Implementing Playful Mathematics Tasks in Calculus Sections

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This preliminary report explores how students experience playful mathematics tasks in sections of a large Calculus I class. We implemented tasks that have been established to foster playful mathematical engagement and learning in 5 sections, and examined whether students experienced playfulness across multiple dimensions. We found that students enjoyed the class, felt engaged, and considered themselves to have understood the content, which they rated as at least somewhat challenging. These findings are being used to design professional development for future Calculus I teaching assistants, in order to examine potential scalability of playful mathematics and learning to lecture-based classrooms.

Keywords: playful mathematics, calculus, undergraduate

There has been increasing interest in examining how playfulness can be incorporated into undergraduate mathematics as the productivity of play for mathematical learning across age groups is becoming more acknowledged. However, the undergraduate research thus far has focused on smaller seminar-style courses (Holton et al., 2001, Ellis et al., 2022), is often conducted with mathematics education majors who are well equipped to handle the ill-defined nature of many mathematical play tasks (Ellis, Horne, & Bloodworth, 2024; Ellis, Bloodworth, & Horne, 2024), and is usually taught by the researcher who is well conversant with supporting mathematical play. This preliminary research report addresses the first year of a multi-year study, in which the authors implemented playful calculus tasks in five Calculus I sections of a large lecture class, as the first step in understanding how to support such students in playing with and learning about mathematics. The second year will focus on conducting professional development with the section TAs (mathematics graduate students) so that they can implement playful calculus tasks in their own sections. We report on the initial results of the first year of the study, and outline our consequent plans and goals for both conducting the professional development workshops and evaluating the success of student learning from the playful mathematics tasks.

Background

Mathematical play has been well valued in early childhood, as a productive and powerful way for young children to experiment with mathematical patterns in the world (e.g., Wager & Parks, 2014). Scholars have theorized that mathematicians and young children both play with mathematics in similar ways (e.g., Williams-Pierce, 2019), and recently, Horne, Bloodworth and colleagues (2022) have empirically confirmed that mathematicians do in fact play with mathematics. They interviewed mathematicians and found evidence of mathematical play while problem solving, including markers of open exploration, enjoyment, investment, perseverance, and flow. An open question, therefore, is how mathematical play can be supported after early childhood with learners who are not mathematicians. Researchers have begun investigating how to do so with older children, but still primarily with youth, not college aged students (e.g., Gresalfi, 2015; Williams-Pierce & Thevenow-Harrison, 2021).

A smaller but no less important community of researchers have begun investigating mathematical play in undergraduate and graduate courses. For example, Ellis and others have designed a variety of tasks specifically for undergraduates addressing covariation and function graphs, calculus topics such as derivatives and integrals, and tropical algebra, and found that playful tasks with intrinsic feedback and ample opportunities for problem posing can foster both playful engagement and learning. Instead of using the term 'mathematical play' in their work, they use the phrase 'playful mathematics,' "to describe the activities…and features of an instructional environment that can facilitate mathematical play" (Ellis, Horne, Bloodworth, & Ely, 2025, p. 2). We consider playful mathematics to be a conceptual subset of mathematical play, and one that is particularly well suited to the complex social learning contexts of undergraduate mathematics courses.

Given this considerable evidence that playful mathematics could be a powerful tool for supporting productive mathematical learning, it behooves us to understand how we could support larger-scale implementation at the undergraduate level. We focus here specifically on Calculus I, because it is commonly taught as a large lecture class, and can be a gatekeeper to STEM degrees and careers. While our long-term goal involves designing and implementing professional development for the TAs who run the smaller sections of the large lecture-based classes, and evaluating the influence of the mathematical play on students' calculus learning, the study we report on here is the foundation of that longer-term work. In particular, we wanted to better understand how students experience playful calculus tasks after being used to more traditional mathematical learning environments. Consequently, we asked: How do undergraduate students in a traditional calculus section experience playful calculus tasks? In particular, we investigated how they self-report on differing dimensions of engaging in playful calculus tasks, ranging from how much they believed they learned to how much they enjoyed (or did not enjoy) the tasks.

