Extending the Theoretical Boundaries of MKT to Mathematics Instruction in Service Courses for STEM Disciplines

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In this theoretical report, we argue for a need to extend the theoretical boundaries of Mathematical Knowledge for Teaching (MKT) beyond its historical applications. Our motivation is the longstanding concern regarding the ability of Calculus as a service course to effectively scaffold non-math-major students' ways of knowing mathematics to support them in transitioning to their primary discipline, particularly in Engineering. We extend the theoretical boundaries of MKT to encompass the knowledge needed to connect Calculus to its ways of use within Engineering in a way that is perceptible and comprehensible to students. Finally, we distinguish this form of MKT from existing work in MKT, and we demonstrate its utility in advancing our understanding of the knowledge required to improving Calculus instruction.

Keywords: calculus, mathematical knowledge for teaching, instruction, engineering

In many U.S. colleges and universities, introductory differential and integral Calculus courses serve students majoring in STEM client disciplines such as engineering, biology, and chemistry, rather than primarily mathematics (Bressoud et al., 2013; Rasmussen et al., 2014). These courses play two roles in tension: "filter" and "scaffold" (Biza et al., 2022). Performance in Calculus acts as a filter as it is a prerequisite to various courses (Bressoud, 2021; Törner et al., 2014) and is associated with retention (or the lack thereof) within STEM majors (Ellis et al., 2016; Rasmussen & Ellis, 2013; MAA, 2015; PCAST, 2012). At the same time, Calculus can support students by scaffolding their transition from high school to college as well as in their transition to the advanced coursework involving mathematics that is required by their major (Biza et al., 2022; Corriveau & Bednarz, 2017).

However, the STEM education community has come to question the efficacy of college Calculus in fulfilling its role as a scaffold. For instance, professionals in mathematically intensive STEM careers such as engineering have reported viewing their undergraduate mathematics experiences as irrelevant to their current work (Quéré, 2017; Härterich et al., 2012; Pearson, 1991). Likewise, student engineers have been reported to find university mathematics in conflict with mathematics use in engineering (Alpers, 2017; Bingolbali et al., 2007; Britton et al., 2005; Gainsburg, 2007; Jablonka & Bergsten, 2022; Wood et al., 2008). One hypothesis regarding the cause of this phenomenon is that Calculus courses, like other mathematics courses, are typically taught from a purely mathematical perspective. Applications to other fields often appear as optional examples that only come after relevant mathematics has been addressed (Kent & Noss, 2002) and serve to ultimately reinforce the mathematical perspective (deemed "applicationism" by Barquero et al., 2013). These concerns were one of the core motivations with the MAA's (2004) CRAFTY initiative which urged mathematics faculty to provide opportunities for students to "see mathematics in context" and to explain "the hows and whys" (pp. 3-4) in solutions over 20 years ago.

We offer a second hypothesis that builds on the first: that our current conceptualization of the *mathematical understanding* required for *mathematical knowledge for teaching* Calculus (Ball, Thames & Phelps, 2008) is premised almost exclusively on ways of knowing mathematics within its own discipline. This is an issue for students, given that many essential concepts in the Calculus sequence are framed and understood differently by engineering faculty (Willcox & Bounova, 2004; Flegg et al., 2012). In this way, mathematical understanding for teaching students of client disciplines is related to but qualitatively distinct from disciplinary mathematics (e.g., Alpers, 2010; Diaz Eaton & Highlander, 2017). For Calculus courses to be a scaffold for engineering students, we must first understand the mathematical knowledge required for instructors to teach Calculus so that it connects naturally to STEM coursework in client disciplines.

In this theoretical report, we address the question: How does one define *the kind of mathematical knowledge needed by Calculus instructors* to support students in connecting mathematics to client disciplines? Similarly to Biza et al. (2022), we center our illustrative examples and literature review within this report on the client discipline of Engineering due to the relative wealth of research in this area regarding associations between student retention and mathematics. However, our intention is for our treatment of this conceptualization to serve as a model for those interested in examining mathematical knowledge for teaching Calculus to students in any client discipline.

Conceptual Framework

Here we conceptualize the key construct of this report: the kind of mathematical knowledge needed by Calculus instructors to support students in connecting mathematics to client disciplines. To do so, we first delineate this domain from three related domains: (1) mathematics as its own discipline; (2) mathematical knowledge for teaching (MKT) as it has been typically represented; and (3) mathematics as it is used in and for engineering (MEng). We then show how each of these three are conceptually related to the key construct, which we will refer to as MKT×MEng -- a notation that shows the intertwining of mathematical knowledge for the purposes of teaching in and for engineering.

