PSTs' Symbolic Forms During Mathematizing: The Case of Mia and Ava and the Laundry Task

Afaf Bahajj
Utah State University

Sindura Kularajan
Utah State University

This study investigates the symbolic forms pre-service teachers (PSTs) drew upon while mathematizing a real-world laundry task. Drawing from Sherin's (2001) framework of symbolic forms, we analyzed how two PSTs, Mia and Ava, coordinated mathematical symbol templates with conceptual reasoning to model when clothes should be washed based on bacterial load thresholds. Our analysis identifies both singleton forms: threshold and exponentially growing, and networks of forms: rate, contributing factors, and exponential growth with a threshold. We end with implications for research on student cognition during modeling.

Keywords: symbolic forms, mathematical modeling, mathematizing, pre-service teachers, student thinking

Introduction

Mathematical modeling entails using mathematical structures and relationships to represent real-world situations. As such, modelers are required to engage in certain processes to have successfully modelled (Niss, 2010). Within the mathematical modeling (hereafter referred to as modeling) education research literature, the process of using mathematical structures to formulate a mathematical model for real-world situations is referred to as *mathematizing*. In essence, mathematizing is a process by which an individual uses mathematics to make sense of reality (Freudenthal, 1968). While there exist various modes of mathematical representations that can be used during mathematization, such as graphs, diagrams, and tables (Duval, 2006), modeling from first principles often requires modelers to represent real-world situations using symbols, expressions, and equations. When a modeler models from first principles, the goal is to align a mathematical representation with a scientific interpretation of the situation (Greca & Moreira, 2002). As mathematizing has been documented as notoriously challenging for learners of all levels (Stillman et al., 2010; Roan, 2024), many research programs have developed around the central goal of supporting students' mathematization. Examples of these research programs include theoretical frameworks for what it means to have successfully mathematized (Niss, 2010), how to support students through scaffolding to mathematize (e.g., Stender & Kaiser, 2015), identifying the blockages students' experience during mathematization (Klock & Siller, 2020; Schaap et al., 2011), and cognitive constructivist perspectives to construct accounts of students thinking around mathematizing, which is relevant to our study.

The primary goals of research programs that adopt a cognitive constructivist perspective to study students' mathematizing are to (a) build asset-based accounts of how and why a student did a certain mathematizing move, and (b) use those student-centric accounts to guide instructional design and intervention. In this study, we aim to contribute to the knowledge on goal (a). To do this, we adapt Symbolic Forms as a theoretical construct that is rooted in Knowledge-in-pieces (KiP) (diSessa, 1993). Particularly, in this paper, we contribute to the literature surrounding modeling and cognitive constructivist theories by documenting the symbolic forms our participants, pre-service teachers (PSTs), drew upon as they constructed a mathematical model for the Laundry Task scenario.

Theoretical and Empirical Background

Mathematizing

We situate our work within the cognitive perspective of modeling (Kaiser, 2017), where the focus is on investigating modelers' cognitive processes during model construction. Within this perspective, mathematizing is typically considered as transforming a real model (i.e., a modeler's idealized conception of the situation) into a mathematical model (i.e., a modeler's organization of their conception of the real-world in mathematical terms). While the process of modeling includes other competencies during modeling, such as understanding the situation, simplifying the situation by making assumptions, validating the process and results, and interpreting the results, the model construction process can be clustered into two major phases: pre-mathematization and post-mathematization (Niss, 2010). The compartmentalization of the modeling process, as before and after the model is formulated, implies the centrality of mathematization to modeling. For successful mathematization, it is suggested, modelers have to engage in both cognitive, meta-cognitive, and anticipatory meta-cognitive activities (Galbraith, 2017; Niss, 2010; Stillman, 2011). Yet, despite this centrality of mathematization to modeling, the process of mathematization has only been investigated to the extent of the kinds of mathematical approaches modeler's use during mathematization (e.g., Anhalt et al., 2018; Larson, 2013), the cognitive blockages students' experience during mathematization (e.g., Klock & Siller, 2020), and the modes of representations through which modelers externalize their mental representations (e.g., Duval, 2006). While this research illuminates important facets of students' mathematization activities, it does not attend to the cognitive resources (Hammer, 2000) that students draw upon during mathematization. Understanding these resources is critical for identifying instructional interventions that go beyond instructing students what to do to diagnosing students' blockages and prescribing ways where students can overcome the blockages to progress in their modeling. In this study, we focus on the cognitive resources PSTs drew upon to mathematize the real-world situation through equations.

