To Whom Are Instructors Obligated?: An investigation of Individual Obligations in Quotient Group Instruction

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Teaching is a complex process in which mathematicians are expected to make many decisions before, during, and after the instructional time. These decisions can be shaped by various factors such as pedagogical or content knowledge, professional obligations, or the personal beliefs and values of the mathematician. In this study, we draw on the theory of Practical Rationality as a means to examine the role that professional obligations play in the pedagogical decisions that mathematicians make. Specifically, we examine the role that individual obligations play in instruction noting that the way in which the obligation is viewed can lead to different decisions.

Keywords: Teacher Decision Making, Undergraduate Instruction, Professional Obligations

Teaching is a complex process in which instructors are expected to make a myriad of decisions before, during, and after instruction. Researchers at the K-12 literature have suggested that teacher decision making is a crucial area to better understand instruction and its impact on student learning (Fennema, et al., 1989). The decisions instructors make are affected by various factors such as pedagogical and content knowledge and belief systems (e.g., Ball, et al., 2008). In addition to individual factors (knowledge and beliefs), societal and institutional factors play a substantial role in instructor decision making. That is, instructional decisions are also informed by professional obligations that inform, justify, and constrain said decisions (Chazan et al., 2016).

In contrast to the K-12 setting, less research has explored instructor decision making at the undergraduate level. The work that does exist has primarily attended to the decision to lecture or using inquiry materials (e.g., Fukawa-Connelly et al., 2016) or inquiry practices (Shultz, 2022), and decisions made during a mandated inquiry-based linear algebra course (Mesa et al., 2020). Lande and Mesa (2016) have also examined decisions by community college faculty shaped by their status as full-time or part-time status. In general, little is known about the day-to-day decision making, especially in advanced undergraduate courses. In this report, we begin to unpack this level of decision making through a study with 13 abstract algebra instructors. We provided the instructors with five distinct images of quotient group instruction and analyzed their reflections on instructional decisions using the construct of professional obligation. In particular, we ask: In what ways do individual obligations influence the content coverage decisions made in Abstract Algebra courses?

Theory

Practical Rationality holds that instructors exist in complex environment with four types of stakeholders (e.g., Chazan et al., 2016) leading to a set of professional obligations that influence instruction. The four professional obligations are disciplinary obligations, individual obligations, institutional obligations, and interpersonal obligations. Disciplinary obligations stem from the perspective of knowledge being a stakeholder within the environment. This poses on the instructor that they are "obligated to steward a valid representation of the discipline of mathematics" (Herbst & Chazan, 2011, p. 450). This obligation holds that instructors hold their students to certain standards of proficiency, that the authentic mathematical knowledge is taught,

and that they steward authentic mathematical values and practices. Individual obligations are derived from a client perspective in which students are a stakeholder in their own education. This lends itself to the idea that the instructor is obligated to do what is best for the student (Herbst & Chazan, 2011). The notion of "best" has the potential to vary based on factors such as cultural or historical contexts (Chazan, et al., 2016). Institutional obligations stem from the institution in which the instructor is situated. This places obligations on the instructor that they attend to various policies from the institution, course schedules, and so on. Lastly, interpersonal obligations stem from the perspective that society has a stake in education. The instructor is then "obligated to share and steward their medium of interaction with other human beings in the classroom" (Herbst & Chazan, 2011, p. 450). Additionally, as is consistent with past studies on teacher-decision making (e.g., Lande & Mesa, 2016), we hold that in addition to professional obligations shaping the decisions made, the beliefs, values, and knowledge of the instructor also plays a role.

Instructional decisions are shaped by the professional obligations at play. At times, professional obligations may be in conflict. At this point, an instructor may attend more substantially to one or more obligation to make their choice. For example, a mathematician's obligation to the mathematics discipline may conflict with the required tasks in a given curriculum, an institutional obligation (Chazan et al., 2015). Similarly, a desire to follow a students' reasoning (obligation to the individual) and to enculturate students into proof norms (obligation to the discipline) may be in tension leading to what Herbst (2002) calls the double bind.

