When We Say "Rigor" in Undergraduate Mathematics, What Do We Mean?

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One equity-oriented Networked Improvement Community (NIC) aimed to interrogate their use of "rigor" and its potential ramifications for the undergraduate mathematics program. In this preliminary report, we use critical discourse analysis on journal and interview data from this NIC to uncover how overloaded the word "rigor" is in mathematical spaces. These preliminary results reveal that rigor is conflated with difficulty, deep understanding, and other notions associated with teaching and learning mathematics. Findings suggest that there is a need to clarify and agree upon what we mean when we say "rigor" within our mathematical circles. We end with discussion and implications for practice.

Keywords: rigor, power, discourse analysis

"Mathematics is a language" is often an adage said by mathematics instructors to frame their course as one where students will be tasked (or challenged) to learn how to "speak math" (Wakefield, 2000, p. 272). As mathematicians, we may often obsess over the clarity or specificity of our phrasing and word choice when we speak or do mathematics, especially when writing proofs. This obsession perhaps demonstrates a desire to be understood exactly as we intended. Often there is an unsaid trust within mathematical circles: that when mathematicians talk, especially when using their mathematical register (e.g., Herbel-Eisenmann et al., 2015), our message is interpreted how we intended. Discourse beyond mathematical proofs, though, is not as clear-cut as mathematical logic. We should examine the assumptions we make when this unsaid trust spills into other discourses. Do we agree on what we mean when we use certain words? Many phrases, even in the RUME community like 'active learning,' are interpreted, conceptualized, and operationalized differently depending on who and where phrases are used (see Williams et al. (2022) for an analysis of various conceptualizations of 'active learning').

Amongst all the fascinatingly complex, intellectually-stimulating words to unpack, we highlight "rigor" in this study for the alarming ways rigor has been used to justify a gatekeeping logic in undergraduate mathematics (Hagman et al., accepted; Leyva et al., 2021) and the drastically different experiences with and operationalizations of "rigor" even amongst the authors. As we discuss in this report, there are also positive aspects to rigor; this report is not meant to vilify rigor. However, if mathematics departments and instructors place a large value on enacting rigor, then it is paramount that we, educators and instructors, interrogate the use of the word "rigor". Therefore, we ask the following research questions: When individuals in mathematical spaces use the word "rigor", what other words are used in relation to it and what do these other words reveal about the diverse ways in which rigor is conceptualized?

Literature Review

At the heart of this report is a desire for *transparency* in the sense of Gutiérrez (2013) where we interrogate taken-for-granted constructs and dominant notions in our mathematical culture, such as mathematical rigor. This does not mean we aim to construct a singular definition; as Riley (2017) argued, definitions of rigor are socially constituted and thus reflect a myriad perspectives on and experiences of rigor. As evidence of this myriad of conceptualizations, Draeger et al. (2013) proposed a model of academic rigor from the point of view of college faculty which posits meaningful content, active learning, higher-order thinking, and expectations comprise the "anatomy" of academic rigor. Later, Draeger et al. (2015) drew upon student data to develop a model of academic rigor from the perspective of students whose anatomy of rigor involved grades, difficulty, high workload, and interest.

As early as the 1920s, mathematicians like Pierpont (1928) have discussed that "absolute rigor"—mathematicians' goal as they scrutinized the foundations of mathematics in light of recent paradoxes at the time—is unattainable (p. 23). Almost a century later, our understanding of rigor has grown from a topic of concern to theoretical mathematicians to a central idea shaping classroom practices and education policy. Leyva et al. (2021), Marsellas (2021), and McNeill & Gresalfi (in press) interrogated rigor in relation to inclusion, care, and equity (or lack thereof) in the mathematics classroom. In policy making, matchmaking K-12 students into different course sequences comes with language that highlights the varying levels of "rigor" present in sequences (Oaks & Guiton, 1995; Herbel-Eisenmann et al., 2018).

Our goal for transparency around *rigor* is to reveal the ambiguity surrounding this word's meaning; when someone says rigor, they may not refer to the same things as others in the same spaces. We contribute to the emerging body of work on rigor and its continued presence in mathematics circles through uncovering the connections "rigor" has with other concepts and notions popular in mathematics instruction discourse. Our overall goal is to start conversations, so that when we say "rigor" in undergraduate mathematics, we can know what we mean.

Conceptual Framework

We conceptualize rigor as part of some mathematical cultures, where culture is broadly composed of the behaviors and values which are learned, shared, and exhibited by a group of people (Hagman et al., accepted; Yosso, 2005). In our view, rigor is a central cultural value within many mathematical cultures in postsecondary contexts. Rigor informs the expectations mathematics instructors have about their students' learning (Draeger et al., 2013) and what teaching practices are valued or necessary for the course to be mathematically rigorous (Leyva et al., 2021). It also informs the interpretation of workload and difficulty of the course for the students (Draeger et al., 2015). Broadly, we approach rigor as deeply involved in different aspects of mathematics teaching and learning—from the perspectives of faculty and students.