Symbolic Forms

The fundamental premise of the KiP framework is that one's knowledge is a complex network consisting of knowledge ontologies and knowledge systems that work in coordination to make sense of a given situation (Swanson, 2025). One example of knowledge ontology—fine-grained knowledge elements that get activated when a situation is presented—that is particularly useful for mathematizing, is symbolic forms (Sherin, 2001). Symbolic forms build on the idea that scientific construction is an epistemic game in which the players' (or learners' or modelers') moves are guided by pre-existing knowledge templates (Collins & Ferguson, 1993). Thus, symbolic forms constitute both a symbol template and a conceptual schema. These constructs have been neatly illustrated by Jones (2013) with the following example. Consider the equation $V = V_0 + \Delta V$. This equation has the symbol template ■ = ■ + ■, and the conceptual schema associated with this template is total equals based plus change. At the same time, consider the equations $T = P + K$, where T represents the total energy of an object and P and K represent potential and kinetic energy of an object, respectively. While this equation also has the symbol template ■ = ■ + ■, it has the conceptual schema parts of a whole. In his work with students working on physics concepts, Sherin (2001) documented the various symbolic forms his students used while constructing equations to represent physical situations.

Few scholars have used symbolic forms and an analytical tool to construct explanatory models of students' mathematizing activities. For example, Roan and colleagues (Roan, 2024

Roan et al., 2025) used symbolic forms together with quantitative reasoning (Thompson, 2011) to explain the many meanings of multiplication students attribute to $\blacksquare \times \blacksquare$, while using it to model real-world scenarios such as interaction between species. The underlying premise is that symbolic forms blended with quantitative reasoning (Brahmia & Thomson, 2025; Roan, 2024) are powerful tools to go beyond describing what the students did to explaining why the students did what they did, providing prescriptive capabilities for educator intervention (Czocher, 2025). With this hope, in particular, we address the following research question: *What symbolic forms did the PSTs draw upon as they mathematized the laundry task?*

Methods

Participants and Setting

The participants were recruited from a mathematics content course required for K-8 pre-service teachers at a large public university in the western United States. All participants had completed prerequisite coursework in college algebra. A total of four students participated, working in two pairs. The research team met with each pair for 60 to 90 minutes for a total of ten sessions. Sessions were held in a quiet room on campus and were audio and video-recorded. The PSTs were given the freedom to conceive the task however they wished and use any mathematics they thought best represented their conception of the task. The researcher's role was to ask clarifying questions and probe the PSTs' thinking. In this paper, we focus on the symbolic forms of one pair of PSTs' (pseudonyms Mia and Ava) working on the Laundry Task. The Laundry Task is presented in Table 1. Mia and Ava's work on the laundry task was reminiscent of what Collins and Ferguson (1993) would call an epistemic game. First, the PSTs conceived that bacterial growth could either occur linearly or exponentially. Next, they modelled each linear and exponential growth, abiding by the "rules" of how each growth occurs. For this reason, we chose to explore Mia and Ava's work, the Laundry Task, for further analysis. In this paper, we report on a portion of those findings.

Table 1. The Laundry Task (abridged version). Task description taken directly from Smith (2023).

On January 10, 2023, TikTok user Allison Delperdang sparked a lively debate when she shared a video about wearing the same pajamas multiple times before washing them. The video garnered millions of views and countless comments, with some agreeing that frequent washing isn't necessary, while others argued it was unhygienic. Experts weighed in, explaining that how often clothes should be washed depends on several factors, including personal hygiene habits, activity levels, and the type of clothing. Dermatologist Dr. Anthony Rossi emphasized that our cultural norms often lead to "over-washing," even though not all clothes need frequent cleaning. At the same time, wearing hygiene clothes can cause skin infections due to bacterial overgrowth in the unwashed clothes. Research indicates that bacterial loads exceeding 1000 colony-forming units per square centimeter (CFU/cm^2) can pose a risk, particularly for pathogenic bacteria like Staphylococcus aureus. Develop a mathematical model for determining how long clothing can be safely worn before washing?

