

Student Agency in the Collaborative Development of Novel Interactive Textbook Questions in Calculus and Linear Algebra

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In this preliminary report, we present results of examining the collaborative process used by teams of students, teachers, authors, and researchers to develop novel interactive questions for free, open calculus and linear algebra textbooks. Of concern is student agency as both a temporally-based social process and an ecological phenomenon as collaborators understand and construct the role of student. Discursive practices of the collaborators contributed to student positioning. In some cases, this resulted in an absence of student voice in the development of content, while in others, student agency yielded meaningful pedagogical changes in the development of questions.

Keywords: role construction, positionality, agency, undergraduate mathematics textbooks

In this time of rapid development of free, open curriculum materials, particularly textbooks, effective methods are needed for generating content that supports both student engagement and teacher pedagogical needs. Attending to the reality that teachers and students interpret and mediate the use of curriculum resources to produce an enacted curriculum different from the intended one (Rezat et al., 2021), the PROTEUS project positions Students and Teachers alongside textbook Authors and educational Researchers, in a “STAR team,” to propose, revise, and evaluate novel textbook questions that will engage a wide range of math learners. The success of this collaborative approach to generate content differently than the author would generate in isolation depends upon the various stakeholders drawing on their lived and practice-based experiences in mathematics classrooms. We are interested in the agency of the students in this unique collaboration fraught with differentials in content expertise and echoes of classroom power dynamics.

Theoretical Framing

Discursive moves can position students as agents in the context of the undergraduate mathematics classroom Mesa and Chang (2010). While the PROTEUS project uses collaboration outside the classroom, we acknowledge that participating teachers’ and students’ positioning is inevitably linked to social structures forged in the classroom. Thus, our conception of student agency in this context draws upon Emirbayer and Mische’s (1998) definition of agency as a process shaped by past, present, and projected (future) social engagement. While our design has attempted to position the students as experts of their own (past) experiences, the students navigated uncertain positioning and power dynamics as they worked alongside content experts who in other contexts hold positions of authority relative to students. We furthermore rely upon Biesta and Tedder’s (2007) expansion of agency as an ecological phenomenon wherein the interaction between an individual and their context-for-action leads to the transformation of environmental structures. Our design aspires to make equitable space for all stakeholders, students in particular, to shape the very processes engaged in by the team. Power differentials among stakeholders naturally make such a realization of student agency troublesome.

Methods

PreTeXt-Runestone: Open Textbooks Engaging Undergraduates in STEM (PROTEUS) is a collaborative project led by the four organizations, one of which serves as the independent evaluator. Together, these organizations coordinate the development of interactive questions for PreTeXt-authored textbooks (<https://pretextbook.org/>) in calculus and linear algebra, ensure their integration into the XXX online textbook platform, and evaluate their effectiveness in authentic classroom settings. In the content development stage, we seek to answer the question: *What are the strategies and heuristics used by teams of stakeholders to create content for three types of interactive questions (Reading Questions, Parsons Problems, Matching Exercises), for calculus and linear algebra that will engage students with learning the material?*, while in this report, we address the subquestion: *How is the role of the student constructed by the teams?*

This study uses qualitative research methods within a phenomenological paradigm to explore how students' roles are constructed and understood during collaboration (Ernest, 1997; Wood, 2017). To capture authentic experiences and perspectives, we employed bottom-up research methods that allow codes and themes to emerge directly from the data rather than imposing a priori categories (Blair, 2015). Specifically, we investigated students' utterances in team meetings to develop interactive questions to be added to four textbooks for calculus (*Active Calculus* [AC], Boelkins, 2022; *APEX Calculus* [APEX], Hartman et al., 2018) and linear algebra (*A First Course in Linear Algebra* [FCLA], Beezer, 2022; *Understanding Linear Algebra* [ULA], Austin, 2022).

Participants

Each STAR team was composed of one or two students, two teachers, the author of the respective participating textbook, and one or two researchers. The eight teachers were recruited from the authors' networks because of their familiarity with the textbooks and their experiences teaching with them. Three educational researchers, who specialize in mathematics education at the undergraduate level, participated in two STAR teams each. Researchers coordinated the team meetings and oversaw the design-based research process (Bakker & van Eerde, 2015) of iteratively developing and testing new textbook questions.

