Reflections on Creativity and Affect as a Vehicle for Writing in a Mathematics Course

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We explore how one instructor used reflective prompts in a proof-based Elementary Number Theory course to add a writing component to the course. Students consistently reflected on their creative process and affective domain throughout the course. We share the design of the course, samples of the reflective prompts, and our deductive coding of them using two codes: creative process and affect. A preliminary examination of written responses from two students indicates that reflecting on their creative process had led them to be more engaged in the proving process in this course and other courses. They also reported that the readings and reflections helped them recognize and validate their mathematical actions, processes, and affective outcomes.

Keywords: Writing in Mathematics, Mathematical Creativity, Affect, Reflections

Writing has long been considered a unique and valuable mode of learning (Emig, 1977). To promote writing at the tertiary level, institutions have utilized two ways of fostering students' writing skills: writing in the discipline (WID) and writing across the curriculum (WAC). In WID courses, "writing is occurring in some form as assignments in subjects or courses in one or more disciplines" (Thaiss & Porter, 2010, p. 538). WAC typically is meant to "assist teachers across disciplines in using student writing as an instructional tool in their teaching" (Thaiss & Porter, 2010, p. 538). Using both types, instructors can tailor writing assignments to develop the specific content of a discipline as well as promote students' curiosity, creativity, metacognition of processes, affect, and expression of target material through a student's individual framework.

In this study, we focus on writing in a proof-based undergraduate Elementary Number Theory course. We report on how one instructor designed and implemented various forms of writing assignments in this course. Writing is particularly suited to this course because it offers students opportunities to write proofs through generating examples, using mathematical definitions and theorems, utilizing various symbolic representations, connecting assumptions to conclusions, and explaining reasoning. While infusing writing in proof-based courses has been researched and documented, we highlight how a focus on students' creative processes and affect can both increase student engagement with content and enhance their emotional value in process, while serving as a vehicle for transforming this course into a writing course. Specifically, we address the following research questions: *How to infuse written reflective assignments on creativity and affect into a proof-based math course? How do students interact with and perceive these assignments?*

Background Literature and Theoretical Framing

Writing in Mathematics

It has been reported that writing in mathematics courses provides students with a deeper understanding of mathematical concepts (Rose, 1989; Beidelman et al., 1995; Burns, 2004). Burns (2004) stated, "Writing in math class supports learning because it requires students to organize, clarify, and reflect on their ideas" (p. 30). Burns added, "when students write, their papers provide a window into their understandings, their misconceptions, and their feelings about the content they're learning" (p. 30). Existing literature shows that writing in mathematics courses (e.g., Urquhart, 2009; Taylor & McDonald, 2007; Graham et al., 2020) can take various forms. Writing assignments can range from short reflective questions (e.g., El Turkey et al., 2023) to daily journals in which students keep their reflections and insights on the course topics (Borasi & Rose, 1989). Other forms include summary writing, writing portfolios, and expository essays. For example, Omar et al. (2019) used writing portfolios in a Combinatorics course where students communicated their solution attempts as well as their reflections on their problem-solving process and creativity. Expository essays are usually assigned so students explain mathematical concepts to others, most often to the teacher.

Many studies explored the use of writing in various mathematics courses. Some discussed writing assignments in general education mathematics courses (e.g., Pinter, 2014), History of Mathematics courses (e.g., Loats et al., 2014), Abstract Algebra (Czerwinski, 1994), Modeling (Linhart, 2014), Complex Analysis (Habre, 2019), or Combinatorics (Omar et al., 2019). In Omar et al.'s study (2019), students were prompted to reflect on their creative process, but writing was not the focus of the study. Beyond that study, we did not find literature focused on infusing writing in a mathematics course using reflections on creativity and affect.

We situate our discussions of using reflections and writing as modes of learning and integratory tools within expressionism (Elbow, 1991), as writing provides space between thought and expression to process the emotional value of information (Flower et al., 1992; Overstreet, 2022). This emotional value is essential to both motivating students to examine target material, and in spurring them to creative thought within a discipline (Auger & Woodman, 2016).

Mathematical Creativity

The creative process for problem solving was described by Wallas (1926) through a four-stage process: preparation (understanding the problem and gathering relevant information for a proof), incubation (taking a break and letting the mind work on a proof subconsciously), illumination (AHA moment, insight, or generating an idea), and verification (verifying the idea is a solution or a proof). Later, Guilford (1950) developed three components of creativity: fluency (number of ideas), flexibility (changing ideas or approaches), and novelty (unique or original ideas/approaches). Torrance (1974) added a fourth component of creativity, elaboration (describing or elaborating on those ideas). These earlier studies led to a plethora of definitions and perspectives of mathematical creativity (Mann, 2006). Some focused on the creative process (Kozbelt et al., 2010) while others discussed a product view (Runco & Jaeger, 2012). Additional studies discussed mathematical creativity that is relative to students versus absolute creativity relative to the field of mathematics (Beghetto & Kaufman, 2013; Leikin, 2009).

