

Approaching Zero: Exploring Calculus Students' Conceptions of Zero

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Although it is perceived as trivial, zero plays a crucial and nuanced role in many topics of mathematics that often goes unnoticed by students. Zero's roles include that of a placeholder and an identity, but for most students zero is perceived simply as a number representing nothing. Previous studies have suggested that a conception of zero rooted in nothing is quite nuanced (Seidelmann, 2004; Vest & Alibali, 2024), but it may not align with all situations involving zero. Studies have also shown that teacher conceptions of zero vary greatly but are very similar to those of their students (Wheeler & Feghali, 1983). Ultimately, researchers have suggested that students' and teachers' understanding of zero is limited or incomplete (Inhelder & Piaget, 1969; Tsamir, Sheffer, & Tirosh, 2000; Seidelmann, 2004). These studies, of which there are very few, have yet to consider how one's conception of zero may be a fragmented combination of many different conceptions. By reframing conceptions of zero this way we can consider how students operationalize different elements of their conception of zero based on problem types and context.

In this study, I consider the many conceptions of zero held by calculus students and how they employ those conceptions based on different tasks. Calculus students have been chosen as the population of study because of the central role of zero in calculus and the fact that students have undergone many mathematical domains in which zero played an important role. Studying calculus students also presents an opportunity to uncover how early or traditional conceptions of zero do or do not align with calculus understanding and procedures. In this work I am guided by the following research questions:

1. In what ways do calculus students conceptualize the concept of zero and its many roles in mathematics?
2. In what ways do calculus students employ varying conceptions of zero to navigate tasks and mathematical situations?

To answer these questions, I have designed a two-stage mixed-methods study with high school and undergraduate calculus students as the population of interest. Students from two high school classrooms and two undergraduate classrooms will respond to a questionnaire that is meant to uncover how students conceptualize zero as an object as well as in the context of traditional arithmetic problems and calculus situations. I will conduct a thematic analysis (Braun & Clarke, 2006) with a series of a priori frameworks from existing literature as a guide to establish students' conceptions of zero. A subset of five to seven students with diverse conceptions of zero will be selected from each population to participate in a follow-up interview. The interview will consist of mathematical tasks and reflection questions that expand upon the topics covered in the questionnaire. A thematic analysis (Braun & Clarke, 2006) of the interview data will enrich my understanding of the conceptions of zero held by students and the ways in which they navigate tasks with these conceptions.

Although the primary data collection is set to take place in the spring, the pilot data suggests that students are comfortable using zero in mathematical situations but may not have a strong understanding of why zero behaves as it does. When some students describe zero, they find that their conception does not align with how they use zero to operate in domains such as multiplication, division, and limit calculations.

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