Preservice Teachers' Reasoning About Area and Volume Measurements

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This preliminary research report examines how undergraduate elementary preservice teachers (PSTs) reason about area and volume measurements prior to receiving instruction in mathematics content course in algebra, geometry, and measurement topics. This is a pilot study for a larger project. The preliminary analysis results from the pre-assessment data of 27 PSTs indicate that they lack productive reasoning about area measurements and their conversions, and they also employ direct linear dimensions conversion strategies on the volume task. This highlights the need for instructional unit to focus explicitly on developing PSTs' productive reasoning about area and volume measurements.

Keywords: area measurement, preservice teachers, quantitative reasoning, volume measurement

In most countries' curricula, area and volume are fundamental measurement concepts that students must learn and apply. Both the Common Core State Standards for Mathematics (CCSSM, 2010) and researchers (Battista & Clements, 1996; Huang & Wu, 2019; Panorkou, 2021) have emphasized the importance of deep and progressive meanings for these concepts. From early elementary grades through college level, students are expected to develop productive meanings for area and volume measurements. Such a foundation, which emphasizes visualization and reasoning over rote memorization (CCSSM, 2010; Huang & Wu, 2019; Panorkou, 2021), is crucial for more complex geometric reasoning and problem-solving topics like optimization, integrals, related rates, volumes of solids of revolution in their later school years (Dorko & Speer, 2013, 2015).

Research has consistently shown that K–12 students are not being provided sufficient opportunities to develop more productive meanings for area and volume measurement. Students frequently struggle to visualize and mentally structure 3D arrays, leading them to count visible faces in an image instead of imagining the total number of unit cubes (Battista, 1999; Battista & Clements, 1996; Eames et al., 2013; Sisman & Aksu, 2016; Vasilyeva et al., 2013). Further difficulties include misapplying or confusing volume formulas with those for surface area or height, and conflating volume with weight (Curry et al., 2006; Ergene & Isiksal Bostan, 2022; Ho & McMaster, 2019; Kim & Oláh, 2019; Seah & Horne, 2018; Sisman & Aksu, 2016; Vasilyeva et al., 2013). Given the challenges faced by K–12 students, there is an expectation that preservice teachers (PSTs) will be equipped to support students in developing productive meanings for these concepts. However, research on PSTs (e.g., Chamberlin & Candelaria, 2018; Hong & Runnalls, 2022; Zembat, 2010) shows significant similarities between the challenges they encounter in developing meanings for area and volume measurements and those faced by children. These findings collectively emphasize the importance of providing instructional opportunities to help PSTs (and students generally) develop more productive meanings for area and volume measurements. Considering the challenges faced by PSTs, it is essential to implement targeted instructional strategies in teacher education programs that foster a deeper conceptual grasp of these mathematical concepts. By closing these knowledge gaps, we can better prepare PSTs to support their future students in gaining more productive meaning for area and volume measurement. In this preliminary report, we address the research question: *How do PSTs reason about area and volume measurement before and after receiving instruction focused on developing meanings for* *area and volume as quantities?* Data collection is on-going in the Fall 2025 semester, so in this paper, we only present results on PSTs' meanings for area and volume by examining their responses before the short instructional unit. We will have additional data and further analysis completed prior to the 2026 RUME conference. Further, we note that the results of this study will serve as a foundation for the first author's dissertation work.

Theoretical Framework

Quantitative reasoning entails conceiving situations in terms of quantities and relationships between them. To conceptualize a quantity, one must identify a particular measurable attribute of an object, recognize an appropriate unit of measure, and conceive of a measurement process (Smith III & Thompson, 2008; Thompson, 2011, 2023). For area and volume, students must first conceive of a 2D or 3D space as a measurable attribute, respectively. With respect to area, students can use either standard units (e.g., square units; Cullen et al., 2018) or non-standard units (e.g., rectangles; Simon & Blume, 1994) to cover the measurable 2D space or a dynamic sweep of a line segment in a perpendicular direction through a distance to generate a rectangular area (Panorkou, 2020). The process of covering the space with the unit is the measurement process described by Thompson and colleagues. With respect to volume, students could also use standard or non-standard units. This may entail conceiving of a 3D object with an array of 2D cubic units iterated in the third dimension (Curry & Outhred, 2005; Curry, Mitchelmore, & Outhred, 2005, 2006) or a dynamic sweep of an area through a length (Panorkou, 2020, 2021).

Research on PSTs' Reasoning about Area and Volume Measurement as Quantities

Research on PSTs' reasoning about area and volume measurement reveals significant difficulties in relation to the three criteria Thompson described for quantities construction. With respect to conceptualizing a measureable attribute, several researchers have argued that PSTs often define area as "length × width" rather than the amount of space within a region, thus lacking meanings for these procedures that connect to a process of measuring an attribute (Livy et al., 2012; Ma, 2020; McNeal et al. 2024; Simon & Blume, 1994).