Data Analysis

The data were transcribed and analyzed using MAXQDA, a qualitative data analysis software, to support the identification of PSTs' symbolic forms. Our analysis proceeded in three stages. First, we used Sherin's (2001) framework on symbolic forms as a foundation to develop a shared criterion for identifying symbolic templates and their coordination with conceptual reasoning. We conducted an open reading of the transcripts, supplemented by repeated viewing of video recordings to become familiar with the data while attending to moments where PSTs expressed their reasoning through both inscriptions and verbal utterances. During this round, we used Sherin's framework (see Sherin, 2001, p. 532–537) to identify any symbolic forms. For this analysis, we sought evidence of symbolic forms through (a) symbolic templates that were explicated either verbally or through notations and (b) PST's justification for their proposed templates. For example, when Mia and Ava wrote the expression $(c + h + a + s)^{\#days}$ as their model for exponentially growing bacterial load, we coded this as the symbol template $\blacksquare^{\blacksquare}$. When the interviewer probed for justification for their representation, Ava stated, "I don't know if you need to put it to a power or something…I feel like when I've done exponential kinds of problems in the past…I mean I think it makes sense in the fact that as more days there's more growth." We took this as evidence of a growing exponentially conceptual schema. This is because the PSTs reasoned that the bacterial load grows "with some parameter that appears raised to a positive power in the exponent of the exponential" (Sherin, 2001, p. 536). Both authors independently coded the transcripts for such instances, marking segments where symbolic forms appeared and assigning them to one of Sherin's categories (e.g., *parts-of-a-whole*, *canceling out, exponent*). If there were any forms were not part of Sherin's framework, we created a new code based on the inferred PSTs' conceptual reasoning. We then met regularly to compare interpretations, discuss discrepancies, and refine the coding scheme together. Finally, the authors applied the refined framework to the entire video to ensure consistency.

Findings: PSTs' Symbolic Forms

Our analysis revealed that PSTs used a variety of forms as they worked on the Laundry Task. These forms fell into two categories: *Singleton Forms,* which involved the use of a single symbolic template tied to a conceptual schema, and *Network of Forms,* which reflected the coordination of multiple symbolic templates to organize the situation. Within the singleton forms, we identified the threshold and exponentially growing forms. Within the network of forms, we identified the rate form, the contributing factors form, and the exponential growth with a threshold form. Table 2 summarizes these symbolic forms, their associated templates, and the conceptual schemas that PSTs drew upon. Due to space constraints, we elaborate on the threshold, rate, and contributing factors in symbolic forms.

Table 2. Overview of PSTs' symbolic forms

Form	Template	Conceptual Schema
	Singleton Forms	
Threshold	$\blacksquare < \blacksquare$	One side cannot exceed the other
Exponentially growing	$\blacksquare^{\blacksquare}$	The amount is growing exponentially with time
	Network of Forms	
Rate	$\blacksquare / \blacksquare = \blacksquare$	Instantiating a rate
Contributing Factors	$\blacksquare = \blacksquare + \blacksquare + \blacksquare$	Factors that contribute to a whole
Exponential growth with a threshold	$\blacksquare \leq \blacksquare^{\blacksquare}$	The amount that is growing exponentially with time cannot exceed the other

The Threshold Form

The threshold symbolic form consists of the symbol template ■ < ■ and the conceptual schema that one situational attribute cannot exceed the measure of another. For example, Mia and Ava reasoned that if the bacterial load reaches 1000 CFU/cm^2, then the clothes must be washed. The following conversation between Mia, Ava, and the interviewer illustrates this form.

Mia: I feel like one side of the equation would need to equal this, though [referring to the 1000 CFU]. Because that's a max that you can go.

Ava: Yeah, because that's a risk.