There were seven students across the teams; each was pursuing an undergraduate degree and had recently completed or was currently taking calculus and/or linear algebra. Students varied in terms of race and ethnicity identity (including Asian, Black or African American, White, or Other with two students identifying as Hispanic/Latinx), gender identity (4 women, 1 men, 2 agender), and prior mathematical experiences. Six of the students were between 20 and 21 years of age, with one over 23 years old. Several of the students had experience tutoring and some completed their teaching practicum concurrently with their participation in the study. They were primarily recruited through the teachers in our teams and through networks of instructors in the state of Michigan. Each student was assigned to either a linear algebra team or a calculus team depending on their background and the needs of the study. Students were asked to contribute their authentic perspectives on how learners engage with interactive questions, to represent the kinds of difficulties students encounter, and to provide feedback based on their experiences.

Outside of their activities developing textbook questions as a team, the STAR members contributed their feedback through surveys and interviews with the external evaluation team. All participants provided informed consent and were compensated for their participation.

Data Collection and Analysis

Data for this analysis were drawn from 51 STAR team meetings during the 2024-25 academic year, which were video recorded through Zoom and subsequently transcribed verbatim. To familiarize ourselves with the data, the first three authors read the transcripts multiple times to understand the meeting content and participants' discursive patterns. Using NVivo (<https://lumivero.com/products/nvivo/>) qualitative data analysis software and guided by the central research question of this study, we engaged in iterative cycles of independently coding transcripts and comparing results until we achieved strong agreement on the discursive segments that contributed to the construction of the student role. We concluded that it was important to distinguish between participants' references to the *General Population* of students and to the *STAR Students* themselves. Furthermore, we distinguished between STAR student *References* and *Utterances* in team dialogue, as utterances *made by* students, *directed to* students, or *about* students contributed in different ways to role construction. Finally, we attended to *Critical Events* (e.g., a decision to revise a question) in which to examine these categories of utterances and how they positioned students (Author & Colleague, 2010). We are in the early stages of textual and discourse analysis to identify how references to students were framed, how student voices appeared (or were absent), and how discursive practices shaped the construction of the student role within meetings. This approach has been chosen to move our analysis beyond surface-level categorization and examine the underlying meanings, assumptions, and institutional logics embedded in the conversations (Bloome et al., 2008).

Findings

Critical events were identified during team processes of norm setting, topic selection, proposing and vetting of questions, posing of conjectures about thinking the questions may elicit, and evaluating responses to the questions in both interview and classroom settings. As team members interacted in a variety of ways with students' articulation of difficulties, representations of textbook use, and descriptions of personal classroom experiences, students were positioned by both discourse and team processes. By authentically representing their own experiences, students surfaced conceptions that influenced team decisions about question design.

Positioning Students Through Discourse

Utterances contribute to the constructed position of participants in dialogue (Author & Colleague, 2010), and as the teams held regular meetings, utterances from each of the four STAR roles positioned students and delineated the (often implicit) scope of their expected contributions. Student utterances were predominantly embedded among those of other roles, such as teachers and authors, and occurred most often as responses. Thus, teacher, author, and researcher utterances shaped the discursive environment in which student utterances were elicited, or into which they were injected. For example:

Author: I think there, there are the basic [questions] about understanding how composition works that have no reading at all associated with them. And then the, you know, read the chain rule formula, and then try to use it, and then kind of the more synthetic ones at the end, where you're trying to put a few things together.

Student: I think still having a reading section like the very first section where it discusses a little bit about composition, might be useful for those questions, instead of just having no reading section.

Researcher: I agree, I agree.

Author: So what, what it would be? What do you think it would be? Let me see.

Researcher: ...Do we want to have two batteries of questions, one that is contextual and the other is more related to the, to the actual symbols? That might be a good way.

Student: I like that idea. It's first having them look at it as symbols and then translating it into an actual context. I think it's, it's a lot more difficult to apply it to a, to a full context. So I do think it's important that we actually add a question that speaks on that.

This sequence from a calculus team highlights the “transactional” achievement of agency (Biesta & Tedder, 2007). Both author and researcher position the student as authoritative by being solicitous of and responsive to the student’s experience with learning the chain rule topic.