Methodology

This study collected data from participants in five different sections of two Calculus I courses, taught by the same professor. This 4-credit course at a large public university had a 50-minute lecture three times a week and a 50-minute section twice a week, so students engaged with the course content daily. Each course had 100 to 150 students, and each section was capped at 25 students. Researchers received permission from the professor and TAs to implement playful calculus tasks in seven different sections towards the end of the semester; in two sections, the students all opted out of participating in the study, resulting in data from five sections.

Playful Tasks

The playful tasks used in this study come from previous research (Ellis, Horne, Bloodworth, & Ely, 2025), and address the construction of function graphs as traces of covariation between area and length of swept shapes. For example, an animated growing rectangle would be shown to the students, and they would be asked to 'sketch' a graph on two axes, where y is the area and x is the length, and given no other information. There were no given labels or quantities (i.e., students could not calculate area or length numerically), and students would discuss their reasoning in small groups and then share with the whole class. Each section completed three growing shapes tasks (rectangle, triangle, triangle-rectangle-triangle), and in only one section did we manage to the fourth task, where students would create their own growing shape.

The primary feedback the students received on their work was through their peer discussions, as the researchers limited themselves to asking questions and encouraging comparison between different student graphs. Refer to the above citation for more details on the task design, and how they foster playful mathematics. The research described in this paper is unique from the previous research in that the tasks were implemented in sections of a large lecture course with undergraduates seeking to fulfill benchmark requirements for their degree programs, whereas other research involved small-group middle school teaching experiments (e.g., Ellis et al., 2025), and undergraduate teaching experiments (Ellis, Bloodworth, & Horne, 2024; Ellis, Horne, & Bloodworth, 2024). The students were ostensibly quite familiar with the concept of covariation and the process of constructing function graphs, prior to data collection, given that they were close to finishing a Calculus I course.

Data Collection

The surveys were adapted from Riling, Dietiker, and Gates (2019), who developed a tool for evaluating students' mathematical experiences in individual lessons. These surveys are brief (one page), with 8 different items, specifically geared towards capturing a variety of student perspectives on a mathematical lesson. In Item 1, students are asked to pick up to three descriptions that 'best describe' their view of the lesson, ranging from positive valenced words such as Fun, Interesting, Surprising, to negative valenced words such as Boring, Stressful, Not Special. Item 2 asks them to rate how they felt they understood the topic - that is, evaluating their learning - with three options: Don't Understand, Somewhat Understand, Understand. Item 3 asks them to choose the best option for how they 'felt,' ranging from "Very bored: I was very bored and wish we would do something different" to "Very interested: I was very interested and the lesson was really fun and/or intriguing." Finally, Items 4-8 were 4-item Likerts ("Strongly Disagree" to "Strongly Agree") with the following prompts: Time flew by; The content of this lesson was relevant to my life; Today's math class was like most days in this class; I wish more lessons were like this one; and The content of today's lesson was challenging for me.

We only included very clean data for this analysis, and plan to incorporate the messier remains in later analyses. In particular, we only included surveys that were administered after the first playful task and at the end of class, discarding surveys administered in the middle (due to inconsistencies in data collection across sections), and surveys found after class with no identifiable temporal provenance. We also removed entire surveys when the respondent indicated more than three descriptors in Item 1. When a single item on a survey was left blank, we replaced it with the average value of other respondents; when someone chose a value between two values on the scale, we used the average value of all the respondents to determine whether to round up or down. We are in the process of incorporating the messier data that we removed for this preliminary analysis, for our next phase of analysis. We used the R base package (R Core Team, 2024) to conduct descriptive statistics on the resulting 85 surveys.

We also collected video, audio, and survey data from consenting students in those five sections ($n = 50$). We are still working on the transcription and thematic coding of this data due to unexpected data complexities, so the primary data source for this paper is the survey data.

Results

Item 1: Positive and Negative Valenced Words

This item asked respondents to choose three terms that best described their experience with the lesson. We enumerated the positive and negative valenced words, then summed them across students. In the 85 entries, positive valenced words accounted for 201 of 238 choices, and negative valenced words accounted for the remaining 37. On average, each survey had 2.36 positively valenced words, and the medians of 3 (positively valenced) and 0 (negatively valenced). These results indicate that students overwhelmingly enjoyed the playful tasks.