Mia: But like something equals 1000 colony for me. I don't know. *Interviewer:* What did you mean by that's the max we could go?

Mia: Because it says any bacterial loads exceeding this amount need to be washed basically, because they pose a risk.

Here, Mia and Ava reasoned that once the bacterial load reaches or exceeds 1000 CFU/cm^2, the clothes need to be washed, implying the bacterial loads on clothes must be less than 1000 CFU/cm^2, for the clothes to be safely worn. They initially mathematized the situation as $clothes\ <\ 1000\ CFU/cm^2$. Once the interviewer probed on what the PSTs meant by "clothes", Mia and Ava modified their model as shown in Figure 1. Mia and Ava's expressions, "$clothes$ < 1000 CFU/cm^2" and "$bacterial\ load\ on\ clothes/cm^2$ < 1000 CFU/cm^2", and their reasonings reflect their organization of the Laundry Task scenario by drawing upon the threshold form.

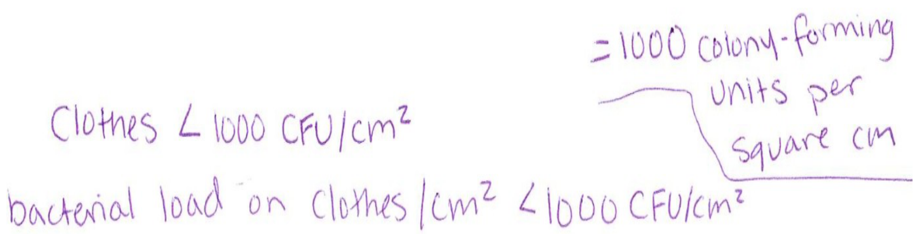

Figure 1. Mia and Ava's threshold symbolic form

The Contributing Terms Forms

Contributing terms symbolic forms consist of the symbol template ■ = ■ + ■ + ■and the conceptual schema that distinct quantities (left side of the equation) contribute to the whole (right side of the equation). This form is considered a network of symbolic forms because it integrates two key components of Sherin's forms: *parts-of-a-whole* form, which represents additive components (■ + ■ + ■) and the *balancing* form (■ = ■) which represents that both sides of the equation should be equal.

Mia and Ava demonstrated this symbolic form as they mathematized the factors that contributed to the rate of bacterial growth on specific clothing pieces. First, they began by identifying specific factors that influence bacterial load, such as type of clothing, personal hygiene, level of activity, and sickness (Figure 2). Mia justified the inclusion of sickness as a contributing factor to bacterial growth as "I just feel like when you're sick, it's like you're trying to get rid of all the germs, so you're trying to wash your clothes more." Next, they conceived that the rate of growth of bacteria per day is composed of the rate of growth of bacteria per day due to the contributing factors. Building on this reasoning about what might cause the bacterial load on specific clothing to reach the threshold of 1000 CFU/cm^2, Mia and Ava mathematized the situation as shown in Figure 2. In this form, the overall rate of growth of bacteria is a whole composed of additive parts, for example, $rate\ of\ bacterial\ growth\ per\ day =$ $rate\ of\ bacterial\ growth\ due\ to\ Factor\ A\ +$ $rate\ of\ bacterial\ grwoth\ due\ to\ Factor\ B\ +$ $rate\ of\ \ bacterial\ growth\ due\ to\ Factor\ C.$

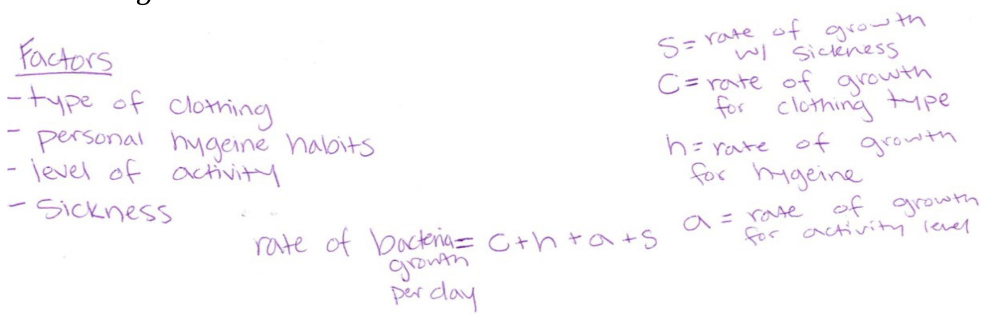

Figure 2. Contributing terms listed by Mia and Ava

The Rate Forms

The PSTs mathematized the number of days specific clothing could be worn as below:
$1000\ \div rate\ of\ bacterial\ growth\ (measured\ in\ days) =\ days\ of\ wear$ (1)

