GTA's Categories of Perception of Precalculus Students' Covariational Reasoning

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Keywords: graduate teaching assistants, covariational reasoning, student thinking

Herbst et al. (2023) published four categories of perception teachers use when considering student contributions: normativity, serviceability, responsiveness, and framing. The authors argue that though attending to the mathematical sophistication of a students' work is important, the task of teaching requires teachers to attend to more, stating that these categories provide additional relevant insights teachers can and should make. Though described with a geometric problem, the authors argue their relevance to other topics. In this study, I used this framework to identify potential categories of perception when interacting with students engaged in tasks offering opportunities to engage in constructing and interpreting graphs via covariational reasoning—reasoning about two quantities changing together. Thompson and Carlson (2017) argued for the importance of this reasoning in supporting students' conceptions of rate of change in precalculus courses and beyond. The Bottle Problem (Carlson et al., 2002), in which students are given a bottle and asked to construct a corresponding volume-height graph, has been widely used to support this end. In this study, I analyzed how graduate teaching assistants (GTAs) reason about various bottle problems and then assess video of precalculus students working on the same problems to understand how these GTAs perceive these students' reasoning.

The group of 11 graduate students were enrolled in a one-credit hour seminar on teaching undergraduate mathematics. In a course assignment, the GTAs first read an introduction to procedural vs. conceptual understanding, which includes an introduction to Carlson's Precalculus Concept Assessment (Carlson, Oehrtman, and Engelke, 2010) and potential student responses to a bottle problem (Abell et al., 2024). Then, the GTAs completed 8 total bottle-graph pairs and submitted a 3-5 minute video of their reasoning. Then, the GTAs watched two precalculus students explain their reasoning on the same problems and submitted their reflections on those videos, including sharing how they would respond to those students. The student videos were strategically selected; one student exhibited amounts of change reasoning, and another student used phenomenological language about "speed".

To attend to the GTAs' responses to students' covariational reasoning, the following analysis occurred. First, GTA responses were coded for correctness (final graph shape), framing (color coding, labeling), normativity (production of reasonable responses), and serviceability (attention to covariational reasoning/mathematical relationships). Second, GTA responses to the precalculus student videos were analyzed according to the same categories of perception.

Results show that the GTAs do attend to various categories of perception, though at varying degrees. The poster will detail overall patterns in the categories of perception as well as consistency between what the GTA attended to in their own work vs. the students' work. Overall, this study points to a continued need to learn about how to support students' covariational reasoning at the undergraduate level by considering what current categories of perception instructors attend to, and particularly how we can support GTAs in intentionally considering a broad range of categories when analyzing student work.

Acknowledgments

Thank you to Jessica Tolchinsky for her ongoing research with GTAs in this project.

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