A Covariational Lens for Proofwriting in Undergraduate Mathematics

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Literature Review

We examine how undergraduate students navigate proof writing through two complementary images of activity: chunky assembly and smooth development, and how students transition between these modes while producing a final proof write-up. While the chunky/smooth framework was originally developed to characterize students' images of change in covariational reasoning, Castillo-Garsow et al. (2013) emphasize that these distinctions also describe how mathematical activity itself is organized, independent of domain.

In this study, we adapt the chunky/smooth distinction as an analytic lens for describing modes of engagement during proof construction. Proof writing often involves alternating between discrete exploratory episodes without attention to the larger argument (chunky) and more continuous episodes in which students attend to the overall logical throughline (smooth). Existing literature documents related distinctions, such as exploration versus validation (Weber, 2008) and dimensions of proof comprehension and coherence (Mejía-Ramos et al., 2012), but offers limited language for characterizing how students move between these modes during authentic proof writing.

By framing proof activity in terms of chunky and smooth episodes, we aim to provide a descriptive vocabulary that complements existing proof frameworks and enables fine-grained analysis of how students organize, refine, and translate exploratory work into coherent written proofs.

Research Questions

RQ1. What episode types (e.g., exploratory "chunky," throughline "smooth," resource-use) arise during students' proof writing, and with what typical sequences?

RQ2. What triggers students' commitment from exploratory "chunky" work to a sustained "smooth" throughline, and what triggers stalls or switches?

RQ3. How do ideas generated during exploration survive (or not) into the final written proof, and how does resource-use (text/web/notes) inform that translation?

Study Design

We conduct exploratory think-aloud interviews (40–60 minutes) with undergraduates enrolled in proof-based mathematics courses. Participants work through two multi-route proof tasks, selected strategically from Hammack's *Book of Proof* (2018), with access to resources as in an authentic homework setting. Data sources include audio recordings, scratch work, and final written proofs.

Episodes will be coded to identify chunky and smooth modes of activity, transitions between them, and their alignment with features of the final proof write-up. Following the interviews, participants complete Likert scale items probing their beliefs about proof writing and what constitutes a "good" proof, allowing us to situate observed activity patterns within students' stated views of proof.

We are beginning the interview phase and anticipate initiating episode coding in January.

References

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