Item 2: Rating their Learning

This item asked students to rate their learning, and whether they considered themselves to have understood the lesson. We assigned values to the three possible responses: 0 (Don't understand); 1 (Somewhat understand); and 2 (Understand). We found that the median response was Somewhat understand, with an average of 1.29, above Somewhat understand. Consequently, we concluded that most students considered themselves to at least mostly understand - in other words, the students generally considered themselves to have learned during the playful task. This result was particularly intriguing when the results of Item 1 are also considered: the students positively described their experience engaging in the tasks, *even though* they only felt like they 'somewhat understood' the mathematics.

Item 3: Rating their Engagement

Item 3 asked students to choose which of four options best described how they felt about the lesson overall: 0 (Very bored); 1 (Uninterested); 2 (Interested); and 3 (Very interested). *Interested* was the most common response by far ($n = 56$), followed by *Uninterested* ($n = 20$), *Very Interested* ($n = 8$), and *Very Bored* ($n = 1$). We considered this to indicate that students tended to be interested in the playful calculus tasks.

Items 4-8: Likert Scales

These five items were developed specifically to triangulate earlier items and test other potential relationships (Riling, Dietiker, & Gates, 2019), and we examined them together. These 4-item Likert scales were forced choice, ranging from Strongly Disagree to Strongly Agree.

Table 1. Response Frequencies of Items 4 through 8. SD = Strongly Disagree; D = Disagree; A = Agree; SA = Strongly Agree.

Prompt	SD	D	A	SA
Item 4: Time flew by	4	19	48	14
Item 5: The content of this lesson was relevant to my life	14	50	21	0
Item 6: Today's math class was like most days in this class	41	38	6	0
Item 7: I wish more lessons were like this one	9	17	51	8
Item 8: The content of today's lesson was challenging for me	15	44	22	4

Students disagreed that the content of the playful calculus tasks were relevant to their lives, disagreed that the content of today's lesson was challenging for them, and tended to disagree/strongly disagree that this was a typical day in the class. On the other hand, students tended to agree that time flew by, and that they wish more classes were like this one. We unpack the complexities of these results in the discussion.

Discussion & Conclusion

In summary, we found that students enjoyed the class (Items 1 and 7) and felt engaged (Items 3 and 4), felt like they somewhat understood the content (Item 2) and agreed that it was at least somewhat challenging (Item 8). They did not consider the content particularly relevant to their lives (Item 5), and they felt that this class session was quite different from other days in section (Item 6). We conclude that our students overall had a positive experience engaging in these tasks, and likely experienced playful mathematics. As a result, we are using these preliminary results to guide the next step in our research.

We are delighted that these playful mathematics tasks appear to have worked as intended outside of their original design context of teaching experiments. In particular, the findings indicate that the students experienced several hallmarks of mathematical play, such as challenge and enjoyment (e.g., Ellis et al., 2022; Williams-Pierce & Thevenow-Harrison, 2021). However, we were unsurprised that students found this class session different from their previous experiences (Item 6). When the first and second authors were originally collecting data, we changed the chairs in each classroom so that students were grouped or facing each other, instead of facing the front of the classroom in neat lines. Several times, students walked in, looked at the chairs, and immediately walked out again to check the room number - in other words, their initial reaction to a non-lecture set up in the classroom was, *this must not be my class*. This anecdotal evidence was supported by the quantitative findings, and if we use Item 6 as a proxy for previous experiences in class, we consider it highly likely that students felt more enjoyment and engagement (as evidenced by Items 4 and 7) with these tasks than in the typical lecture-based section. However, there are numerous potential reasons why they may have experienced these tasks differently, such as the sheer novelty of a different instructor or being part of a research study. Our future research will dig deeper into these potential reasons and other limitations, as well as investigate our professional development efforts with TAs, and the resulting playful mathematics experiences of their students.

Acknowledgments

Many thanks to Dr. Kendall Williams and his TAs for sharing their classrooms, students, and insights! Thanks also to our anonymous reviewers, who were kind even as they critiqued – a wonderfully helpful balance when in the preliminary stages of a research project. Finally, thanks to INFO's HCIM Travel Awards for generous travel funding - much appreciated!

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