Mathematics as its own Discipline

Mathematics can be pursued for its own sake, and the work of professional mathematicians entails its own language, reasoning, questioning, and practices (Kitcher, 1984). Mathematicians' practices often involve developing strategies for generalizing and specializing patterns and structures (Pólya, 1990), potentially to give insight into new ways of thinking (Weber, 2010). Most college Calculus instructors are enculturated to this discipline through their graduate studies, either completed or in progress.

Mathematical Knowledge in and for Teaching (MKT)

Teaching mathematics differs from *doing* mathematics for its own sake as well as doing mathematics in other mathematically intensive professions (Ball et al., 2008). We define *mathematical knowledge in and for teaching (MKT)* as the mathematical skills, dispositions, and reasoning entailed in and developed through the recurrent work of teaching (Ball et al., 2008; Rowland & Ruthven, 2011; Thompson & Thompson, 1996). MKT is a practice-oriented domain (Biehler et al., 2022; Ball & Bass, 2003; Kent & Noss, 2002). As situated knowledge (Lave, 1988), mathematical meanings may be constituted in terms of workplace concerns and interactions. In teaching, these include unpacking student thinking and constructing explanations that render content comprehensible to learners.

MKT is premised in part on the norms and expectations of mathematical practice; teaching mathematics is often conceived as teaching students to learn content in ways aligned with mathematics as its own discipline (e.g., Stein et al., 1996; Greeno & Middle School Mathematics Through Applications Project, 1997; National Research Council, 2001). For instance, studies of instructors' knowledge in Calculus and Pre-calculus focus on mathematical constructs, and applications serve an ultimate purpose of creating mathematical meanings (for reviews, see Bressoud et al., 2016 and Thompson & Harel, 2021). Some exceptions include recent literature in mathematical modeling, but even this scholarship frames results in terms of enhancing teachers' and students' practices for mathematics as its own discipline (e.g., Anhalt & Cortez, 2016; Wickstrom & Jung, 2024). *Due to this inherent theoretical tie to the discipline of mathematics, we propose extending the theoretical boundaries of MKT to align with the norms and expectations of other STEM disciplines. This extending may allow us to better understand the kinds of mathematical knowledge required to teach Calculus effectively in service courses.*

By "kinds of mathematical knowledge" we include specialized content knowledge (SCK) and pedagogical content knowledge (PCK). SCK is defined as ways of knowing mathematics unique to teaching. In elementary mathematics, SCK includes understanding and connecting different fraction representations and the definition of fraction. In a Calculus course whose goal is only to enculturate students to the norms and expectations of math as its own discipline, it may include ways of explaining the concepts of limit, rates of change, and derivative. PCK includes familiarity with student conceptions as well as how students may need content in future courses. *As we will argue, it is reasonable to believe that SCK and PCK must be conceived differently when instruction is predicated on a domain other than pure mathematics.*

Mathematics in and for Engineering (MEng)

We use the term *mathematics in and for engineering (MEng)* to mean the mathematics entailed in recurrent work of engineering. Although this construct has been studied by a variety of researchers (see Alpers (2010) for a review), there was no common term that we found in the literature; researchers variously described "mathematical expertise of mechanical engineers" (Alpers, 2010), "mathematical dispositions of structural engineers" (Gainsburg, 2007), or "mathematical components of engineering expertise" (Kent & Noss, 2002). We use the term "mathematics *in and for* engineering" to acknowledge its practice-oriented nature.

The subject of Calculus provides several core ideas critical to solving problems in a variety of engineering domains such as continuity of functions, derivatives, and polynomial basis functions (Alpers, 2010), as well as minimizing surface area, finite element analysis, and Navier-Stokes Equations (Cardella, 2006).

However, ways of knowing Calculus that are relevant to MEng are distinct from those that would be relevant to mathematics as its own discipline; MEng is constrained and motivated by specific problems encountered by engineers (Alpers, 2010; Gainsburg, 2007; Kent & Noss, 2002). Throughout deployment of MEng, engineers using mathematical ideas may think primarily in terms of engineering objects and tools, and preference industry needs and practicality when evaluating which inputs, outputs, and options are relevant in given scenarios (Alpers, 2010; Gainsberg, 2007; Kent & Noss, 2002). MEng may differ from mathematics as a discipline in part because of the role of generalization. To illustrate, we use this excerpt from Gainsburg's (2007) ethnographic study of two engineering firms:

> The problem-solving conditions for these engineers engendered mathematical practices that often diverged from those in traditional and reform mathematics classes. Popular in reform instruction is the practice of generalizing (Radford, 2000; Schoenfeld, 1994), that is, detecting patterns in data and expressing them as mathematical rules or formulas...Generally, the engineers were expected to adhere to the rules of the industry,

> not invent their own. When invention did become necessary...no data existed from which to generalize a rule anyway. Further, the rare method or representation form that the engineers improvised was so situation-specific that it would have had no life beyond the task for which it had been developed (p. 485).