Literature Review

The literature base on instructional decision-making in undergraduate mathematics has focused on a few studies and reflections. Schoenfeld et al. (2016) and Woods and Weber (2020) examined university mathematicians using resources, orientations, and goals framework. Schoenfeld et al.'s piece argued that the synergy or relationship between pedagogical content knowledge and mathematical content knowledge influenced the decision making of the mathematician. In using this framing, they found that mathematical correctness or precision was a driving factor in decision making for mathematicians. Woods and Weber (2020) interviewed eight mathematicians about findings from five different studies related to instructional practices in undergraduate mathematics. They concluded that goals of content coverage contributed to the use of lecture practices. Mathematicians' orientation towards precision similarly contributed to the traditional "chalk-talk" practices as it was viewed as allowing appropriate formal content coverage.

A well-studied decision is the decision to lecture. Woods and Weber found that while mathematicians recognized that lecture had shortcomings, they felt it allowed them to accomplish the goals that they had for the course such as content coverage goals, modeling authentic mathematical practices, or preparing future mathematicians. In a large-scale survey of abstract algebra instructors, Fukawa-Connelly et al. (2016) and Johnson et al. (2018) found that 85% of mathematicians chose to use lecture over non-lecture curricula. Further questioning showed that mathematicians often cited time constraints or lack of resources as common reasons why. Johnson et al. suggested institutional obligations accounted for mathematicians' decision to lecture such as time constraints or lack of support from the department. In spite of reporting these perceived constraints, the participants additionally explained that they did not feel pressure to cover specific content and that their departments would possibly support curricular redesigns

suggesting the link between institutional obligations and the decision to lecture was not clear cut. Several other studies have considered some of the decisions instructors make in the context of inquiry learning. For example, Mesa et al. (2020) identified three tensions that mathematicians experienced during the implementation of a mandatory IBL Linear Algebra curriculum. These tensions were between perceptions of IBL and institutional expectations; instructor knowledge and the textbook; and a final tension with pacing of the course. These tensions were underpinned by obligations to the discipline and institutional obligations conflicting with one another. Mesa et al. reported that disciplinary obligations guided a large portion of the decisions made. While the mathematicians were constrained by the curriculum imposed by their department, the mathematicians were in turn selective about definitions, examples, and application-based problems that they used. Mesa et al. did not identify any individual and interpersonal obligations in their data, and there were no claims that could be made regarding how those obligations contributed to double-bind situation. Johnson et al. (2013) studied some of the instructional decisions made by three instructors voluntarily teaching an inquiry-oriented abstract algebra curriculum. They found that the instructors differed in the weight they gave to content coverage aims and the engaging students in authentic mathematical practice. This could reflect a tension related to disciplinary aims (both practice and content) and institutional aims (expected course coverage). The role of course coverage was even more pronounced in Mesa's (2014) study of community college trigonometry instructors whose agency to make such decisions was limited.

Across these studies, disciplinary and institutional obligations are most salient. Our knowledge of these obligations is largely in the context of major instructional choices such as the decision to lecture. The purpose of this study was to examine instructional decisions as a more micro-level by focusing on the specifics of instruction related to quotient groups with attention to obligations to the individual.

Methods

This data comes from a larger dissertation study that examined the pedagogical decisions and approaches related to quotient group instruction. Data collection for this portion of the study involved 13 mathematicians from 6 universities across the United States. These mathematicians were selected via a combination of convenience and snowball sampling (mathematicians suggested other instructors who would be good to interview either because they were thoughtful about their practice or they approached instruction differently). Mathematicians additionally met the criterion of having taught an abstract algebra course since 2019. This criterion was added for the sake of recall. These mathematicians all took part in an initial semi-structured interview where they described their approaches to teaching quotient group. From these interviews five distinct profiles of instruction were developed and shared during the second interview. It is this interview that is the focus of the present paper.