In this paper, we adopt a process-oriented relativistic perspective that defines mathematical creativity as a process of offering new solutions or insights that are unexpected for the student, with respect to their mathematics background or the problems they have seen before (Liljedahl & Sriraman, 2006; Savic et al., 2017). Thus, we acknowledge that students "have moments of creativity that may, or may not, result in the creation of a product that may, or may not, be either useful or novel" (Liljedahl, 2013, p. 256). This view of mathematical creativity aligns with the developmental theoretical perspective emphasizing that a person's creativity can and does develop over time and a person's process and actions, as opposed to the product created, should be the focus of investigation (Kozbelt et al., 2010).

Affect

We approach affect broadly in this study following recommendations from the literature (e.g., Tang et al., 2022). Grootenboer and Marshman (2016) acknowledged the difficulty in defining the affective domain but conceptualized it largely as beliefs, values, attitudes, and emotions that overlap. For this study, affect encompasses feelings (McLeod, 1988) and other constructs such as beliefs, attitudes, emotions, motivation, engagement, and so forth (Middleton et al., 2017). We also adopt Tang et al.'s (2022) definition of an affective outcome as a "collective of any emotions, beliefs, attitudes, and values that students reference" (p. 31). Tang et al. additionally highlighted the importance of examining the intersection of students' mathematical creativity and their affect, which we undertake in this study through the design and implementation of reflective prompts attending to both constructs.

Methods

Study Setting

The study took place in an Elementary Number Theory course at a comprehensive northeastern university in the United States. Ten students were enrolled in the course which met three times a week (50 minutes) on Zoom in Fall 2023. The instructor (first author) characterizes the classes as interactive lectures, each of which began with time for student questions, a brief introduction of new content (on OneNote using a stylus), followed by elicitation of target material through student discussion. In this course, the instructor created and assigned three types of written assignments: pre-class assignments (PA) (N=35), homework assignments (HW) (N=10), and a complete proof portfolio for all proofs from class (N=1). In addition, there were two exams and a final exam. Five of the PAs (PA 8, 11, 18, 23, 35) included reflective questions whereas each HW assignment had the last question as a reflection prompt. Reflective questions were graded based on completion, with some comments from the instructor.

Data

The data collected for this qualitative study includes the assignments, students' work on them—including their answers to reflective questions—and end-of-term interview data with two students. We focus in this paper on the assignments themselves to explore the instructor's design intentions as our unit of analysis to document how the instructor promoted writing within this course. Additionally, we share sample written responses from the two interviewed students.

Data Analysis

Reflective assignments. Using deductive coding (Patton, 2002), we coded the reflective questions (N=15) on the pre-class assignments and the HW assignments. The codes used were creative process and affect. The creative process code encompassed any prompts that referenced actions from Creativity-in-Progress Reflection Tool (Savic et al., 2017) (7 prompts), mathematical creativity in general (3 prompts), Wallas's four-stage process (2 prompts), and AHA/memorable moments (2 prompts). The affect code encompassed any prompts about feelings (6 prompts), attitudes (1 prompt), math story/journey (1 prompt), and teaching actions related to affect (2 prompts). It is important to note that some prompts were coded with both codes. Below we provide a sample of reflective questions with their codes in brackets.

Students' written responses. We report on written data from two student participants who also volunteered for the end-of-semester interview: Amity[1] (pronouns she/they; mathematics major, senior, final grade A) and Smiley[2] (pronouns she/her; actuarial science major, senior, final grade B). In a word count of all their responses to the reflective questions, Amity used around

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[1] Self-selected pseudonym

[2] Self-selected pseudonym

2700 words while Smily used around 1600 words. These were in addition to the mathematical writing that they submitted on the various assignments. In the following, we share our deductive coding of their written responses to three reflective questions. We underline certain parts of their responses to capture the creative process theme and use *italics* to capture the affect theme.

HW 9 [Creative Process & Affect]: Becoming more aware of creativity and how it functions can lead to deeper immersion in problem-solving or proving processes. Has reflecting on your creative process made you feel more engaged in these processes? Explain (Using specific examples is encouraged.)

