Is it a Simple Mistake?: Tutors' Perceptions of Student Errors


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*Undergraduate math tutors frequently interact with students who are currently at a point of impasse or who have experienced failure and are seeking help. This affords tutors the opportunity to impact students' understanding of mathematics as well as redirect their study efforts. Helping students accurately classify their mistakes is a worthwhile effort, but is challenging and requires training and support. However, to provide this training for tutors, we need to know how tutors view student errors. This report details preliminary results from a survey given to current tutors, asking them to classify authentic student mistakes on a test as "simple" or "not simple." We discuss multiple reasons tutors give for their classifications, including some which stand in contrast to those we might use as researchers.*



*Keywords:* error analysis, tutoring, college algebra, simple mistakes


Experiences in first year mathematics courses influence persistence in Science, Engineering, Technology, or Mathematics (STEM) majors (Seymour et al., 2019). The presence of a math tutoring center, outside the classroom, has been shown to be significantly related to a positive student attitude toward mathematics (Bressoud et al., 2019). A key element of tutoring is identifying student mistakes and addressing students' misconceptions. Consider the following student work from a College Algebra exam.

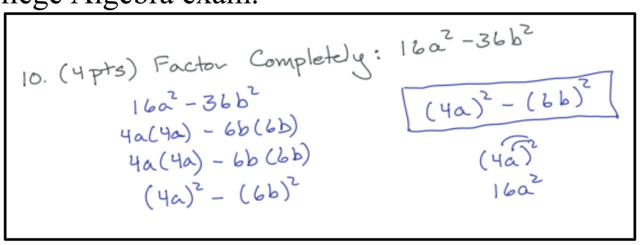


*Figure 1. A student's work on a College Algebra exam problem asking students to complete factor $16a^2$-$36b^2$.*

When presented with the student work (Figure 1), some undergraduate math tutors classified the mistake as being simple and others did not. A tutor who felt that the mistake was simple stated, "The student recognized that both 16a^2 and 36b^2 are both perfect squares…I think this mistake is just a lack of pattern recognition and can be solved with just more reps and familiarity." In contrast, another tutor felt that the mistake was not simple, stating "I don't think they understood what the question was asking and knowing what to do when asked to factor is important." When people in support roles, such as tutors, provide assistance to students, their perceptions of students' errors can impact the type of assistance they decide to give. However, tutors are often expected to provide guidance and instruction without training in how to analyze student work or respond to it. This preliminary report discusses how tutors classify College Algebra students' errors on an exam, specifically addressing the question, *What justifications do undergraduate math tutors provide when classifying a student's error as simple or not simple?*

## Literature and Theoretical Perspective

Errors are typically described as a deviation between "the current state and the desired state" or a lack of adherence to convention (Tulis et al., 2015). Mathematical mistakes may be a result of misconceptions or “inadequate cognitive schemes” or due to a mental “slip” (Molina et al., 2017). We distinguish between a “simple mistake,” which is “one that could be made accidentally, would likely not be repeated, or violates a mathematical convention rather than a rule” and a “not simple mistake” which provides evidence of “lack of conceptual understanding” (Ryals et al., 2020). Prior work has shown that students and teachers view mistakes differently (Lannin et al., 2007), and students are more likely than researchers to categorize their own errors as simple mistakes (Ryals et al., 2020).

Our study is motivated and guided by our valuing the “constructive error climate,” which Benecke and Kaiser (2023) describe as “one where both teachers and students acknowledge errors as potentially helpful” and “errors are analyzed and used as learning opportunities” (p. 164). The traditionally negative view of mistakes as something to avoid, is shifting toward a more positive view of mistakes as contributing to learning (Borasi,1987; Dweck, 2006). Reflecting on mistakes helps learners establish correct mental models (Kapur 2010; Kapur 2014; VanLehn et al. 2003). Still, students do not often use errors to learn, as learning from errors is challenging and more demanding than simply looking at a correct solution (Kyaruzi et al, 2020; Heinze et al., 2012). Students need guidance in learning from mistakes. While researchers have shown that beliefs and self-regulation strategies are important, teachers still concentrate primarily on content rather than on problem solving, metacognition, and beliefs (Schoenfeld, 2022). They typically do not direct students’ attention to ways to avoid similar errors in the future (Rach et al., 2013). We therefore recognize an important opportunity for tutors to support individual students to analyze, understand, and learn from their errors.

While new concepts are typically introduced during class time, it is during office hours and tutoring sessions that students can discuss their mistakes. Undergraduate peer tutors view mistakes from two perspectives; they recognize their own mistakes as students, and they recognize common mistakes they see in their students’ work (Ryals et al., 2022). Peer tutors also work in the environment of a tutoring center where the teacher is not present and the student may be more comfortable discussing their mistakes. While tutors have a unique opportunity to help students understand and learn from their errors, and do so in a positive manner, they would likely need training to do so (Johns, 2022). To provide effective training, we first must determine how tutors view students’ mistakes and how those views differ from our own.