Related to the importance of units, researchers have found that many PSTs could not accurately identify the units involved in area measurement tasks because they lack experiences that give meaning to the concept of "square units," (Browning et al., 2014; Ghosh Hajra et al., 2016; McNeal et al. 2024). Similarly, Dorko and Speer (2015) examined students' understanding and use of units when calculating area and volume. They found that only 26.6% of students used the correct units for all tasks, while 73.4% made at least one mistake in applying or omitting units in area and volume calculations. The most common error was using length units (such as cm or m) instead of area units (cm^2) or volume units (cm^3).

The findings from these studies highlight the importance of supporting PSTs to develop more productive meanings for area and volume as quantities (as defined by Thompson) units. Ultimately, strengthening their meanings for area and volume measurements will not only enhance their mathematical reasoning but also enable them to teach these fundamental concepts more effectively to future students.

Methods

In this section, we describe our participants and the setting of the course, followed by an overview of the data sources used.

Participants and Setting

The participants for this study are elementary PSTs taking a required undergraduate mathematics content course within a four-year teacher education program at a university located in the Mid-Atlantic region of the United States. The participants were sophomores and juniors. We invited all 30 participants in the cohort class to take part in a pre- and post-assessment test. The goal of the course is to provide PSTs with the opportunity to develop productive meanings for algebra, geometry, and measurement topics and to learn how these topics are developed throughout the K-8 curriculum. All students in the class will participate in the same instructional activities. Additionally, three PSTs are actively taking part in a short teaching experiment (Steffe & Thompson, 2000), of which the first author is primary teacher-researcher, to explore more in depth the efficacy of the task sequence.

Data Sources

PSTs will take the same assessment both at the beginning and end of the semester, administered online through a Desmos Activity. The pre-assessment consisted of several tasks. Of relevance to this report, the first two tasks (Figure 1) required converting between area measurement units, one involving standard units (Task 1) and the other involving non-standard units (Task 2). Task 3 (Figure 2) required finding the volume of a box involving a non-standard unit and performing unit conversions.

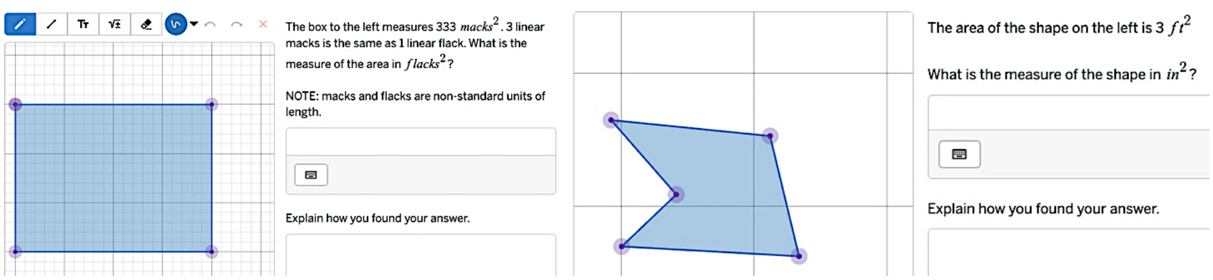

Figure 1. Tasks 1 and 2

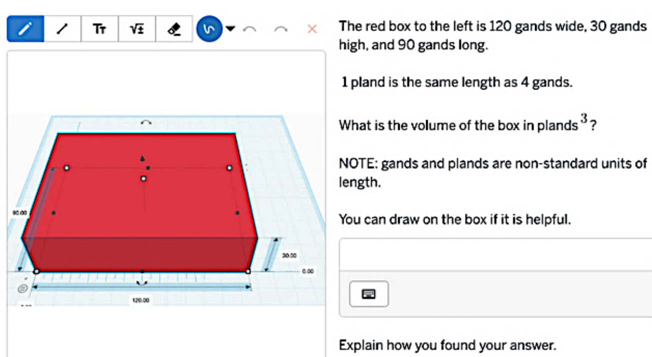

Figure 2. Task 3

Analysis

To code the PSTs' responses to pre- and post-assessment items, we will begin by individually reading through the PSTs' responses and making notes about ways they reasoned about area and volume in each problem. We will then meet to compare observations and develop codes that could apply across problems. We will likely use both a priori codes (e.g., relying on formulas) and emergent codes (e.g., converting linear units before determining area or volume). Once we agree on features for coding, each author will independently code several pre- and post-test responses for each problem, and we will calculate interrater reliability. Whereas we will present the results of this analysis at the conference, due to the time constraints for preparing this report, we will focus on PSTs' numeric responses to the tasks to highlight the ways such analysis is likely to provide powerful insights into their reasoning.

Preliminary Analysis and Results of the Pre-assessment Test

We first present a summary of the PSTs' responses to the three tasks. For each task, we present the PSTs' numeric values, regardless of whether they included units in their response. We note that for the two tasks in Figure 1, only 2 PSTs obtained the correct numeric values for the tasks (37 and 432, respectively); these were the same two PSTs. The majority of PSTs provided a value that was likely determined by multiplying or dividing by the linear conversion fact (e.g., dividing 333 by 3 and multiplying 3 by 12).

