Chunky or Smooth? Students' Covariational Reasoning When Analyzing Phase Plane Trajectories

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Over the last thirty years there has been a trend in the teaching of differential equations to move away from analytic techniques and to focus on graphical and numerical methods (e.g., Rasmussen, 2001). In parallel to this movement, mathematical modeling has gained a lot of importance in biology and the life sciences. Leading professional organizations have called for a reform of the mathematics instruction of biology students in order to include the study of modeling and dynamical systems (e.g., American Association for the Advancement of Science, 2009). The notions of time series and phase plane trajectories are central to the study of modeling and dynamical systems. A pivotal skill is the ability to navigate between these two types of representation of solutions to differential equations.

Previous research has analyzed students' conceptions of solutions to differential equations as well as strategies used to sketch their representations (e.g., Rasmussen, 2001; Trigueros, 2001; Bennoun, 2024). In particular, Bennoun (2024) used the covariational reasoning framework developed by Thompson and Carlson (2017) to analyze students' reasoning when sketching time series and trajectories. Bennoun (2024) observed that students reasoned in majority at the smooth continuous or chunky continuous levels. An important open question is: How do students explain why they use either of these levels when working with time series and trajectories?

To investigate this question, we conducted ten think-aloud interviews with students who had completed a mathematics course focusing on modeling biological systems. Students were given a textbook exercise showing the (smooth) time series of an epidemiological model that depicted the evolution of healthy and infected populations. Then, they were presented with two possible solutions, one chunky continuous trajectory (i.e., with clear corners) and one smooth continuous trajectory. They were asked which one of the two trajectories best corresponded to the given time series and why. We conducted a reflexive thematic analysis (Braun & Clarke, 2006) of the interviews focusing on the reasons the students used to justify their choice as well as the level of covariational reasoning.

Our preliminary findings reveal that while all students picked the smooth trajectory as being the one that best corresponded to the given time series, their reasons varied. Several students reasoned that since the time series given was smooth, the associated trajectory should also be smooth. For example, one student explained that "because these [two functions on the time series] are curved graphs, […] the actual [trajectory] would also be a curved graph." He added that if the functions on the time series were chunky, then the trajectory would also be chunky. Other participants mentioned Euler's method, that they had learned to approximate trajectories, and how a smaller time step means that the trajectory is more accurate. They explained that as one decreases the time step the trajectories look more and more curvy and that therefore the smooth trajectory was more accurate. Some students used arguments relying on more superficial features such as the smooth graph having axes that are easier to read or the chunky graph feeling confusing to them. On the poster, we will describe these themes in more depth as well as discuss implications for practice.

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