, while others perceived it as a destabilizing force for established content and pathways. As one administrator noted, the legislation would break established curricular structures, disrupting what what Diaz-Eaton (under review) identified as "forces that hold the 'canon' sacred." These legislative changes, including significant budget reallocations, functioned as concrete, enacted constraints and new powers, shaping anticipatory positioning around curricular innovations. In this context, the *Modeling Life* approach was viewed by stakeholders as aligning with the direction signaled by the mandate. However, departmental leadership was hesitant to commit sustained resources amid budget pressure and unresolved questions about course evaluation and students' revised pathways. Within this environmental uncertainty, instructors endeavored to predict the eventual audience for the course and align the MOs with their anticipated needs, including retaining, excluding, modifying, or emphasizing MOs from the traditional and modeling focused versions of the course. Overall, *Modeling Life* occupied an ambiguous institutional position: it was sufficiently aligned with emerging priorities to proceed, but its long-term stabilization and the securing of additional, dedicated resources for its full development remained a concern.

At Harvard, a different kind of positioning uncertainty emerges through consideration of students' pathways through mathematical programs. There, *Modeling Life* was conceived as an alternative to the precalculus/calculus requirement, but not as calculus preparation, since the praxis blocks contrast starkly. This reflects institutional uncertainty about how MOs with fundamentally different praxeological structures should relate within mathematical programs. However, the introduction of *Modeling Life* creates system-level effects throughout the mathematical program that can impact other students, as it draws life science students away from traditional courses and subsequently relieves pressure on the precalculus courses regarding what praxeologies are now suitable. Additionally, students' hesitancy to take a course that hadn't been offered before indicates their perception of instability due to the institution's inability to provide clear positioning information about MOs still undergoing noospheric negotiation. These uncertainties both emerge from, and contribute to, broader system-level effects throughout mathematical programs.

Stakeholders across multiple sites evidenced epistemological conflicts between the praxis and logos blocks of *Modeling Life* and institutional commitments to traditional mathematical praxeologies. At UCLA, the initial response from the mathematics department was fierce and vigorous opposition to the new course viewing it as "not real math" or "not rigorous enough." Faculty resistance surged from both the logos blocks of *Modeling Life* positioning mathematics as emergent from biological inquiry, and the praxis blocks using geometric and numerical methods. Chevallard (2015) noted that teaching systems establish an "authorized" version of knowledge and, Diaz-Eaton (under review) argued the language of rigor is often wielded to justify deviations from that authorized knowledge. It is beyond the scope of this paper to argue the mathematical rigor of *Modeling Life*, however we note that its technology integrating feedback mechanisms, state space representations, dynamical systems, and covariational reasoning are all rigorously articulated areas of mathematical practice. Indeed, multiple stakeholders explained that taking a pure mathematics approach as being "not in the right frame of mind" since teaching *Modeling Life* requires "letting go" of the usual mathematical practices. Thus, it seems that productive integrations of this course into the ecology of mathematics departments might further entail an epistemological reorientation towards the course's theoretical underpinnings, namely, positioning mathematics as emergent from biological inquiry rather than as monuments to visit (Chevallard, 2012). Instructors willing to engage in this shift may yet feel "nervous" or "surprised" by the textbook's approach, as did an instructor at Utah State.

These epistemological conditions create institutional pressure on the survival of *Modeling Life*'s praxeologies by demanding faculty orientations that mathematics departments may find difficult to supply, forcing institutions to either modify their implementation approach or develop new human resource strategies to support MOs whose requirements exceed traditional faculty preparation. At our sites, we observed plural strategies to address faculty knowledge and epistemological gaps, with a commitment to maintaining praxeological integrity. At UCLA dedicated teaching lines were authorized to support course development and maintenance. U. Arizona dedicated resources to a co-teaching model in order to induct new instructors to the implementation cycles. This positions co-teaching as a mechanism for developing faculty capacity to implement MOs whose praxeological requirements exceed traditional preparation. Although it is expensive, administrative stakeholders there recognize that courses like *Modeling Life* cannot be sustained with a "rotating cast of characters," and that co-teaching, particularly when paired with a dedicated lead instructor, provides for continuity, recruitment, and is essential for the long-term viability of the course. Additionally, administrative and instructional stakeholders asserted that co-teaching facilitates a collaborative environment which becomes engaging for students. UCLA and Harvard rely heavily on undergraduate learning assistants, but how those resources are used differ. At UCLA, learning assistants have successfully completed the course and can help students who have GTA's that are not as familiar with the interdisciplinary MOs. At Harvard, the learning assistants have not completed the course, but they are drawn from a pool of students with advanced applied mathematics knowledge. Investment of resources in undergraduate learning assistants created a low student-to-teacher ratio, creating opportunities for the instructor to focus on generating pedagogical notes that would make the course "teachable by others."