During the second interview, the five profiles or vignettes (each of roughly one page in length) captured an array of instructional decisions including both decisions related to teaching style and those related to content presentation. Mathematicians read through these during the interview (one at a time) and responded to the vignettes as they went through the interview. By examining the decisions of others, we hoped to perturb the mathematicians to be more reflective of the decisions they make and articulate the obligations behind their choices. The five vignettes will be referred to as IOAA (Inquiry Oriented Abstract Algebra), IBL (Inquiry Based Learning), Worksheet, Equivalence, and DTP (Definition, Theorem, Proof). The inquiry vignettes were

heavily student-centered with the IOAA vignette having students reinvent quotient groups from informal ideas of partitioning, and the second having students work in the formal system to develop proofs of major ideas. The equivalence vignette contained a lecture where ideas of quotient groups were rooted in equivalence relations. The DTP vignette was also a lecture-based image of teaching, but it took a traditional approach of providing major definitions including a process for constructing cosets from a normal subgroup. Finally, the worksheet vignette included a similar approach but provided students time to work on worksheets after introducing the content.

Each vignette-response interview lasted approximately 75 minutes and was video recorded and transcribed. As the mathematicians read the vignettes, they were prompted to discuss anything they noticed (positive or negative) or that stuck out to them. For consistency across interviews, there were several talking points across the vignettes that I would bring up if it did not come up naturally. Those talking points included questions regarding training mathematicians and its relation to the participants job and comparing and contrasting the various informal understandings across the vignettes, examples, or questioning practices during instruction.

The data was analyzed for professional obligations using methods described in Shultz et al., (2019). This involved identifying references to each of the four obligations using codebook descriptions developed by Shultz et al. The first author coded each interview completely. Once coded, 10% of the transcript (for each interview) was sent to a second coder. The portion sent to the second coder was randomly selected post removal of discussion not relevant to the study (e.g., talking about technical difficulties was removed before selection). At this stage, there was mostly consistency (76%) on interpretations of obligations. Any disagreements were discussed and resolved.

Results

As results are discussed, we will present some of the findings of the study related to the individual obligations that mathematicians expressed with regards to content coverage. We begin by looking at the role of equivalence within content coverage followed by an examination of mathematicians' decisions in choosing curriculum. Please note that participants in the study are referred to as Dr. A through Dr. N. The vignette instructor does not have a title.

Obligation to the Individual in Content Choices: Equivalence and Intuition

Mathematicians were presented with various formal and informal understandings of quotient groups. In the IOAA vignette, the instructor emphasized the ideas of partitioning in which quotient groups were created via identifying subsets that build a new group structure. This hypothetical vignette instructor put forth that they made the decision to not emphasize formal equivalence relations as it was unintuitive for students leaving that exploration to homework. This is juxtaposed with the Equivalence vignette in which the instructor emphasized defining quotient groups using equivalence relations and classes first then determining that the equivalence relation being explored leads to classes that form a quotient group.

The decision to focus heavily or lightly on equivalence relations were both motivated by obligation to the individual (the students). For example, Dr. D (lecturer – equivalence)[1] stated "I'm very persuaded by the idea of equivalence relations being far more intuitive than cosets. I a

[1] The tag on mathematicians refers to the vignette that I suspected they most likely identified with based on initial interview.

million percent agree with them [the equivalence vignette instructor]." In this quote, Dr. D expressed that the decision to emphasize equivalence due to obligation to students to teach in an intuitive way. Dr. A (lecturer – worksheet) also expressed that she thinks equivalence relations are somewhat intuitive for students and should be covered stating "I think it's a lot more intuitive for students than this person thinks. How intuitive and how well it fits depends on what they've seen before and what they mean about it." Thus, Dr. A is making the decision to build off what students have covered previously which was equivalence relations. In these quotes from both participants, curricular decisions are being justified by their obligation to the individual to teach in a way that is intuitive for students.