MKT×MEng: Mathematical Knowledge Needed to Support Students in Connecting Mathematics to Engineering

We now introduce the central construct of this paper: mathematical knowledge needed to support students in connecting mathematics to engineering. We use MKT×MEng to refer to this domain, as it is a form of mathematical knowledge for teaching, intertwined with the purpose of fostering learners' understanding of mathematics in and for engineering. *We define MKT×MEng as the MKT needed to connect course content to MEng in a way that is perceptible and comprehensible to students.*

MKT×MEng is distinct from mathematics as its own discipline, from MKT as it has been typically represented and examined, and from MEng. The purpose of MKT×MEng is pedagogical, which separates it conceptually from both mathematics as its own discipline and MEng. It is also different from MKT as often conceived, because the practices that it aims to ultimately support are mathematical practices within MEng. We see MKT×MEng as part of MKT, because it deals wholly with mathematical issues; but we also see it as strongly connected to knowledge outside MKT, because it is motivated by engineering contexts and concerns that are outside mathematics as its own discipline. To demonstrate one way that MKT×MEng is related to but conceptually distinct from MKT, we provide an illustration of how SCK as it is typically conceptualized within MKT does not meet the needs of MKT×MEng. A similar discussion of PCK is possible but is not provided here for brevity.

SCK within MKT includes mathematical knowledge for evaluating the viability of students' alternative algorithms for solving problems, with attention to whether proposed solution strategies are generalizable to other mathematical problems and contexts (Ball et al., 2008). Given the prior characterization of MEng, it seems unlikely that SCK in the domain of MKT×MEng would emphasize the notion of mathematical generalization. Instead, SCK in the domain of MKT×MEng may include an understanding of the contextual factors that determine the viability of various solutions strategies in different applications. For example, in the areas of statics and circuits, continuity is not treated as a property to be evaluated within a given function. Instead, emphasis is placed on how one manages discontinuity within an existing system (Faulkner et al., 2020) so that parameters can be chosen that satisfy the necessary constraints while being viable in the engineering context. In this way, many traditional assignments in single-variable Calculus that ask students to redefine functions at key points or intervals to make them continuous may unintendedly contribute to subsequent confusion for engineering students. SCK in the domain of MKT×MEng may therefore involve instructor knowledge regarding the context-dependent nature of the existence of discontinuities within a handful of real-world systems. Such knowledge could enable an instructor to do more than simply point out an application of the notion of continuity in another field; it could allow them to support and respond to student thinking on Calculus assignments that intentionally interrogate various interpretations of discontinuities and their context-dependent meanings.

We close this section on MKT×MEng with three questions we have encountered about the scope of MKT×MEng and a clarification of our views. (1) Is a focus on MKT×MEng meant to supplant any intention to teach mathematics as its own discipline? (2) Wouldn't MKT×MEng require an engineering degree? (3) Isn't MKT×MEng already encoded in curricular materials that connect mathematics and engineering, and is therefore unnecessary to study? Our view on (1) is that there is active and robust literature on teaching Calculus (and other fields of mathematics) to represent disciplinary mathematics practices. More can be done to close gaps between research and practice, and we support the perspective that one aim of Calculus is to convey mathematics as a discipline. But in this paper, we draw attention to a domain of professional teaching knowledge that has not been extensively studied and whose examination may address the inefficacy of Calculus courses being a scaffold to disciplines outside of mathematics.

With respect to (2), we are not asserting that all Calculus instructors must be fluent in all aspects of MEng. Just as an elementary teacher does not need to know all of mathematics to develop robust MKT, we suggest that a Calculus instructor does not need to know all of engineering to develop the MKT×MEng needed to support freshmen and sophomores who are only beginning their engineering trajectories. The degree of fluency with respect to MEng that is required for such support remains an open question-one whose empirical investigation we intend to support by providing this framework.

Finally, in response to (3), MKT×MEng is *knowledge* and it is distinct from curricular *materials* that connect mathematics and engineering. We develop this argument in the next section.

How Attending to MKT×MEng Advances Existing Work

Here we review literature describing interventions in college mathematics classrooms that are meant to support student understanding of MEng. We have two purposes. First, we underscore the lack of conceptualization on the *knowledge for teaching* required to facilitate such interventions successfully, which we suggest is MKT×MEng. Second, we argue that a theoretical underpinning of MKT×MEng would provide affordances to work that seeks to improve college mathematics outcomes.

