Can x Equal Zero? A Look at Two Calculus I Students' Reasoning About Variables

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Introduction

When solving calculus problems, undergraduate students rely on variables and algebraic manipulation. However, students frequently view variables as symbols to be manipulated rather than as quantities to be related (LaRue & Infante, 2015; White & Mitchelmore, 1996). While algebra education research (e.g., Booth, 1989; Küchemann, 1981; Lucariello et al., 2014) addressed students' interpretation and manipulation of variables, more research is needed to understand how these processes manifest specifically among calculus students. This poster will address several facets of this need.

Context

This work is part of a qualitative research study conducted at a large R1 university on the East Coast of the United States. The study involved students enrolled in Calculus I courses for life and social sciences during the Fall 2024 semester (Abadir, 2025). This poster will highlight a task-based episode from the larger study in which two undergraduate calculus students reasoned about algebraic manipulation and the use of variables while identifying points of local maxima and minima for a given function. In calculating critical numbers, students mistakenly treated the variable as a constant and divided both sides of the derivative equation by it, which led to the omission of a critical number. However, during a debriefing session with the researcher, the two students explained and reflected on their solution process, allowing them to identify their mistake and the conceptual implications of their algebraic manipulation. The students were able to connect algebraic ideas and properties, allowing them to engage with error in a way that facilitated a deeper understanding of the significance of variables as placeholders.

Methodology

Guided by Borasi's (1987) framework "errors as springboards for learning," this poster presentation aims to answer the research question, "In what ways do students' reflections on their algebraic manipulation and errors serve as an opportunity to reason about variables?" This analysis examines how students' engagement with their own errors fosters a deeper conceptual understanding of the role and significance of variables. The primary data sources include students' written work and a video recording capturing both the problem-solving process and the subsequent debriefing session.

Results and Significance

This poster will present detailed data and systematic analysis of the students' algebraic manipulation and error, along with their reflections as they articulated the significance of variables and how they relate to algebraic properties. These findings contribute to the body of research that frames errors as opportunities for deeper learning, specifically by using students' algebraic mistakes to enhance their understanding of the role of variables.

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