In contrast, Dr. B (student centered - IOAA) rationalized the decision to focus on partitioning rather than equivalence relations because:

> You know, I think equivalence relations are very formal and borderline impenetrable for students in terms of them having like a conceptual footing in it and it ends with quotient group structure versus I have students begin with quotient group structures, abstract that idea and then we move to cosets.

This motivation is similarly made via the lens of obligation to the individual and the desire to teach in a way that is intuitive to students. However, the instructors appeared to have different conceptions and evaluations of what is intuitive to students. We hypothesize this may connect to what is valued in the discipline (reasoning from formal definitions) and differing pedagogical knowledge for teaching.

Obligation to the Individual: Teaching Practice and the Choice to Lecture/Not Lecture

As mathematicians engaged with the vignettes, there were often claims made about various teaching styles that extended more broadly than the context of quotient groups. For example, Dr. B (student centered – IOAA) shared their reason for why they adopted student-centered tasks saying, "I don't take curriculum tasks on lightly, without really evaluating whether I think that this is something that makes sense and that this is a way that's likely to be meaningful for my students." Dr. B makes the decision to adopt tasks (in particular, IOAA tasks) based on the perception of students which is an explicit acknowledgement that there is an obligation to individuals. Other instructors commented on the IOAA vignette illustrating something "accessible" for students or providing them opportunity to deeply learn particular content (D4 in this case).

However, unlike what is established in prior research, it was often obligation to the individual that also supported the decision to lecture and not use inquiry approaches. Dr. D (lecturer - equivalence) shared concern that students would not understand the point of an activity without the lecture. Other instructors (Dr. G and Dr. H) explained they thought students would "lose interest" or become bored. Thus, there is an obligation to keep a student engaged and to lead them to a satisfying conclusion, which instructors may see inquiry activities as not meeting.

Finally, we share that for many instructors, the obligation to the individual was not treated as a monolith, but rather the obligation was to support the subset of students continuing to graduate school. Dr I (lecturer - DTP) explained that he felt "inquiry-based learning, especially in something like abstract algebra and deprive[s] anybody, that somebody that wants to go on to grad school." He explained this approach would be good for a class of high school teachers.

In this case, the decision to train mathematicians stems from the academic need of the students going on to graduate school rather than it strictly being a disciplinary obligation to

steward mathematical practice. Dr. G (lecturer – DTP) expressed a similar sentiment with regards to graduate school by saying:

> And of course, if you want your students to have a chance to get into a good graduate program, you have to make sure they are not too much behind compared to students who apply from a school at [University]. You see, Yeah, well, it's difficult because we do get these good students in our classes too. To get them, they are just not as many as at [neighboring school] And so it's hard to cater to them without neglecting the whole rest. At the same time, it's hard to cater to everyone without neglecting the good students. So, it's really difficult.

Unlike Dr. I, Dr. G (lecturer) expressed a tension between meeting the needs of the "good" students without neglecting the other students in this class. As he talked through the amount of material he should cover in class, he seemed to express two separate obligations to two separate groups of individuals – those going to grad school in mathematics and those who are not. Those two sets of obligations created a tension in how he covers and presents material.

In contrast to concerns that the inquiry is not challenging enough for students, some instructors voiced that the inquiry-based learning approach would only work in situations where students came into the course with a strong grasp on proving. Dr. K explained:

> The one thing with our [university] seniors when I'm teaching Modern algebra is, is I also feel to some extent I'm teaching an introduction to proof course. Even though they've already had an introduction to proof it, it tends to be not evident. In terms of what they come with, and one thing that one reason I stick to the equivalence classes is the definition. The definitions of reflexive, symmetric and transitive are things with. Very structured definitions that inform the structure of the proof, so I like them to work with that definition and writing proofs so this may be so there […] I think it would allow me to do a little bit more of just what are the ideas and they can handle the proofs and the definitions later. And it might also it might make this approaching a bit more feasible that I'm saying. Because if they're strong in writing formal proofs, that may be actually decent of formulating definitions for their concepts too.