## Methods

### Data Collection

We designed a survey with six student errors from a College Algebra exam, including the one presented above. We selected these examples strategically, wanting to provide a mixture of examples that we, as researchers, considered simple and not simple errors. We also wanted to choose student work that contained one specific error, rather than several, so that attention could be drawn to that one error for analysis. Each problem itself along with the student work was displayed in the survey, and we provided a short description of the error that was made. We asked the tutor to classify the error as Simple or Not Simple and to explain their choice. We did not give the tutors a definition of a simple mistake and did not provide examples of mistakes we would consider simple or not simple. At the end of the survey, tutors were asked to summarize how they distinguish between simple and not simple mistakes and how the type of mistakes students had made on a recent test would influence how they help a student prepare for future tests.

The survey was distributed to all current tutors employed in the campus Math Learning Centers at two institutions in the Spring 2023 term. Eleven tutors at Institution A and 9 at Institution B submitted the survey. Of these 20 submissions, 11 submissions were complete. We omitted responses to the sixth error because we had provided an incorrect description of that error in the survey. Therefore, our analysis focuses on the categorizations of 5 errors by 11 tutors. We have not yet analyzed responses to the final two questions of the survey, where tutors were summarizing their definitions and discussing how they would assist students who had made particular types of mistakes.

**Analysis**

We began by reading through each tutor's response to each student error. We individually used open coding to begin developing a codebook. We then discussed each response and the codes we had individually developed and came to consensus on which codes to apply for each tutor response. We decided that multiple codes could be applied to a single tutor response. We added these to the first version of our Codebook.

We then individually used this codebook to do a second round of coding and used the constant comparative method to refine and develop new codes (Glaser, 1965). We noted difficulties in applying these codes and met a second time to resolve the codes. In the third round of code development and modification, we began condensing codes and creating subcodes. For example, we had applied the code "Lacking Knowledge" to many tutor responses. We grouped the responses based on similarities and replaced this code with four more specific codes that identified what type of knowledge the student was lacking (e.g., conceptual, specific formulae).

## Results

We begin with a discussion of responses tutors gave to support their classification of a student error being simple. Later we will discuss responses given to support not simple classifications, and finally those that could be used for either simple or not simple. The diagram below (Figure 2) shows each code and whether it was applied more often to responses that classified the error as simple or not simple.