Data

This analysis is within a larger study that investigated Networked Improvement Communities (NICs) under ACT UP Math (EES #2201486). Through this project, education researchers with equity specializations partnered with three mathematics departments to initiate equity-oriented changes, supported by equity-oriented professional development, quantitative and qualitative data, and a broader network of support across the three departments. Hagman et al. (accepted) focused on one NIC at Kappa University and investigated how their NIC's goal of addressing rigor served as fertile ground for reckoning with tensions between the perceived value and operationalization impacts of rigor. The present report uses interview transcripts and journal

reflections to investigate the variation among meanings of rigor that were unearthed through our analysis of rigor as it relates to equitable instruction.

The Kappa NIC was composed of four faculty affiliated with the introductory mathematics program, two graduate students who had taught in this program, and two undergraduates who had taken classes within this program. Research participants opted into the NIC with interests of contributing to equity-oriented change and were all equally paid by ACT UP Math grant. The two NIC faculty co-leaders (Tiersa, Skylar) invited faculty (Anna, Senna) and purposefully selected out of an applicant pool the graduate (Parker, Kiara) and undergraduate students (Ricardo, Timothy). Pseudonyms were chosen by participants. The NIC identified as predominantly white women, and included one woman of color and two men.

Preliminary Nature

In Hagman et al. (accepted), we focused our argument in demonstrating how discourses around rigor reveal a culture of power (Delpit, 1998) in undergraduate mathematics education. This focus resulted in some analyzed data not being appropriate for that argument. We realized there is bountiful discussion to be had with focusing on the data centering only on how the word "rigor" itself was used, how other words served as a synonym or antonym for "rigor", and how "rigor" made distinctions in mathematics education. Therefore, we aim to share our current findings from Kappa, gather feedback from the RUME community towards a more complete manuscript using data from all three NICs, and spur motivation for others to investigate how "rigor" is used in their mathematical spaces.

Analysis

We selected a subset of data for analysis by locating instances of the word "rigor," taken together with relevant context. We leveraged critical discourse analysis—a method that examines how ideologies related to societal power relations shape discourse, both implicitly and explicitly (Fairclough, 1989, 2013). Our multi-stage analysis was methodologically aligned with reflexive thematic analysis, which generally includes six phases: familiarizing yourself with the dataset; coding; generating initial themes; developing and reviewing the themes; refining defining and naming the themes; and writing up the analysis (Braun et al., 2023). First, we read through all responses to the journal prompt "What does mathematics rigor mean to you?" as a basis for our preliminary codebook which included synonyms and antonyms as categories. The second stage consisted of developing our codebook using the subset of data which mentioned "rigor" and its surrounding context. We found "distinctions" as another category which referred to things that are described as distinct from rigor but could not necessarily be interpreted as an antonym. Each excerpt was coded by at least two authors, wherein interpretations of an excerpt were reconciled within partner-to-partner or full author group discussions. In the final stage, we focused on each category of synonyms, antonyms, and distinctions to find themes within the discourses.

Findings

We organize our findings by giving empirical illustrations of words used in relation to rigor: synonyms, antonyms, and distinctions. Distinctions refer to phrases used in relation to rigor not necessarily as similar or as in opposition. For synonyms and distinctions, similarly-used words or phrases were grouped together. There was only one instance for antonym.

Rigor as a Synonym

Deep/true understanding, justification, analytical/logical. When speaking about rigor, all

faculty in the NIC highlighted the educative benefits of rigor for students from their perspective. Senna, Skylar, and Tiersa each said that deep or true understanding is itself rigor or is required for rigor in their responses to "what does math rigor mean to you." For example, Senna shared, "I would say that something is mathematically rigorous if it requires a student to truly understand why something is true, and to be able to effectively communicate their ideas behind the concept." In her journal, Anna focused more on rigor's place in argumentation saying that rigor "means clearly constructed arguments toward a finding or conclusion. It means clearly constructed analytical/logical thought about the concepts over completing mechanics." As was in Draeger et al. (2013), faculty members of the NIC understood rigor through a positive lens of benefiting students in providing higher-order thinking or meaningful learning. This is not to say students are also not making these positive connections. For example, Ricardo shared,

> … [A] rigorous class would normally have expectations that students can extrapolate based on what they've learned, reaching conclusions that might go beyond what's being explicitly taught in lectures, which is a positive, challenging, and potentially rewarding experience.

However, overall, the faculty tended to speak about rigor more positively than students.

Difficult(y) and challenge. As for the students in the NIC, "difficult(y)" and "challenge" were used in relation to rigor. As a representative example, Timothy shared in his journal,

> I think that mathematical rigor is like difficulty and challenge in mathematics. And when I think of that rigor a lot of different things come to mind. I picture myself struggling in some of my math classes, not really understanding the content, um, feeling challenged, feeling like I don't, um, know what's going on, feeling like I'm not doing well.