In response, both students highlighted how reflecting on their creative process has led them to be more engaged in the proving process. Amity wrote, "trying to find different creative ways to do problems definitely makes me *feel more engaged* in trying to figure out the solutions or proofs and helps me understand it better." Smiley explained, "reflecting on my creative process has made me *feel more engaged* in the proving process. Sitting and actually thinking about how I am solving problems makes me *feel like I am actually apart* [sic] *of the proving process* instead of just skating through it." She then added, "Reflecting gives me the chance to edit and revise my proving process… For example, I now sit in the incubation stage more when I am proving, and I have found that it helps out a lot."

PA 11 (part 5) [Affect]: Please discuss an affective outcome you felt while working on a problem or during class.

On this assignment, the students were asked to read parts of the Tang et al. (2022) article about students' affect when doing mathematics. In response to the above prompt, Amity wrote, "I *feel a lot of enjoyment* while doing work for this class. Figuring out how to prove something always feels like a puzzle to me, and when I finally figure it out it's *so satisfying* and *makes me excited* to do more work for the class." Smiley wrote, "I have *felt confidence* in class when you ask me a question that I am actually able to get right. It makes me *feel good* to know that I am understanding the concepts that are being taught."

PA 35 (part 5) [Creative Process & Affect]: Do you think that creativity affected your actions in the course, e.g. what you do during or out of class when working on math, how you problem solve or prove, etc?

Both students indicated how being aware of their creative process helped them in the course as well as other courses. For example, Amity responded, "the creative mindset I've developed from this class has assisted me in my other math classes as well." She elaborated, "In my Complex Variables class, I have found myself becoming much more flexible in my problem solving and finding solutions when I get stuck on something." Smiley wrote, "creativity affected how I problem solve the most. Creativity allowed me to try numerous ways to solve a problem whether it was attempting different proof methods, finding counter examples, or just using trial and error." She added, "As I do problems in the future, I definitely feel like I will be more creative because of this course."

In one of the questions on this last reflective assignment, the instructor asked students about some takeaways they learned from the reflective questions. Amity wrote, "I learned that there are names for the different mental processes I use and how to better make them work together." Smiley stated, "my biggest takeaway was learning about the four stages of solving a problem." She then elaborated on the preparation and incubation stages of Wallas's four-stage creative process. She concluded, "I will definitely use this four-stage process in my other courses."

Preliminary Results

In addition to fostering students' writing in an advanced mathematics course, two main design intentions or instructional goals were captured in the reflective questions: 1) increasing students' awareness of their creative process; and 2) providing students with opportunities to recognize and acknowledge their affect while doing mathematics. The written responses from the two students indicated a recognition of their own creative process. They elaborated on how reflecting on their processes allowed them to better understand the content and be more engaged in the proving process. Their recognition of the creative process through reflections has also promoted flexibility in trying different attempts or proofs, which has assisted them in other math classes. In terms of affective outcomes, both students reported positive feelings such as enjoyment, excitement, feeling good, and confidence. They also linked the creative aspect of writing proofs to their positive feelings which aligns with findings in Tang et al.'s study (2022). We claim that our reflection prompts provided students an opportunity to closely examine their affect while doing mathematics, a practice that is uncommon in mathematics teaching and learning.

Discussion

The instructor designed numerous reflective prompts utilizing the Creativity-in-Progress reflection tool, Wallas's four-stage creative process, and affect. Through the various assignments, the instructor attended to both WID and WAC components of writing courses. While the context of the study was Elementary Number Theory, we hypothesize that the reflective prompts can be adopted and adapted to any upper-level mathematics course that involves problem solving or proving. It is important to note that the consistency in assigning reflective prompts throughout the semester established a norm and an expectation for students to 'sit and actually think' (Smiley) about how they solve problems. In addition, most of prompts were very specific and aimed at focusing students' attention on a specific mathematical action, an affective outcome, or a moment in the course or their mathematical journey.

In terms of limitations, the online nature of the course added a layer of difficulty in having in-class discussions about creativity and affect. While the instructor integrated conversations about creativity and affect during the Zoom class sessions, there would have been more engaging conversations if the class was in person. Additionally, only two students volunteered for interviews, which limited capturing various students' perspectives about the reflective questions.

Conclusion

Writing skills are central to most professional careers, especially in STEM disciplines. According to the CUPM Curriculum Guide, "Clear communication in oral and written forms is essential; students should develop these skills as undergraduates both through writing courses and by preparing written class and project reports" (CUPM, 2015, p. 47). As a mechanism to encourage writing in mathematics, reflections play an important role in mathematics learning as just completing tasks is insufficient (Wheatley, 1992). With these goals in mind, this study explored the design goals of reflective questions that prompted students to read, think, and write about their mathematical creativity, creative process, and affect. In the future, we will examine the students' interviews to learn about how they perceived those reflective questions. We also plan to provide some recommendations for instructors who plan to integrate writing into their mathematics courses.

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