Table 1. PSTs' Responses to Pre-Assessment Tasks on Area and Volume

Task 1 (*n* = 27)		**Task 2** (*n* = 27)		**Task 3** (*n* = 24)	
Answer	№ of ans. (as %)	Answer	№ of ans. (as %)	Answer	№ of ans. (as %)
37	2 (7.4%)	432	2 (7.4%)	5,062.5	13 (54.1%)
111	22 (81.5%)	36	23 (85.2%)	81,000	6 (25.0%)
1110	1 (3.7%)	32	1 (3.7%)	1,296,000	2 (8.3%)
100	1 (3.7%)	9	1 (3.7%)	972,000	1 (4.2%)
99.9	1 (3.7%)			5,062.3	1 (4.2%)
				32	1 (4.2%)

In contrast, a majority of PSTs (54%) obtained the correct response (5062.5). The most common incorrect answers were probably obtained by multiplying the three given dimensions (120×30×90), then dividing or multiplying this value by the given linear conversion factor, thus obtaining 81,000 or 1,296,000, respectively.

We were initially surprised by the large discrepancy between the number of PSTs who correctly answered Task 3 (Figure 2) compared to Tasks 1 and 2. However, as we preliminarily reviewed the PSTs' written responses (which we will more formally code once both pre- and post-assessments are complete), a possible explanation became apparent. Rather than determining a conversion factor to determine an answer (i.e., there are 4×4×4 plands^3 in 1 gland^3), we found that many PSTs directly converted the three given linear measures before multiplying; this strategy was not available to them in the area problems (Tasks 1 and 2), as no dimensions were provided. For example, one PST on Task 3 justified her correct answer by saying:

> I divided each measurement by 4 to convert gands into plands, because if 1 pland is the same as 4 gands, there will be a smaller amount of plands to take up the same amount of space. Then I multiplied the length times width times height to get the volume.

On Task 2, the same student explained how she determined there were 36in^2 in 3ft^2 said:

> There are 12 inches in a foot, so for every 1 foot^2, there are 12 inches^2. There are 3 ft^2, so 3 times 12 inches per 1 foot is 36 inches^2.

Hence, the PST was able to determine a correct solution to the volume problem by converting linear measures and then multiplying, a strategy not available in Task 2, as no linear measures were provided. We observed numerous other similar responses from PSTs who successfully converted linear dimensions in Task 3 but not in Tasks 1 and 2.

Discussion

Our preliminary findings corroborate what other researchers have noted. Namely, prior to intervention the PSTs' meanings for area and volume seem to rely on procedures (Chamberlin & Candelaria, 2018; Hong & Runnalls, 2022; Livy et al., 2012; Koçak et al., 2017; Ma, 2020; McNeal et al. 2024; Simon & Blume, 1994; Zembat, 2010). With respect to Tasks 1 and 2, these procedures do not seem to reflect a robust meaning of either area as entailing a process of using a unit to measure an attribute (e.g., reasoning about using different units to cover a space; Thompson 2011, 2023). However, our prompt for Task 3 may be limiting our ability to explore PSTs' meanings for converting between units of volume. We plan to add an additional volume question to the post-assessment test without linear dimensions provided to better explore their meanings for volume conversions.

As we continue to code the pre- and post-assessment responses, we will further analyze the data as described in the methods. We anticipate several codes as being relevant based on the literature (e.g., Dorko & Speer, 2015), such as "omission of unit, but correct numerical value," "use of wrong unit, but correct numerical value," "omission of unit and wrong numerical value," and "use of wrong unit and wrong numerical value." This coding process will provide a clearer picture of the changes in their PSTs' meanings for area and volume measurements, and after the course. We are hopeful that the instructional unit will support them in developing such productive meanings, and these meanings will be reflected in students' post-test responses.

Implications and Questions for Feedback

Based on the preliminary analysis and results of our pre-assessment test, we highlight two main implications that can be drawn from these preliminary findings. First, a significant number of PSTs answered Tasks 1 and 2 incorrectly, indicating that the PSTs' meanings for area measurements and their conversions are not robust enough to address a variety of tasks related to area. This implication highlights the need for instructional unit to focus explicitly on developing productive reasoning related to area and volume measurements. Second, the findings revealed that PSTs employed direct conversion strategies in the volume task (Figure 3), which were not easily applied in the two area tasks due to dimensions not being apparent. This highlights the need to further explore and develop PSTs' meanings for both quantities as measurable attributes entailing a unit and measurement process.

Based on these preliminary findings and implications, we intend to ask the audience for the following in their feedback:

1. In what ways do you think this study can be expanded upon? Are there other critical ideas to explore that support PSTs in developing productive meanings for area and volume measurements?
2. In what other directions can the analysis focus, especially when both pre- and post-assessment data sets are available after the course?
3. Apart from quantitative reasoning, are there other theoretical frameworks to consider grounding the study in?

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