We identified this mathematization as PSTs drawing upon their rate form. A rate symbolic form consists of the symbol template $\blacksquare/\blacksquare = \blacksquare$ and a conceptual schema instantiating a rate. We identified this as a network of forms as PSTs combined multiple symbolic forms simultaneously: (i) canceling form with symbol template $\blacksquare/\blacksquare$ and conceptual schema that identical symbols appear in the numerator and denominator, and which represents the rate relationship, and (ii) balancing form with symbol template ■ = ■ and conceptual schema both sides of the equation are equal. We interpret that the PSTs drew on both these symbolic forms to mathematize the number of days one could safely wear a clothing piece before it needs to be washed. We elaborate on this interpretation below.

For the PSTs', one side of the equation was number of days. We make this conclusion due to the task prompt and the models PSTs constructed for this task. So, for the PSTs, the game was: *if one side of the equation was number of days, what should the other side be so that both sides are equal*. To attain this, as shown in Figure 3(a), the PSTs simplified the scenario by assuming that the total rate of bacterial growth was 100 CFU/cm^2 per day. Next, drawing on their balancing form that two sides of an equation should equal, the PSTs sought out what operation would produce number of days to balance the other side of the equation. Given that, 1000 CFU/cm^2 was the threshold bacterial load before the clothes required washing, to balance both sides of the equations to number of days, the PSTs drew on their canceling form and divided the threshold bacterial load by the bacterial growth per day to effectively cancel identical symbols. They verified their actions through considering different values for bacterial growth per day, such as 200 CFU/cm^2 (see Figure 3(b)). Finally, as shown in Figure 3Figure 3(c), the PSTs generalized their reasoning into equation ($1000\ \div rate\ of\ bacterial\ growth\ (measured\ in\ days) =$ $days\ of\ wear$ (*1*).

We nominalized this form as a rate form because Mia and Ava purposefully operated on the threshold bacterial load, 1000 CFU/cm^2, and the bacterial growth per day, for instance 100 CFU/cm^2 per day, to yield the number of days. This operation mirrors Thompson's (1994) notion of the quantitative operation instantiation rate. In a similar manner, we hypothesize, Mia and Ava constructed to a network of symbolic forms to mathematize their conception of the laundry scenario.

(a)	(b)
100 CFU/cm^2/day $1000 \div 100 = 10$ days	200 CFU/cm^2/day $1000 \div 200 = 5$ days of wear

1000 ÷ rate of bacterial growth (days) = days of wear

(c)

Figure 3. Network of symbolic form examples

Discussion

In this study, we found that our PSTs drew on symbolic forms in multiple ways, alone or in coordination as a network of forms. Our study is a first step towards building a repertoire of modelers' constructive resources (Sherin, 2000)—cognitive resources used by modelers as they mathematize real-world scenarios such as the Laundry Task. As mathematizing is not limited only to the construction of equations, our study's focus does not provide a complete picture of students' constructive resources for modeling. Researchers interested in modeling-centric cognition should draw on other representational forms, such as graphical forms (Rodriguez & Jones, 2024), to understand modelers' representational activities during modeling. Building such fine-grained accounts of modelers' knowledge enables educators and researchers to (a) diagnose the blockages modelers may experience during modeling (Roan, 2024), (b) provide modeling support that attends to and responds to modelers' cognition, and (c) develop modeling task characteristics from the perspective of the students (Roan et al., 2025). This repertoire of students' thinking can be used in modeling specific professional development programs to support teachers in designing and implementing modeling tasks that are guided by modelers' cognition (Franke & Kazemi, 2001).