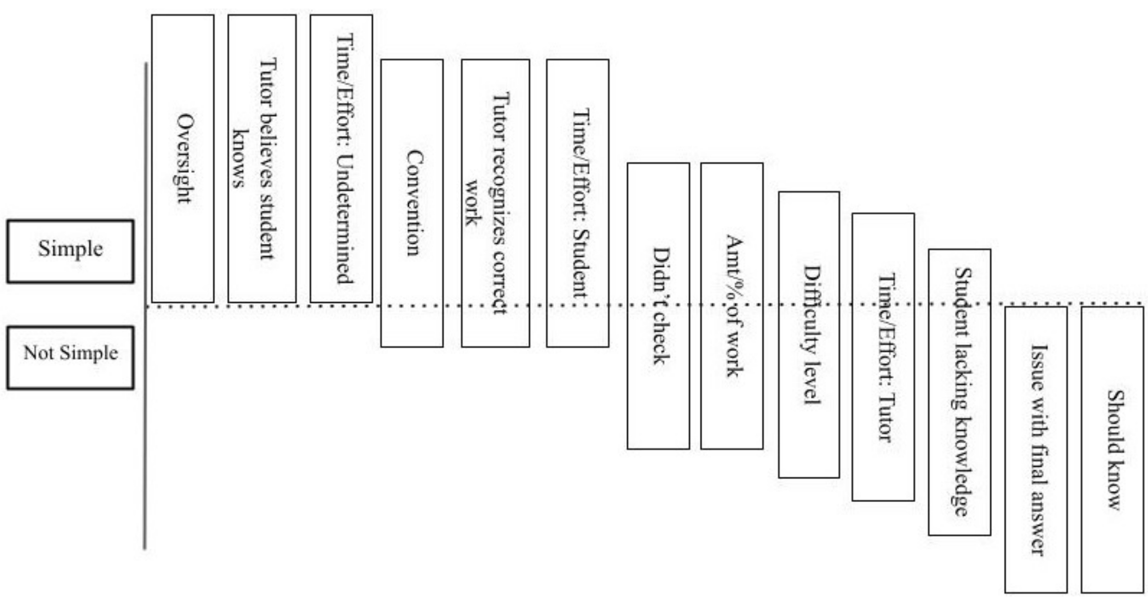


*Figure 2. Codes Describing Reasons Tutors Provide for Error Classification*

**Codes for Simple Mistakes**

There were three codes we developed that were applied only to responses where tutors had classified an error as simple. The first was *oversight.* We applied this code when the tutor claimed the mistake was simple because the student had overlooked something or worked too quickly. The second was *tutor believes student knows*. This code was used when tutors stated or implied that despite the error, the student understands the concept or knows the procedure. We also created a code, *time/effort: undetermined* that we used when tutors indicated the mistake was simple because not much time or effort would be required to prevent the error. When this code was applied, it was unclear whether the tutor was referencing time and effort on the part of the tutor, student, or both.

Other codes were used almost exclusively for mistakes tutors classified as simple. Tutors sometimes claimed that the student had not made an error, but rather had failed to adhere to classroom expectations or convention, such as simplifying an expression to a certain point. To these we applied the code *convention*. There were also tutor explanations that focused on the work completed in the problem. We applied the code *tutor recognizes correct work* when tutors explained the student had successfully completed the difficult part of the problem or noted students had demonstrated the knowledge needed to avoid that error somewhere else in the problem. Finally, sometimes tutors focused not on the work done in the problem but what could have been done prior to the exam to prevent the error. Multiple responses indicated an error was simple because the *time/effort* required by the *student* to avoid the error was relatively low.

**Codes for Not Simple Mistakes**

Several reasons were used by tutors exclusively to justify classifying errors as not simple. The first of these was that the student *should know* how to solve this problem correctly. In cases where we applied this code, tutors discussed their expectation that students in the course should not have made this error or this was not an understandable mistake. A second code, *issue with final answer*, was applied when a tutor claimed an error was not simple because the error caused the student's answer to be significantly different from the correct response.

Many tutor responses attributed errors to the *student lacking knowledge*. We identified four types of knowledge they referenced; two of these were applied only in situations where the tutors were claiming an error was not simple: *lacking conceptual knowledge* and *lacking knowledge of specific content or rules*. When the code lacking conceptual knowledge was applied, tutors described a lack of understanding, reasoning, or fundamental knowledge. The code, lacking knowledge of specific content or rules, was used when tutors had indicated the mistake was not simple because the student was unfamiliar with or had forgotten a rule or formula.

**Codes for Both Simple and Not Simple Mistakes**

We developed two other codes to use when tutors referenced a lack of knowledge. In the first, *structure and strategy*, tutors attributed errors to an inability to recognize the problem structure or identify an appropriate strategy for solving the problem. Closely related was the code *goal state issues*. This was applied when tutors indicated the student could not identify what the problem was asking or what type of answer they were expected to provide. Both these codes were applied at times to justify classifying a mistake as simple, and others to justify the choice of not simple. Other codes related not to the knowledge tutors believed students had but to student behavior while solving the problem. These include *didn't check* and *amount/percent of work*. Tutor responses with the code didn't check indicated the error would have been avoided if the student had checked their work. Responses coded as amount/percent of work focused on the amount of work shown or the percent of the problem that was successfully completed. Again, these codes were sometimes used to support classifying a mistake as simple and at other times to support classifying a mistake as not simple.

Two additional types of responses were used to justify both simple and not simple classifications. In the first, tutors would describe the amount or type of assistance they would need to provide the student for them to avoid the error. This was coded *time/effort: tutor*. Not simple errors were ones that would require significant intervention by the tutor. In the second type of response, the tutor focused not on the student's answer but on the *difficulty level* of the problem itself. It was common for tutors to associate problems they perceived as easy with simple mistakes, though this wasn't always the case.

### Codes About the Tutors' Reflective Process

The codes discussed above were reasons tutors gave to explain their classification of an error as simple or not simple. However, three additional codes were created that describe the tutors' reflective process as they determined whether students' mistakes were simple or not. While these codes do not directly address the research question, we wanted to capture recurring themes in the tutor approach to analyzing student work that may be useful in future analysis. The first of these codes was *more information*. This was applied to responses where tutors expressed uncertainty about their decision to classify the error as simple or not simple. The second was *tutor notes multiple reasons* and was applied if the tutor gave multiple possible explanations for the student's mistake. Finally, the code *tutor references past experience* was applied if tutors made mention of a mistake being common or seeing this issue with students in the past.