Interventions may be organized according to the depth of resources (class time, preparation, etc.) required to facilitate them. On one end lies successful resource-intensive approaches, such as those reported by Johnson et al. (2024) who increased student retention by revamping differential and integral Calculus courses to center ways of knowing Calculus that would better prepare students in client disciplines. For instance, they emphasized R to gain intuition about Calculus concepts and de-emphasized concepts viewed as necessary only to mathematics as its own discipline (e.g. l'Hopital's Rule). Similar resource-intensive approaches include team teaching between mathematicians and instructors from other STEM fields (e.g., Quintanilla et al., 2007), coordinated mathematics and engineering courses (e.g., Laoulache et al., 2001; Lowery et al., 2010; Schroeder & Carpenter, 1999), and specialized sections of Calculus courses for engineering majors (e.g., Horwitz & Ebrahimpour, 2002; Sathianathan et al., 1999).

These efforts require significant coordination and knowledge-sharing between mathematics and engineering departments. However, literature reporting on such interventions has tended to focus on the materials themselves rather than the MKT×MEng acquired by the mathematicians to decide how to sequence course topics, frame mathematical ideas, and interpret and respond to student thinking. For instance, in the area of pedagogy, Johnson et al. (2024) only report that their revamped Calculus course involves student board work with instructor feedback. One wonders, then: in what ways did mathematics faculty need to know Calculus differently to effectively respond to student board work and support connections to their client disciplines? Similarly, Horwitz and Ebrahimpour (2002) describe a variety of projects created for engineering-intensive sections of a differential Calculus and integral Calculus courses and provide more than enough information regarding the knowledge that *students* needed to know to complete the projects. Almost no information is provided regarding the mathematical knowledge required of the Calculus instructor to support students in their investigations. Some projects required students to assess the extent to which experimental data can be approximated by mathematical formulae. The mathematical knowledge required to make such an assessment is not part of the standard Calculus curriculum and thus must have been modeled by the mathematics instructor. However, the following remains unknown: what knowledge of Calculus as it is applied in experimental fields was required for that instructor to effectively *teach* this to students? Questions such as these concerning MKT×MEng are critical to consider when thinking about how one might translate these successful interventions to other institutions.

Moderate resource-intensive approaches focus on in-depth assignments, rather than a total course overhaul. Perhaps the most notable example is the work of Schmidt and Winsløw (2021) who, over two decades, created in-depth assignments for service mathematics courses in contexts such as Halbach magnet arrays and red blood cells. In all tasks, the “central challenge” was mathematical, and context and discourse were true to engineering (p. 270). Their views are representative of other assignments with in-depth applications of mathematics to engineering (e.g., Anderson & Nguyen, 2024; Anderson & McCusker, 2019; Barquero et al., 2013; Härterich et al., 2012; Lowery et al., 2010; Peters, 2023; Rønning, 2023; Neubert et al., 2014). Given their more specific nature, literature regarding these assignments typically has the space to describe the MEng required for an individual to complete the task successfully. For instance, Anderson and Nguyen (2024) provide a rigorous, step-by-step overview to support the reader in “mastering the mathematical foundations of the linear algebraic” tool (p. 809) that underlies their in-class intervention. This information is necessary to understand the intervention, but it may be sufficient for supporting instructors in responding to the demands of teaching it. We propose that an investigation of the MKT×MEng employed by instructors, including in facilitating such projects, would provide theoretical support for implementation in other contexts.

Finally, low resource-intensive approaches focus on shorter examples infused into existing Calculus content throughout a term. For instance, Kerrigan and Prendergast (2021) solicited examples from engineering faculty for a flipped Precalculus course. Burrill et al. (2023) reported a similar effort to redesign linear algebra, with the effect that the “students appreciated the content much more and enrollment ... grew vigorously while that of the analogous class from the school of engineering dropped precipitously” (p. 800). Although these examples take up little class time, their effective facilitation may still require a particular type of mathematical knowledge to situate the examples. The relationship between resource intensity of an instructional intervention and degree of MKT×MEng required for effective facilitation remains an open and important question for future research.

Implications for Further Research

In summary, we propose a theoretical construct -- MKT×MEng -- that builds on existing work in MKT and whose further development will respond to ongoing calls for Calculus to fulfill its promise as a scaffold for STEM client disciplines. While mathematics education literature abounds with various examples, activities, and projects for use in courses such as Calculus, we are lacking in systematic studies of the mathematical knowledge required to teach with such materials in mathematics courses.

In broaching and delimiting the construct of MKT×MEng, we support researchers in undergraduate Calculus education in investigating questions such as: What ways of knowing Calculus within MEng are relevant for college Calculus instructors? By what epistemological principles (core assumptions, concepts, and norms) do engineering faculty interpret student errors and evaluate student solutions involving Calculus concepts, and how do these principles compare to those prevalent in current work in MKT? What are features of contexts where Calculus within MEng are likely to arise, and what knowledge is needed to detect and explain these forms of MEng so they are visible to students? In investigating such questions, we advance not only our theoretical understanding of mathematical content for teaching in service courses, but also our potential for implementing the wealth of existing materials into Calculus courses beyond the institutional context in which they were developed.

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