Investigating students' symbolic forms and other constructive resources will pave the way to conceptualize modeling as a goal oriented, epistemic game, where students have the autonomy to choose their next move based on the resources available to them. In this view, modelers' actions are driven by "*I need to get X, what can I do to get X?*" For example, in the threshold form we illustrated, Mia and Ava's modeling actions were guided by the question, "*how can I mathematically represent that one side of an expression cannot exceed* 1000 CFU/cm^2." Similarly, in the rate form, Mia and Ava's actions were driven by "*what operations can perform to make one side of the equation the same as the other, which is number of days of wear*." Thus, adapting this perspective implies that facilitating students' modeling activities entails creating opportunities for students to activate certain resources, as well as designing learning ecologies that occasion the construction of certain resources which can later be activated for model formulation. Students' cognition during modeling is idiosyncratic and complex; we contend that using one theoretical framing alone is inadequate in capturing the intricacies in modelers' mathematics. Therefore, we call for more work in connecting multiple theoretical framings (Radford, 2008) that work synergistically to provide rich descriptions of modelers' cognition.

References

Anhalt, C. O., Cortez, R., & Bennet, A. B. (2018). The emergence of mathematical modeling competencies: An investigation of prospective secondary mathematics teachers. *Mathematical Thinking and Learning*, 20(3), 202–221. https://doi.org/10.1080/10986065.2018.1474532

Brahmia, S. W., & Thompson, P. W. (2025). Modeling with quantities in calculus and physics: A conceptual framework of the fundamental theorem. *arXiv* preprint *arXiv:2501.04219*. https://doi.org/10.48550/arXiv.2501.04219

Collins, A., & Ferguson, W. (1993). Epistemic forms and epistemic games: Structures and strategies to guide inquiry. *Educational psychologist*, 28(1), 25-42. https://doi.org/10.1207/s15326985ep2801_3

Czocher, J. A. (in press). Informing scaffolding for stem undergraduates' modelling with explanatory accounts of their reasoning. In T. Ikeda, A. Saeki, V. Geiger, G. Kaiser (Eds.). *International horizons in mathematics modelling education.* Springer

diSessa, A. A. (1993). Toward an epistemology of physics. *Cognition and Instruction*, 10(2-3), 105–225. https://doi.org/10.1080/07370008.1985.9649008

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61, 103–131. https://doi.org/10.1007/s10649-006-0400-z

Franke, M. L., & Kazemi, E. (2001). Learning to teach mathematics: Focus on student thinking. *Theory into practice*, 40(2), 102-109. https://doi.org/10.1207/s15430421tip4002_4

Freudenthal, H. (1968). Why to Teach Mathematics So as to Be Useful. *Educational Studies in Mathematics*, 1(1/2), 3–8. https://doi-org.dist.lib.usu.edu/10.1007/BF00426224

Galbraith, P., Stillman, G. A., & Brown, J. P. (2017). The primacy of 'noticing': A key to successful modelling. In G. Stillman, W. Blum, & G. Kaiser (Eds), *Mathematical modelling and applications: Crossing and researching boundaries in mathematics education* (pp. 83-94). Springer. https://doi.org/10.1007/978-3-319-62968-1_7

Greca, I. M., & Moreira, M. A. (2002). Mental, Physical, and Mathematical Models in the Teaching and Learning of Physics. *Science Education*, 86(1), 106-121. https://doi.org/10.1002/sce.10013

Hammer, D. (2000). Student resources for learning introductory physics. *American journal of physics*, 68(S1), S52–S59. https://doi.org/10.1119/1.19520

Jones, S. R. (2013). Understanding the integral: Students' symbolic forms. *The Journal of Mathematical Behavior*, 32(2), 122-141. https://doi.org/10.1016/j.jmathb.2012.12.004

Kaiser, G. (2017). The teaching and learning of mathematical modeling. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 267–291). The National Council of Teachers of Mathematics, Inc.