The positioning of *Modeling Life* within degree plans and mathematical programs also places demands that create incoherence, where praxis blocks diverge from the philosophy enacted through the logos blocks. For example, at Harvard, concern for students' success in later physics courses necessitated inclusion of some tasks from traditional calculus praxeologies, such as trigonometric functions, with companion differentiation and integration techniques. The instructor reflected that these changes were "not very closely tied to anything else we're doing," suggesting that the praxis block was not aligned with *Modeling Life*. Similar adjustments were made to *Modeling Life* at Utah State to satisfy constraints attributable to downstream statistics courses. An instructor at U. Arizona observed that these kinds of modifications "weakened the course a little bit." Interpreting their comments in terms of the ATD, the ecologies in these institutions created a divergence where the logos block supporting the traditional calculus tasks was largely absent (see BarbÉ et al., 2005) and, when obliged, in conflict with the praxeologies of *Modeling Life*. These adaptations preserve prerequisite functions while introducing praxeological tensions between modeling-centered and traditional procedural approaches.

Resource constraints systematically modified the computational praxeologies of the MOs. At Utah State, stakeholders explained that time limitations inhibited integrating coding into the course work because there were only three (credit) hours a week allocated to the course. The full realization of the computational praxis block would require a 4 or 5 credit-hour course, however an existing mathematics course was repurposed to realize *Modeling Life* there. Thus, to suit the institutional conditions, the computational praxeology was removed entirely. Yet, the instructor excluded treatment of limits, despite protestations from colleagues, to avoid "just teaching typical calculus." They opposed specific praxis blocks to preserve logos blocks they perceived as fundamental to the course's philosophy. Student population characteristics also drove systematic adaptations to praxis blocks as institutional conditions forced modifications to task types and techniques. Institutions expanded praxis blocks to include foundational quantitative reasoning tasks (proportions, percents) or adjusted the pace of technique development to accommodate local student preparation, demonstrating how demographic constraints shape praxeological boundaries during external transposition. At UCLA, programming components were "softened" with labs becoming "much easier on the coding" because they could be "brutal" for students with no prior coding experience. Here, the expectation shifted from students writing Python code to modifying provided code to run simulations. This transformation altered both the task types (T) from code generation to code interpretation and modification, and the associated techniques (τ) from syntax mastery to parameter manipulation. The logos block adapted correspondingly, with the technology (θ) repositioning computational work from teaching coding to using simulations to emphasize important ideas from the course. Harvard adopted Chat GPT for coding to bypass syntax learning constraints. These modifications preserve the logos block's emphasis on the value of simulation while adapting the techniques to institutional realities.

UCLA differed from the other sites in that sustained noospheric activity located *Modeling Life* within the Life Sciences (LS) Core, a separate academic unit. There, faculty possessed epistemological commitments to the praxeologies underlying *Modeling Life*, such as praxis blocks centered on biological questions giving rise to logos blocks positioning mathematics as a tool. Within the unit, resources were allocated towards course releases for material development, funding for ongoing evaluation, and strategic hiring of quantitative biologists dedicated to course maintenance. This permitted both the iterative refinement of MOs as specific praxeologies were adopted and discarded, and the production of concrete instructional materials to unify multiple course sections. These artifacts then served as a reference MO (BarbÉ et al., 2005), at least provisionally as new instructors were inducted to the course and the faculty of the LS Core sought to secure ongoing resource allocation. As a result, many of the tensions observed at other sites were attenuated. The pre-med and life science majors' willingness to satisfy degree requirements with *Modeling Life* further legitimized it. A secondary consequence was that downstream courses could build upon the praxeological foundation of *Modeling Life*, reinforcing the institutional integration of its MOs.

Implications & Future Directions

Our analysis identifies four interconnected patterns of praxeological uncertainty when institutions adopted *Modeling Life*'s MOs. Institutions experience positioning uncertainty as they struggle to locate *Modeling Life* within existing academic structures. Epistemological conflicts emerge between faculty commitments to traditional mathematical praxeologies and modeling-centered approaches. Novel MOs create human resource demands that exceed faculty's preparation. Finally, institutional constraints drive systematic adaptations to MOs as they strive for ecological fit within the local conditions. These findings demonstrate transposition instability creating praxeological uncertainty about *what* is being adopted, *how* it should function institutionally, and *for whom* it was offered. The literature identifies broad challenges to to innovative mathematical education such as "monument" paradigms (Barquero et al., 2015), institutional inertia (Bosch et al., 2020), and faculty preparation gaps (Díaz Eaton, 2017), but provides limited analysis of underlying mechanisms. Our findings reveal specific processes through which these barriers operate. *Modeling Life*'s interdisciplinary MOs create institutional tensions operating across multiple codetermination levels. Unlike content-focused innovations that accommodate existing structures, integrating dynamical systems with biological reasoning requires coordinated changes spanning disciplinary, pedagogical, and institutional levels. UCLA's LS Core created dedicated institutional space where hybrid MOs could develop without competing against traditional departmental expectations, addressing what Winsløw et al. (2014) identified as constraints from mathematical domain isolation. Yet, at each of the sites, pressures to adapt *Modeling Life* arose from the noosphere. ATD permitted insights into how and why challenges to innovative MOs operate through specific praxeological mechanisms, rather than general resistance patterns (e.g., Shadle et al., 2017). We acknowledge that some praxeological uncertainty emerged from the interdisciplinary characteristics of *Modeling Life*'s logos blocks rather than its shift in mathematical content. This presents challenges to potential instructors trained in analytic and symbolic techniques, at the pedagogical level of codeterminacy, that could be addressed through collaboration with life science colleagues (Bennoun, 2022). It is yet unclear what features of collaboration could support a bridge between disciplinary domains.