We point out that such responses reflect an obligation to the student (and that this obligation is shaped by the particular population of students at a given institution) and the decision to lecture is supported by the desire to provide students with material they can succeed at.

Discussion

Due to space constraints, we shared only some of the data related specifically to individual obligations. Our guiding research question informing this report was: In what ways do individual obligations influence the content coverage decisions made in Abstract Algebra courses? We begin by discussing variation in pedagogical decisions that stem from individual obligations followed by a brief discussion about the composition of individual obligations and their role in decision-making.

Individual Obligations on a Spectrum

As a result of the data analysis, we noted that individual obligations seem to exist on a spectrum which allows for the possibility of opposing decisions to be made with the same justification. While mathematically equivalent, a major distinction between the mathematicians was how they defined quotient groups: either as a collection of cosets along with a binary operation that partition a group G (building from informal) or imposing an equivalence relation ~

onto a group G (building from formal). These alternative approaches caused a divide between what participants emphasized. Justifications regarding this divide stemmed largely from obligations to the individual. Recall that mathematicians such as Dr. A or Dr. D felt that equivalence was an important topic that required more attention than just being a portion of the homework. However, on the other end of that spectrum, Dr. B felt as if it was a better decision to not emphasize formal equivalence relations outside of the homework. All of these instructors' justifications were rooted in ideas of what is intuitive and accessible for students. That is, the same perceived obligations can lead to different outcomes, and this influences not just general approaches to instruction, but also smaller content decisions.

Tensions within Individual Obligations

Previous research such as Woods and Weber (2020) noted that mathematicians viewed training mathematicians as an important part of their job, and this goal led mathematicians to engage in lecture practices. While the data supports this conclusion, it also provided more insight into the role that individuals have in the decision-making process. As participants discussed their curricular decisions and what approaches to take, it became apparent that participants viewed their classes as having two competing groups of individuals that had a stake in what was taught in the course. As Dr. G expressed, there were students in the class who were going on to pursue more mathematics and then students who were not. In a sense there was an obligation to train or prepare the students going on to graduate school. However, the students not going to graduate school in mathematics presented a different set of needs. Dr. I framed these individuals as teachers/ those not pursuing more mathematics and those going on to graduate school. Regardless of the title given to the groups of individuals, the two groups imposed competing choices on the mathematician with respect to the content that they covered. In some cases, the instructor prioritized the students going to graduate school and in other cases, the tension remained unresolved as to what was the best course of action to take. We emphasize that the decision to lecture or not lecture was very much shaped by obligations to the individual not just towards obligations to the discipline or to the institution.

Conclusion

In this report responses from a study including 13 mathematicians regarding instructional decisions were discussed. By and large, the mathematicians' choices were influenced by obligations to the individual (the students). However, this obligation appeared to play out differently related to several dimensions including: what the instructor perceives as intuitive to students (which we conjecture may be related to both pedagogical content knowledge and distinctions between intuitive, accessible, and its meaning in deductive mathematics), what group of students is prioritized (those going to graduate school or the broader class), and instructor beliefs about what students are capable of. (such as elite students needed to engage in IBL).

As we consider future directions, we intend to further examine the role that professional obligations play in the teaching of quotient groups and abstract algebra. We note a limitation of this analysis and future analysis is that the data set is limited in terms of reflection rather than in the moment teaching decisions. Even so, we intend to expand from individual obligations to also examining the role of disciplinary, interpersonal, and institutional obligations. Furthermore, we feel that it is worth examining how these decisions impact student learning. We also intend to examine the role of intuition further. This includes investigation into what mathematicians consider to be intuitive and how that is related to the decisions that they make.

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