Klock, H., & Siller, H. S. (2020). A time-based measurement of the intensity of difficulties in the modelling process. In G. A Stillman, G. Kaiser, & C. E. Lampen (Eds), *Mathematical modelling education and sense-making* (pp. 163–173). Springer. https://doi.org/10.1007/978-3-030-37673-4_15

Larson, C. (2013). Modeling and quantitative reasoning: The summer jobs problem. In R. Lesh, P. L. Galbraith, C. R. Haines, A. Hurford (Eds.), *Modeling students' mathematical modeling competencies* (pp. 111–118). Springer https://doi-org.dist.lib.usu.edu/10.1007/978-1-4419-0561-1_9

Niss, M. (2010). Modeling a crucial aspect of students' mathematical modeling. In R. Lesh, P. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modelling students' mathematical competencies* (pp. 43–59). Springer. https://doi-org.dist.lib.usu.edu/10.1007/978-1-4419-0561-1_4
Radford, L. (2008). Connecting theories in mathematics education: Challenges and possibilities. *ZDM–Mathematics Education*, *40*(2), 317–327. https://doi.org/10.1007/s11858-008-0090-3
Roan, E. (2024). Explaining mathematical expressions undergraduate STEM majors construct to model dynamic scenarios [Unpublished Doctoral Dissertation]. Texas State University.
Roan, E., Kularajan, S., & Czocher, J. A. (2025). Reconsidering task classification systems for situations that can be modelled with multiplication. *ZDM – Mathematics Education, 57,* 411–424. https://doi.org/10.1007/s11858-025-01657-4
Rodriguez, J. M. G., & Jones, S. R. (2024). How students understand graphical patterns: Fine-grained, intuitive knowledge used in graphical thinking. *Journal for Research in Mathematics Education*, 55(2), 96-118. https://doi.org/10.5951/jresematheduc-2022-0167
Schaap, S., Vos, P., Goedhart, M. (2011). Students overcoming blockages while building a mathematical model: Exploring a framework. In: G. Kaiser, W. Blum, R. Borromeo Ferri, G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (vol 1, pp. 137–146). Springer https://doi-org.dist.lib.usu.edu/10.1007/978-94-007-0910-2_15
Sherin, B. L. (2001). How Students Understand Physics Equations. *Cognition and Instruction*, 19(4), 479–541. https://doi.org/10.1207/S1532690XCI1904_3
Stender, P., & Kaiser, G. (2015). Scaffolding in Complex Modelling Situations. *ZDM Mathematics Education*, 47, 1255-1267. https://doi.org/10.1007/s11858-015-0741-0
Smith, J. (2023, February 2). How many times can you wear pajamas, jeans and other clothes before washing them? CNN. https://www.cnn.com/2023/02/02/health/how-many-times-rewear-clothes-before-washing-wellness/index.html
Stillman, G., Brown, J., & Galbraith, P. (2010). Identifying challenges within transition phases of mathematical modeling activities at year 9. In R. Lesh, P. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies* (pp. 385–398). Springer. https://doi-org.dist.lib.usu.edu/10.1007/978-1-4419-0561-1_33
Stillman, G. (2011). Applying metacognitive knowledge and strategies in applications and modeling tasks at secondary school. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G.Stillman (Eds.), *Trends in teaching and learning of mathematical modeling* (pp.165–180). Springer. https://doi-org.dist.lib.usu.edu/10.1007/978-94-007-0910-2_18
Swanson. H. (2025). Modeling the knowledge underlying student engagement in theory-building practices. In the *Proceedings of the 19th international conference of the learning sciences - ICLS 2025*. International Society of the Learning Sciences.
Thompson, P. W. (1994). Students, functions, and the undergraduate curriculum. In E. Dubinsky, A. H. Schoenfeld, & J. J. Kaput (Eds.), *Research in Collegiate Mathematics Education*, 1 (Issues in Mathematics Education Vol. 4, pp. 21-44). American Mathematical Society.
Thompson, P. W. (2011). Quantitative reasoning and mathematical modeling. In L. L. Hatfield, S. Chamberlain, & S. Belbase (Eds.), *New perspectives and directions for collaborative research in mathematics education*, (WISDOMe Monographs Vol. 1, pp. 33–57). University of Wyoming.