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References

BarbÉ, J., Bosch, M., Espinoza, L., & GascÓn, J. (2005). Didactic Restrictions on the Teacher's Practice: The Case of Limits of Functions in Spanish High Schools. *Educational Studies in Mathematics*, *59*(1-3), 235-268. https://doi.org/10.1007/s10649-005-5889-z

Barquero, B., & Bosch, M. (2015). Didactic Engineering as a Research Methodology: From Fundamental Situations to Study and Research Paths. In A. Watson & M. Ohtani (Eds.), *Task Design In Mathematics Education* (pp. 249-272). Springer International Publishing Switzerland. https://doi.org/10.1007/978-3-319-09629-2_8

Barquero, B., Bosch, M., & Romo, A. (2015). *A study and research path on mathematical modelling for teacher education* CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Prague, Czech Republic. https://hal.archives-ouvertes.fr/hal-01287246

Bennoun, S. (2022). Transforming a Calculus for Life Sciences Course: Moving From Procedural Calculus to Studying Dynamical Systems and Bifurcation Theory. *Primus*, *32*(2), 246-259. https://doi.org/10.1080/10511970.2020.1827104

Bergsten, C., Jablonka, E., & Klisinska, A. (2010). A remark on didactic transposition theory. In C. Bergsten, E. Jablonka, & T. Wedege (Eds.), *Mathematics and mathematics education: Cultural and social dimensions* (pp. 58-68). Svensk förening för matematikdidaktisk forskning, SMDF.

Bosch, M. (2014). Research on University Mathematics Education Within the Anthropological Theory of the Didactic: Methodological Principles and Open Questions. *Research in Mathematics Education*, *16*(2), 112-116. https://doi.org/10.1080/14794802.2014.918346

Bosch, M., & Gascon, J. (2006). *Twenty-Five Years of the Didactic Transposition* ICMI Bulletin,

Bosch, M., Hausberger, T., Hochmuth, R., Kondratieva, M., & Winsløw, C. (2021). External Didactic Transposition in Undergraduate Mathematics. *International Journal of Research in Undergraduate Mathematics Education*, *7*(1), 140-162. https://doi.org/10.1007/s40753-020-00132-7

Bosch, M., & Winsløw, C. (2020). The external didactic transposition of mathematics at the university level: dilemmas and challenges. *Educação Matemática Pesquisa: Revista do Programa de Estudos Pós-Graduados em Educação Matemática*, *22*(4), 373-386. https://doi.org/10.23925/1983-3156.2020v22i4p373-386

Brewer, C. A., & Smith, D. (2011). *Vision and change in undergraduate biology education*. http://www.visionandchange.org/

Chevallard, Y. (2002). Organiser l□'étude 3. Écologie & régulation. In J. L. e. a. Dorier (Ed.), *ctes de la 11e école de didactique des mathématiques*. La Pensée Sauvage.

Chevallard, Y. (2012). *Teaching Mathematics In Tomorrow's Society: A Case For An Oncoming Counterparadigm* 12th International Congress on Mathematical Education, Seoul, Korea.

Chevallard, Y. (2015). On Didactic Transposition Theory: Some Introductory Notes.

Diaz-Eaton, C. (under review). The uncomfortable symbiosis of calculus and biology in biology undergraduate studies.

Leech, B. L. (2002). Asking Questions: Techniques for Semistructured Interviews. *PSOnline*, 665-668.

National Research Council. (2013). *The Mathematical Sciences in 2025*. National Academies Press. https://doi.org/10.17226/15269

Ragin, C. C. (2006). How to Lure Analytic Social Science Out of the Doldrums: Some Lessons from Comparative Research. *International Sociology*, *21*(5), 633-646. https://doi.org/10.1177/0268580906067834

Shadle, S. E., Marker, A., & Earl, B. (2017). Faculty drivers and barriers: laying the groundwork for undergraduate STEM education reform in academic departments. *Int J STEM Educ*, *4*(1), 8. https://doi.org/10.1186/s40594-017-0062-7

Steen, L. A. (2005). The 'Gift' of Mathematics in the Era of Biology. In *Math and Bio 2010: Linking Undergraduate Disciplines* (pp. 13-25). Mathematical Association of America.

Strømskag, H., & Chevallard, Y. (2024). Didactic transposition and the knowledge to be taught: towards an archeorganisation for concave/convex functions. *International Journal of Mathematical Education in Science and Technology*, 1-28. https://doi.org/10.1080/0020739x.2024.2305879

Winsløw, C., Barquero, B., De Vleeschouwer, M., & Hardy, N. (2014). An institutional approach to university mathematics education: from dual vector spaces to questioning the world. *Research in Mathematics Education*, *16*(2), 95-111. https://doi.org/10.1080/14794802.2014.918345