Student Agency Through Team Processes

Participation in structured team processes both supported and evidenced student agency. All team members, students included, took turns taking meeting notes, allowing the documentation of team discussions and decisions to reflect the differing focus, perspective, understanding, and voice of the team member whose turn it was. In this excerpt from a linear algebra team, the student offers a clarification of terminology as the team makes decisions about their development process:

I defined what a cycle is right here, but if somebody would like to change what the definition of a cycle is, from—I was explaining that one cycle is that, like, it’s written or edited, and then it has gone through the, through the interviews. That is one cycle. And then, and if we edit it again, and then it goes through a cycle, or an interview, that’s a second cycle.

The responsibility of note taking served to elicit a glimpse of the STAR student’s perceived capacity to take action (“I defined”) and their sense of immersion as a team member (“we edit”), empowering them to shape the team’s operational environment (Biesta and Tedder, 2007).

Student Agency in Surfacing Conceptions

One of the linear algebra teams had a single student member, (alias) Melody. During the team’s work to develop novel questions for the eigenvector section of their textbook, Melody proposed the short-answer question: *Describe the relationship between Eigenvectors and Eigenvalues*. A practice promoted by the researchers within each team was to listen to the question proposer’s rationale for their proposed content. Melody explained their inspiration:

I know there’s a formula, and I can’t name it off the top of my head right now, for the relationship between eigenvectors and eigenvalues. But I sometimes think that it’s forgotten, the why behind what is happening. And it’s just memorized what you have to do.

Their proposed question differed categorically from those of the author and teachers, each of whose proposed questions in the same topic probed a relationship between an eigenvector and a *matrix*. Furthermore, their verbal justification brought to the team’s attention the student experience of fixating on a process without internalizing a relationship. The author on the team suggested that Melody’s proposed question begin the question set for the textbook section. After team discussion and revision, the altered form of the question read: *What does it mean for a vector v to be an eigenvector of a matrix A ?* Notably, all members of the team continued to refer to this fundamentally altered question as “Melody’s question,” positioning the student as agent not only of surfacing student conceptions, but as an agent influencing the textbook content that would address those conceptions.

Agency as Social and Ecological Phenomenon

Evidence of the highly contextual nature of the constructed student roles came at the very end of the 2024-25 academic year, when the four separate STAR teams joined together in a week-long workshop in Ann Arbor. The students' roles, previously constructed in the liminal environment of the Zoom meeting and delineated primarily through discourse (or its absence), were given an opportunity for physical embodiment, such as spatial positioning of students with respect to other members of the teams, behaviors and mannerisms accompanying the distribution of tasks, and differentials in the enacted use of students' time and resources and content experts' time and resources. Perhaps most importantly, interpersonal norms were drastically rewritten in the in-person setting. Marked changes in student roles emerged during the workshop, underscoring both the sensitivity of role construction to environmental factors (Biesta and Tedder, 2007) as well as issuing from activities of social engagement (Emirbayer and Mische, 1998).

Implications and Future Work

While the findings presented here are preliminary in nature, early stages of analysis provide evidence that discursive practices positioned students differently across teams. Some students were positioned so that they exerted agency in developing novel content and providing feedback to questions that others produced, whereas others were positioned so that they only provided feedback for questions that others produced. In this latter situation, it appeared as if students' agency was made invisible relative to how the teams were expected to operate.

Limitations of the study include the less consistent attendance of STAR team meetings by students than by other members, inevitably diluting student voice in the process and impacting the construction of students' roles. Furthermore, one might argue that the first act of role construction was the decision by the research team to associate members of the STAR teams with their experience-wise labels (Student, Teacher, Author, Researcher) as they were emplaced in their team environments. However, beyond this structural necessity, specific roles, in the functional sense, were not imposed on members of the teams. The design choice to ask students (relative content novices) to participate as equals alongside authors, teachers, and researchers rather than to replicate classroom positioning (Wagner & Herbel-Eisenmann, 2008) was, to varying extent, a source of tension in the teams.

Our larger investigation of question development heuristics will be informed by what we learn about how teams used student expertise and it will allow us to see how the work in the teams helped subvert the social construction of institutionally-constructed roles of teachers and students. We suspect that in this setting the students' perception of the textbook's authority (Wagner & Herbel-Eisenmann, 2014) is being challenged as well.

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