## Discussion

In this study we asked undergraduate math tutors to classify student errors on a College Algebra test as simple or not simple and provide justifications for their decisions. We saw that at times tutors provided similar distinctions between simple and not simple errors that we had previously made ourselves (Ryals et al., 2020). For example, tutors typically said that mistakes made in-the-moment due to *oversight* or errors that could be attributed to not conforming to *convention*, such as simplification, were simple. Tutors also tended to classify a mistake as simple if the student had demonstrated the necessary knowledge or skills elsewhere in the problem. In contrast, the tutors were much more likely to consider a mistake not simple if they attributed the mistake to a *lack of knowledge*.

Some of the tutors' distinctions between simple and not simple stood in contrast to our own. For example, tutors sometimes attended to features of the problem itself, rather than the student work. In other cases, they focused on what would need to be done, either by the student or tutor, to prevent the error. If the time and/or effort required was small enough, tutors considered the mistake simple. We also saw cases of the tutor deciding whether a mistake was simple based on how much the error impacted the student's final answer.

We have more data to analyze from the completed survey and would like to distribute the survey more widely. We pose the following questions for discussion:

1. How might we modify the format or administration of the survey in future iterations to collect data we can use to further refine our current codebook?
2. How does the experience level of the tutor impact/influence their categorizations about the simple/not simple distinction? Can we answer this using our current data?
3. How can we use these results to provide training for new and experienced tutors?

## References

Benecke, K., & Kaiser, G. (2023). Teachers' approaches to handling student errors in mathematics classes. *Asian Journal for Mathematics Education*, *2*(2), 161-182.

Borasi, R. (1987). Exploring mathematics through the analysis of errors. *For the learning of Mathematics*, *7*(3), 2-8.

Bressoud, D. M., Mesa, V., & Rasmussen, C. L. (Eds.). (2015). Insights and recommendations from the MAA National Study of College Calculus. MAA Press.

Dweck, C. S. (2006). *Mindset: The new psychology of success*. Random house.

Glaser, B. G. (1965). The constant comparative method of qualitative analysis. *Social Problems*, *12*(4), 436-445.

Heinze, A., Ufer, S., Rach, S., & Reiss, K. (2012). The student perspective on dealing with errors in mathematics class. *Learning from errors at school and at work*, 65-79.

Johns, C. (2022). Research-based training for undergraduate mathematics tutors. *International Journal of Mathematical Education in Science and Technology*, 1-26.

Kapur, M. (2010). Productive failure in mathematical problem solving. *Instructional science*, *38*, 523-550.

Kapur, M. (2014). Productive failure in learning math. *Cognitive science*, *38*(5), 1008-1022.

Kyaruzi, F., Strijbos, J. W., & Ufer, S. (2020, September). Impact of a short-term professional development teacher training on students' perceptions and use of errors in mathematics learning. In *Frontiers in Education* (Vol. 5, p. 559122). Frontiers Media SA.

Lannin, J. K., Barker, D. D., & Townsend, B. E. (2007). How students view the general nature of their errors. *Educational Studies in Mathematics*, *66*, 43-59.

Molina, M., Rodríguez-Domingo, S., Cañadas, M. C., & Castro, E. (2017). Secondary school students' errors in the translation of algebraic statements. *International Journal of Science and Mathematics Education*, *15*, 1137-1156.

Rach, S., Ufer, S., & Heinze, A. (2013). Learning from Errors: Effects of Teachers Training on Students' Attitudes towards and Their Individual Use of Errors. *pna*, *8*(1), 21-30.

Ryals, M., Hill-Lindsay, S., Burks, L., & Pilgrim, M. (2020) *Metacognition in College Algebra: An Analysis of "Simple" Mistakes*. Proceedings of the 23rd Conference on Research in Undergraduate Mathematics Education. Boston, MA.

Ryals, M., Hill-Lindsay, S., Pilgrim, M, & Burks, L. (2022) *College Algebra Students' Definitions of 'Simple Mistakes' Through A Causal Attribution Lens.* Proceedings of the 24th Conference on Research in Undergraduate Mathematics Education. Boston, MA.

Schoenfeld, A. H. (2022). Why are learning and teaching mathematics so difficult?. In *Handbook of cognitive mathematics* (pp. 1-35). Cham: Springer International Publishing.

Seymour, E., Hunter, A. B., & Harper, R. P. (2019). Talking about leaving revisited. *Talking About Leaving Revisited: Persistence, Relocation, and Loss in Undergraduate STEM Education*.

Tulis, M., Steuer, G., & Dresel, M. (2015). Learning from errors: Process and contextual conditions. *Funktionen von Fehlern im Kontext individueller und gesellschaftlicher Entwicklung*, 53.

VanLehn, K., Siler, S., Murray, C., Yamauchi, T., & Baggett, W. B. (2003). Why do only some events cause learning during human tutoring?. *Cognition and Instruction*, *21*(3), 209-249.