As was in Draeger et al. (2015), students in the NIC understood rigor through a lens that they are being challenged and their experiences in class are made difficult for the mission of rigor.

Unnecessarily hard. Related to difficulty and challenge was the phrase "unnecessarily hard" which many NIC members picked up on from their department's ACT UP Math data. This had been particularly moving for the faculty members of the NIC who had not realized that mathematics at Kappa had such a negative reputation with the undergraduate students. During an interview, Skylar shared her dream with respect to the NIC's goal of transformation,

> My dream would be to change the narrative of "Kappa Math," so that it is no longer perceived as "unnecessarily hard." I think we are working towards this as a NIC, and that key steps will be interrogating our own perceptions of rigor and students as experts, as well as trying to better understand the source of this narrative for students so that we can address it.

We highlight here the importance of the descriptor "unnecessarily" and how Skylar (and also Tiersa) adopted the phrase from students. The sobering effect of "unnecessarily" gave a clear target for the NIC to strive for in their change efforts.

Clarity and precision. We found that students and faculty aligned in their perception that rigor is (or adds to) the clarity and precision that is required in discussing mathematics. Tiersa, for one, highlighted how rigor is linked with mathematical notation, stating that "I also feel that notation and precision in language are important." Kiara understood rigor as "being very focused on precision and careful detail checking." She added that, "it's all done meticulously and within a very set way of communicating in a set way of doing the things, um, that's like agreed upon." Again, we see that rigor is discussed in relation to clearly communicating ideas to others with a pronounced focus on precise notation as it relates to proof writing.

Rigor as an Antonym

Manipulating formulas. The idea that rigor is in diametric opposition from the manipulating formulas was the only instance of rigor being described using antonyms. When describing what

mathematical rigor meant to her, Skylar said it required students to "truly understand why something is true, and be able to effectively communicate their ideas behind the concept" and added that "**by contrast,** most of my students come from high school very adept at manipulating formulas." By contrasting manipulating formulas with rigor, Skylar echoed the associations mathematicians and faculty make with rigor and higher-order thinking (Draeger et al., 2013).

The two undergraduate students Timothy and Ricardo also echoed this diametric opposition. Timothy, in his response to "what does mathematics rigor mean to you," described experiences in high school as different from those at Kappa, "[rigor is] also **not like math in a high school** sense… where you're just given formulas and you have to plug them in and call it a day." For Ricardo, he understood that mathematical rigor is associated with the depth of content "**as opposed to** being explicitly told how to solve specific types of problems." Both students understood that the rigor they are experiencing at Kappa not only goes deeper but is disjointed from manipulating formulas. So, while faculty in the NIC began to understand the association of rigor with "unnecessarily hard" as a synonym, students also began to understand the association of rigor with "deep understanding" by viewing manipulating formulas as an antonym for it.

Distinctions

Excerpts coded as distinctions clearly described something as different from rigor, but not framed in opposition, i.e., an antonym. For example, the question of inclusivity in mathematics classroom was brought up in the context of rigor, "How can we keep rigor in the classroom, but in a way that doesn't feel exclusionary or daunting to the students?" (Senna). With regards to the NIC's change efforts, Skylar further discussed in an interview that she hoped to make "Kappa Math" a "kinder, gentler, more welcoming" department after hearing students' perspectives on rigor. And in her journals, Tiersa reflected on care and rigor (challenge),

> I think that before spending time with focus group data, I might have argued that "rigor" in our courses is part of what makes them rewarding. After looking at the data, I think that in rigorous courses with high expectations, students may need additional care and support in order to appreciate the challenge.

Thus, what we saw here are faculty members of the NIC making a distinction between rigor, inclusivity, and care—that both can exist simultaneously in a way that they hope would improve the undergraduate mathematics experience.

Implications & Discussion

As we aimed to demonstrate, "rigor" is an ambiguous word in our spaces. Draeger et al. (2013) and Draeger et al. (2015) already showed the different perspectives on rigor from a faculty perspective versus from a student perspective, and here we confirm this difference in the context of one undergraduate mathematics program. As demonstrated by this NIC, it is imperative for mathematical spaces to discuss these differing perspectives—wherein faculty in the NIC learned that their "Kappa Math" had a negative reputation while students in the NIC learned that rigor could be positively enacted and rewarding. Moreover, as in Hagman et al. (accepted), the interrogation of rigor can lead to an interrogation of power. In the case of Kappa, their interrogation of rigor led to student perspective and voices to be valued as their change and improvement efforts emphasized rewriting the "unnecessarily hard" narrative of Kappa. Thus, we urge the RUME community to interrogate our use, conceptualization, and operationalization of "rigor," so that when we say "rigor" in undergraduate mathematics, we know what we mean. So, our initiating discussion question for RUME is: *what does mathematics rigor mean to you?*

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