Unpacking Students' Intuitive Limit Views Using Graphical Representational Approach

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Students' intuitive reasonings of the limit of a function at a point play a role in their overall conceptual understanding of the limit concept. Some researchers have defined intuitive limit views as students' understanding of the limit concept based on their everyday experiences (e.g., Adiredja, 2014; Adiredja et al., 2020; Rathnayake & Jayakody, 2023). In this study, I characterize limit views that are based primarily on reasonings with less emphasis on *mathematizing* those ideas as *intuitive*. Research shows that intuitive views are closely linked to students' graphical reasoning (Adamoah & Strayer, 2025). Building on the findings of Adamoah and Strayer (2025), this study explored how students' intuitive understandings of limits are connected with graphical representations. The results of this study highlight an instructional approach to help overcome what past research identified as students' misconceptions about the difference between $lim_{x \to a} f(x)$ and $f(a)$ (Bezuidenhout, 2001; Denbel, 2014; Fernández, 2004). Specifically, I addressed the research question: *How do students' intuitive limit views influence their **reasoning with** and **use of** graphical representations in calculus?*

This poster presentation shares a portion of the results of a larger study that framed students' limit views as *intuitive*, *informal*, and *formal* and their representational approaches as *graphical*, *numerical*, and *algebraic* to understanding the limit of a function at a point. This study was conducted at a large public university in the southeastern United States in Fall 2024 with 63 Calculus II students. First, all the 63 participants completed a pre-assessment survey that focused on the three limit views and three representational approaches. Afterwards, I chose four participants for each limit view type (intuitive, informal, and formal), and that resulted in 12 participants purposively selected for the structured task-based interviews where they explained more fully how they reason about limits with each of the three representational approaches.

In this study, I found that students' *intuitive* limit views were linked with the graphical representational approach in two ways: (1) viewing limits with graphical representation as an *object* and (2) viewing limits with graphical representation as a *tool*. Graphical representation as an object implies students' belief that "*it shows me*" and graphical representation as a tool implies students' belief that "*I'm using it to show*". Below was Jake's explanation of how the graph with removable discontinuity showed and helped him to distinguish $lim_{x \to 2} f(x)$ and $f(2)$.

> *The graph helps me understand that the limit isn't going to just be like a point* [referring to the (2, 5) on the graph]. *We're going to get to, so it's not going to be the value of F of 2 is 5. This helps me understand* [referring to the graph] *we're getting closer and closer to a number, just as like we're evaluating a function as it gets closer and closer to a point.*

Jake's explanation above characterizes students' reasoning with graphical representation as an *object*. On the other hand, students' view of graphical representation as a *tool* was seen as they used it intuitively to prove the formal conventional $\varepsilon - \delta$ definition as well as using graphs to visualize and validate analytic properties of limits. The results of this study offer calculus instructors ways to integrate and effectively use graphical representations either as an *object* or as a *tool* to help deepen students' understanding of limits including the formal limit definition.

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